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Second Exercise Sheet on Relativity and Cosmology I
Summer term 2009

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Exercise 4 (6 points): *Motion in the gravitational field*

The equation of motion for a test particle in the gravitational field is given by

$$\ddot{x}^i + \Gamma^i{}_{kl}\,\dot{x}^k\dot{x}^l = 0, \tag{1}$$

where $\dot{x}^i = dx^i/ds$ and $\Gamma^i{}_{kl} = \frac{1}{2}g^{ij}(\partial_l g_{jk} + \partial_k g_{jl} - \partial_j g_{kl})$. 1. Repeat briefly the derivation of (1) from the variational principle $\delta \int ds = 0$ as presented in the lecture course. Why can the derivation not be used for photons? 2. Derive (1) from the alternative variational principle

$$\delta \int g_{ik}\,\dot{x}^i\dot{x}^k\,d\lambda \equiv \delta \int \mathcal{L}\,d\lambda = 0,$$

where λ is an affine parameter, and $\dot{x}^i = dx^i/d\lambda$ .Show that the derivation holds also for photons and determine $\mathcal{L}$ for the solution of (1).

Exercise 5 (6 points): *Christoffel symbols*

Derive the transformation property of the Christoffel symbols

$$\Gamma_{ikl} = \frac{1}{2}(g_{ik,l} + g_{li,k} - g_{kl,i})$$

under a coordinate transformation $x^{i'}(x^a)$ The result shows that they do not form a tensor.

Exercise 6 (4 points): *Rotating reference frame*

Calculate in the Newtonian approximation the Christoffel symbols for a system which rotates with constant angular velocity ω around the z-axis and formulate the geodesic equation (1) for this case. Identify the centrifugal and the Coriolis force in the resulting equation of motion.

Exercise 7 (4 points): *Freely falling observer*

The equation of motion of a mass point in a (flat) 1+1-dimensional Minkowski space be given by the example $m\ddot{x} - m\,g = 0$. In analogy to the equation of motion (1) we set $\Gamma^1{}_{00} = -g$ and $\Gamma^i{}_{kl} = 0$ otherwise. On physical grounds it is obvious that there should exist a reference frame in which the Christoffel symbols vanish and the equation of motion for a free mass point therefore reads $m\,\ddot{x} = 0$. Find such a coordinate system by "integrating" the Christoffel symbol.