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05/18/2004

Exercise 5 of Theoretische Physik II: Elektrodynamik
multipole expansion, cylindrical capacitor

Submission date: 05/26/2004

Problem 1 (3 points): *gravitational forces*

This problem aims at showing that gravitational forces, which dictate physics on large scales and which are thus the only significant forces in cosmological problems (with the exception of the Big Bang and black holes), can be neglected for almost all problems one is concerned with in electrodynamics. To this end, consider the hydrogen atom in a purely classical way. Calculate the ratio of gravitational to electrostatic forces between a resting electron and a (resting) proton. Now consider two bodies with charges $+e$ and $-e$. How large must their masses be so that gravitational and electrostatic forces are of equal strength (for simplicity, consider two equal masses)? Compare this with the electron or proton mass.

Problem 2 (3 points): *multipole expansion*

Show the following properties:

(i) The odd (even) multipole moments vanish for a symmetric (antisymmetric) charge distribution with respect to the symmetry point. (1,5 points)

(ii) If the monopole and dipole moments of a charge distribution vanish, then the quadrupole moment is independent of the choice of the origin (this fact can be generalized to the multipole moment of nth order!). (1,5 points)

(Hint: choose the multipoles in appropriate coordinates!)

Problem 3 (10 points): *charged circular ring*

Consider a homogeneously charged circular ring in the x-y plane with its center in the coordinate origin, with total charge Q and Radius a.

1. What is the charge distribution $\rho(\mathbf{x})$ for this situation? Calculate both the electrostatic potential $\phi(\mathbf{x})$ and the electric field $\mathbf{E}(\mathbf{x})$ on the z-axis. (3 points)

2. Calculate the monopole, dipole, and quadrupole moment for this charge distribution. (3 points)

3. Calculate the potential $\phi(\mathbf{x})$ on the z-axis in quadrupolar approximation and compare it with the exact result of part 1 by making a sketch. Assess the approximation in the near and far zone. (2 points)

4. Give a charge distribution composed of four charges which lead to the same result as the quadrupolar approximation of the potential given in part 3. (2 points)

Problem 4 (4 points): *cylindrical capacitor*

Consider two concentric cylinders with radii R_1 and R_2 and length h. The cylinders carry homogeneously distributed total charges Q and $-Q$.

1. Calculate the electric field $\mathbf{E}(\mathbf{x})$ as well as the electrostatic potential $\phi(\mathbf{x})$ in the regions $\varrho < R_1$, $R_1 < \varrho < R_2$ and $R_2 < \varrho$ (ϱ, φ, z cylindrical coordinates). Make an ansatz for the electric field and use Gauß's law of electrostatics. Construct the potential considering the continuity and boundary conditions of the problem. Neglect boundary effects which are due to the finite length of the cylinders. (3 points)

2. Determine the voltage between the two cylinders and hence the capacity C of the capacitor. (1 point)