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Exercise 12 of Theoretische Physik II: Elektrodynamik
velocity addition theorem, acceleration

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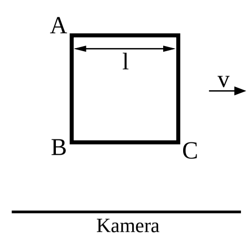
Problem 1 (6 points): *general velocity addition theorem*
Consider a point mass which is moving with velocity $\mathbf{v}'$ in the frame I'. With which velocity $\mathbf{u}$ is the point mass moving, seen in the frame I, wenn I' is moving with a velocity $\mathbf{v}$ with respect to I? Discuss the special cases $\mathbf{v} \parallel \mathbf{v}'$ and $\mathbf{v} \perp \mathbf{v}'$. Show that

$$\mathbf{u}^2 = 1 - \frac{(1-\mathbf{v}^2)(1-\mathbf{v}'^2)}{(1+\mathbf{v}\mathbf{v}')^2},$$

and discuss the limit $|\mathbf{v}'| \to 1$.

Problem 2 (5 points): *invisibility of the Lorentz contraction*
1. Consider a cube with side length l which is passing a camera at a very large distance with the velocity $\mathbf{v}$. At the moment of passage, a snapshot is made. Show that the photographic image of the cube does not look contracted but rotated by an angle $\varphi = \arcsin(v)$. (2 points)



2. Give a general proof of the invisibility of the Lorentz contraction. To this end, consider the world lines

$$x_A = k\lambda_A + d_A$$
$$x_B = k\lambda_B + d_B$$

of two photons A and B, where k is the wave vector of the photons and $\lambda_{A/B}$ parametrizes their world lines. Demand that both photons hit the camera perpendicularly and at the same time. From this, derive the Lorentz-invariant relation $(x_A - x_B)^2 = (d_A - d_B)^2$, which is valid for two arbitrary points on the world lines, and interpret the result.

(3 points)

Problem 3 (9 points): *acceleration*
1. Prove that the modulus square of the 4-acceleration a^i is given by

$$a^i a_i = \gamma^6 \left[(\mathbf{v} \times \mathbf{a})^2 - \mathbf{a}^2 \right],$$

where $\mathbf{v}$ is the 3-velocity and $\mathbf{a}$ the 3-dimensional acceleration. Is a^i spacelike, timelike or null? (3 points)

2. A particle with the 4-velocity u^i and the 4-acceleration a^i is moving according to the equation

$$\frac{\mathrm{d}a^i}{\mathrm{d}\tau} = \alpha^2 u^i,$$

where α^2 is a scalar. Prove that this leads to

$$a^i a_i = -\alpha^2, \alpha^2 = const.,$$

i.e., that the proper acceleration α is a constant (hyperbolic motion). (3 points)

3. Determine explicitly the curve of motion of a particle with constant proper acceleration for the special case of planar motion, i.e. $\mathbf{v} \parallel \mathbf{a}$, and draw a space-time diagram. (3 points)