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Exercise 7 of Theoretische Physik II: Elektrodynamik
law of induction, Maxwell's equations

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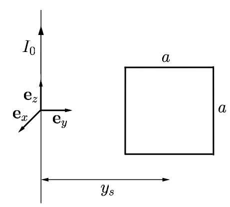
Problem 1 (3 points): *Maxwell's equations*
Maxwell's equations of electrodynamics were introduced in the lecture. You should now get accustomed to them by performing the transformation between the integral and differential formulations of the equations. Discuss the physical meaning of the laws.

Problem 2 (6 points): *conducting loop in a magnetic field*
A rigid quadratic conducting loop with side length a and Ohmic resistance R is situated close to a current-carrying wire with current density $\mathbf{j}(\mathbf{x}) = I_0\delta(x)\delta(y)\mathbf{e}_z$. Let the center of mass be $\mathbf{x}_s = y_s\mathbf{e}_y$ with $y_s > a/2$. The normal vector of the enclosed area points in the direction of the x-axis.



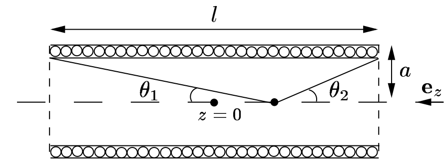
The conducting loop is moved away from the wire with a constant velocity $\mathbf{v}_0 = v_0\mathbf{e}_y$.

(i) Calculate the current which is induced in the conducting loop. Make a sketch of the direction of flow of the current and discuss this using Lenz's law. (3 points)

(ii) What force must be applied to the loop so that it retains a constant velocity $\mathbf{v}_0$? (3 points)

Problem 3 (6 points): *finite solenoid*
1. Calculate the magnetic induction $\mathbf{B}(\mathbf{x})$ on the z-axis of a current-carrying circular loop of radius a and current I which is located in the $x-y$-plane with its center in the origin. (2 points)

2. Using this result, derive $\mathbf{B}(\mathbf{x})$ for a solenoid of length l and loop density $n = \frac{N}{l}$ with its axis along the z-axis. Express the result in terms of the angles θ_1 and θ_2. (2 points)



3. Calculate approximately the magnetic induction in the middle ($z = 0$) and at the end ($z = l/2$) of the solenoid, and consider the limiting case $l \to \infty$ (discussion!). (Cf. exercise 6, Problem 3) (2 points)

Problem 4 (5 points): *curvilinear coordinates*
1. Derive the representation of the δ-distribution in curvilinear coordinates (u, v, w) from its properties in Cartesian coordinates (x, y, z). What does $\delta(\mathbf{x})$ look like in spherical and cylindrical coordinates? (2 points)

2. When transforming between current density and current elements in curvilinear coordinates, extra factors appear. Since the consideration of non-Cartesian coordinates is very important in magnetostatics, these factors will be derived in the following.

To this end, consider a current density which is limited to a path C, which is described by $\mathbf{x} = \mathbf{x}(s)$, where s is the arc length. The new coordinates (u, v, w) are chosen so that u is the arc length s, and v and w are constant along the path. The direction of $\mathbf{j}(\mathbf{x})$ is thus that of $\mathbf{e}_u = \frac{\partial \mathbf{x}}{\partial u}$. Make the ansatz

$$\mathbf{j}(\mathbf{x}) = A(u, v, w)\mathbf{e}_u\delta(v - c_1)\delta(w - c_2)$$

and determine the factor A. Specify your result in spherical and cylindrical coordinates for a current density of a current-carrying circular loop. (3 points)