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Exercise 4 of Theoretische Physik II: Elektrodynamik
electrostatics,method of images

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Problem 1 (2 points): *surface integrals, line integrals*

1. Calculate the flux of the vector field $\mathbf{A}(\mathbf{x})$ through the surface of a sphere of radius R: (1 point)

$$\mathbf{A}(\mathbf{x}) = \frac{(x, y, z)^T}{\sqrt{\alpha + x^2 + y^2 + z^2}}$$

2. Calculate the following line integral along a unit circle γ in the x-y plane (choose an appropriate parametrization): (1 point)

$$\int_\gamma [(x^2 + y)dx + (y^2 + z)dy + (z^2 + x)dz]$$

Problem 2 (13 points): *electrostatics*

1. Calculate the electrostatic potentials $\phi(\mathbf{x})$ for the following charge distributions $\rho(\mathbf{x})$ with the help of the Laplace equation. Make an ansatz which takes into account the symmetry of the problem. Consider the boundary and continuity conditions to obtain the potentials. (9 points)

(i) Charged (infinitely extended) sheet with constant area charge density σ in the y-z plane:

$$\rho(\mathbf{x}) = \sigma\delta(x)$$

(ii) Charged spherical shell of total charge Q, radius R, and surface $F = 4\pi R^2$:

$$\rho(\mathbf{x}) = \frac{Q}{F}\delta(R - r)$$

(ii) Charged solid sphere of total charge Q, radius R, and volume $V = 4\pi R^3/3$:

$$\rho(\mathbf{x}) = \frac{Q}{V}\theta(R - r)$$

(iv) Charged (infinitely long) infinitely thin, straight wire with constant line charge density κ:

$$\rho(\mathbf{x}) = \frac{\kappa}{\pi}\frac{\delta(\varrho)}{\varrho}$$

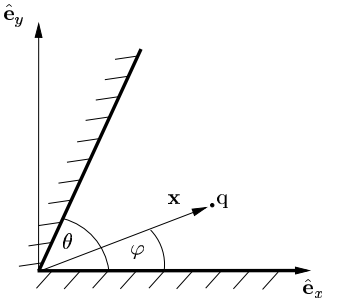
Here, (ϱ, φ, z) are cylindrical coordinates.
(Note that $\int_\alpha^\beta dx\, \delta(x - a)f(x) = \frac{1}{2}f(a)$ if $a = \alpha$ or $a = \beta$.)

2. Calculate the electric fields for the situations given above. (2 points)

3. Make sketches of the lines of constant potential and the electric field lines for the situations above in a plane which takes into account the symmetry of the problem. (2 points)

Problem 3 (5 points): *method of images*

Consider two grounded metal plates which enclose an angle of $0 < \theta < 2\pi$, the intersection of which is the z-axis. A charge q is located between the two plates at $\mathbf{x} = (r\cos\varphi, r\sin\varphi, 0)$. Let $0 < \varphi < \theta$.



1. For which of the following cases can the potential $\phi(\mathbf{x})$ be calculated with the help of the method of images? Explain your answers and, in the cases where it is possible, give the position and magnitude of the charges. (2 points)

(i) $\theta = \frac{\pi}{3}, \varphi = \frac{\pi}{4}$ (ii) $\theta = \frac{2}{3}\pi, \varphi = \frac{\pi}{3}$ (iii) $\theta = \frac{3}{4}\pi, \varphi = \frac{\pi}{3}$

2. Construct the potential $\phi(\mathbf{x})$ with the help of the method of images for the case $r = \sqrt{2}$, $\theta = \frac{\pi}{2}, \varphi = \frac{\pi}{4}$. Show that $\phi(\mathbf{x})$ vanishes on the metal plates. (2 points)

3. What is the surface charge density $\sigma(\mathbf{x})$ on the metal plates in the case above? (1 point)