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Exercise 11 of Theoretische Physik II: Elektrodynamik

dielectrics, special relativity

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Problem 1 (5 points): *force on a dielectric*

A perfectly fitting dielectric is inserted a distance $h < a$ into a parallel-plate capacitor, consisting of two parallel, quadratic conducting plates with edge length a and plate separation d. The plates carry charges $Q > 0$ and $-Q < 0$, respectively.

1. Calculate the electric field $\mathbf{E}(\mathbf{x})$, the dielectric displacement $\mathbf{D}(\mathbf{x})$, and the potential $\phi(\mathbf{x})$ in the whole gap between the plates, neglecting boundary effects. Consider a homogeneous isotropic medium with dielectric constant ϵ. (2,5 points)

2. Calculate the capacitance C of the configuration, the electrostatic field energy W_{el}, and hence the force $\mathbf{K}(\mathbf{x})$ acting on the dielectric. (2,5 points)

Problem 2 (4 points): *space(and)time*

1. Explain the term *inertial frame* and give examples of inertial frames. (2 points)

2. Compare the terms *space* and *time* in Newtonian mechanics with those in special relativity. (2 points)

Problem 3 (5 points): *time dilation*

1. Consider a configuration consisting of two mirrors parallel to the x-axis, which are moved with a constant velocity v compared to an external observer in the x-direction. Calculate the time which the light takes to make a round trip between these two mirrors, in the rest frame of the moving mirrors ($\{x'^i\}$) as well as in the rest frame of the external observer ($\{x^i\}$), using only geometric considerations. Show that the relationship between the measured time intervals is given by (2 points)

$$\Delta t' = \sqrt{1-(v/c)^2}\Delta t.$$

2. The energetic part of the secondary cosmic radiation which reaches the surface of the Earth from the higher layers of the atmosphere consists mainly of fast-moving muons. Muons at rest have a mean lifetime of approx. $\tau_0 = 2.2 \times 10^{-6}s$. Now consider a stream of muons which originates from a height $h = 10km$ above the surface of the Earth and approaches the Earth's surface perpendicularly with a constant velocity $v = 0,98c$. The intensity of the stream at height h is given by I_h and at the Earth's surface by I_0. Calculate the ratio I_0/I_h under the assumption that the number of muons is only reduced by spontaneous decay. Which result would a nonrelativistic calculation yield? (3 points)

Problem 4 (6 points): *Lorentz contraction*

1. Following the problem above, consider the path which a light ray traverses which runs between two mirrors located perpendicularly to the x-axis, moving with a constant velocity v compared to an external observer in the x-direction. Derive the relationship between the path length in the rest frame of the moving mirrors ($\{x'^i\}$) and that in the rest frame of the external observer ($\{x^i\}$), and show that it is given by (2 points)

$$\Delta x' = \frac{1}{\sqrt{1-(v/c)^2}}\Delta x.$$

2. With which velocity must a ladder of rest length L_0 be carried into a garage of rest length $G_0 < L_0$, if one wants to close the garage door before one notices the ladder hit the back of the garage? Discuss the problem in the rest frame of the garage as well as that of the ladder. In both cases, sketch the world lines of the ladder ends and those of the garage door and back in a spacetime diagram. (4 points)