black holes

in string theory

Christian Schell (cschell@smail.uni-koeln.de)

04.11.2010

outline

1. string theory
   - fundamental objects
   - some facts about string theory
2. string theory and black holes
   - black hole entropy calculation in string theory
   - correspondence principle
3. information loss problem
   - violation of ordanary physics?
   - further correspondence

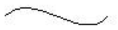
main references

- Horowitz, G.: Black Holes, Entropy, and Information, arXiv:0708.3680v1 [astro-ph]
- Horowitz, G.: Quantum States of Black Holes, arXiv:gr-qc/9704072v3
- Kiefer, C.: Quantum Gravity, 2^{nd} edition, Oxford University Press, Oxford 2007
- Kiritsis, E.: String Theory in a Nutshell, Princton University Press, Princton 2007
- Peet, A.: TASI lectures on black holes in string theory, arXiv:hep-th/0008241v2
- Klemm, D.: Black holes and singularities in string theory, arXiv:hep-th/0410040v2

fundamental objects

fundamental objects are strings:

- open
- closed



usual worldline becomes a *worldsheet*
on this worldsheet we have left- & right-moving components
$\Delta x \to 0$, but cutoff at small, nonzero value $\Rightarrow \Delta p \nrightarrow \infty$
modified uncertainty relation $\Delta x = \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar}$

fundamental objects

parameter α' related to *string tension* T_S by $\alpha' = 1/(2\pi T_S)$
minimum distance given by *string length* $l_S \sim \sqrt{\alpha'}$
for open strings there are two types of boundary conditions

- *von Neumann* free endpoints
- *Dirichlet* fixed endpoints

string entropy

- quantizing strings in flat spacetime leads to an infinite tower of massive states
  for each level of excitation N there are highly degenerated states with $M^2 \sim \frac{1}{\alpha'}(N-1)$

string entropy

level density d_N of highly excited string state
divide string with energy M into two parts, s. th.
$M = M_1 + M_2$
$\Rightarrow d_N(M) = d_N(M_1)d_N(M_2) = d_N(M_1 + M_2)$
this leads to the exponential function for d_N:
$d_N \sim e^{4\pi\sqrt{N}} \approx e^{M/M_0}$ with $M_0 := \frac{1}{4\pi\sqrt{\alpha'}}$
from statistical mechanics one has for the string entropy
S_S: $d_N = e^{S_S}$

$$\Rightarrow S_S \sim M \sim \sqrt{N}$$

string entropy

interpretation [Mitchell, Turok 87]: string as random walk with step size l_S
energy after n steps $E \sim n/l_S$
k possible directions
$\Rightarrow$ total number of configurations k^n
$\Rightarrow$ entropy for large n proportional to energy

[Mitchell, Turok 87] Mitchell, D., Turok, N. 1987: Statistical Mechanics of Cosmic Strings, Phys. Rev. Lett., **58**, 1577-1580

string coupling

- string interactions governed by the *string coupling constant g*
  determined by scalar field ϕ called dilaton ($g = e^{\langle \phi \rangle}$, $\langle \cdot \rangle$ vacuum expectation value)
  Newtons constant in D dimensions: $G_D \sim g^2 l_S^{D-2}$, for four spacetime dimensions $G_4 \sim g^2 l_s^2$
  so G is determined by dynamic field, but often g just treated as a parameter in the theory

curvature condition

- classical spacetime only well defined when curvature less than $1/l_s^2$
  low energy approach breaks down if curvature approaches string scale

SUSY

- string theory implies *supersymmetry* (SUSY): fundamental symmetry between bosons and fermions, which transforms one into the other
  important consequence: bound of total mass M of all states by their charge Q: $M \geq Q$
  bound is called *BPS bound* (Bogomolnyi, Prasad, Sommerfield)

SUSY

BPS state invariant under non-trivial subalgebra of full SUSY algebra
mass fixed in terms of charges (no quantum corrections) and spectrum preserved while going from weak to strong coupling

dimensionality

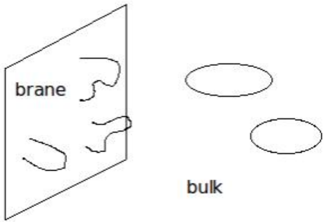
- spacetime has more than the usual four dimensions; 26 in bosonic string theory, 10 in supersymmetric theories (to get rid of negative norm ghost states)
  idea proposed in 1920's by Kaluza and Klein
  explanation, why we usually do not see extra dimensions:
  - *compactification* $\Rightarrow$ *Calabi-Yau mannifolds*
  - extra dimensions are large, but we are confined to live on 3+1 dimensional submanifold, called *brane*, space outside of brane called *bulk*

D-branes

- string theory includes more objects than just one-dimensional strings, but also higer dimensional branes, nonperturbative objects with mass $M \sim \frac{1}{g l_S}$ gravitational field $GM \sim g l_S$ ($\to 0$ for weak coupling, so there is a flat spacetime description)
these branes just can be seen as the surfaces, where open strings can end

D-branes

the dynamic of these branes at weak coupling is described by open strings which the two endpoints are stuck on certain surfaces ($\rightarrow$ brane)



condition at end of string, keeping it on the surface, is given by Dirichlet boundary condition ($\Rightarrow$ *D-brane*)

D-branes

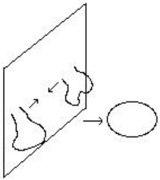
all particles of standard model (quarks, leptons, gauge bosons,...) are believed to come from these open strings and are confined to the branes

more exotic particles like the graviton come from excitation states of closed strings which are free to move in the *bulk*

$\exists$ many types of D-branes, of various dimensions, each carries charge

D-branes

BPS state: D-branes are flat (i.e. form an extremal surface) and have (in the groundstate) no open strings attached
on excited D-branes (open strings attached) two open strings can approach each other and form a closed string
the later one can leave the D-brane and take some of the branes energy with it



fundamental questions

fundamental questions of black hole dynamics:

- origine of black hole entropy?
  usually, thermodynamics is an approximate theory of underlying fundamental statistic description
  there, entropy is a measure (log) of the number of microstates and therefore for information
- black hole evaporation $\Rightarrow$ information loss?
  this would imply a violation of the unitarity of quantum mechanics

extremal black hole

extremal black hole has minimal possible mass that is compatible with given charge and angular momentum ($M \sim Q$)

often supersymmetric (i.e. invariant under several supercharges) as a consequence of BPS bound

then stable and no emission of Hawking radiation

breakthrough

breaktrough 1996 by Strominger and Vafa with charged five dimensional extremal black holes [Strominger, Vafa 96]

astrophysically not very interesting, but theoretically since they satisfy BPS bound $M \geq Q$

$M = Q$ are extremal black holes with zero Hawking temperature (equivalent to strong coupling analogs of BPS states)

[Strominger, Vafa 96] Strominger, A., Vafa, C. 1996: Microscopic Origin of the Bekenstein-Hawking Entropy, arXiv:hep-th/9601029v2

breakthrough

procedure:

- start with extremal (BPS states) black hole in 5 dimensions and $N = 4$ SUSY $\Rightarrow S_{BH}$
- reduce as a gedankenexperiment string coupling g to obtain weakly coupled system of strings and branes with equal charge; number of BPS-states $N_{BPS} = e^{S_S}$

microscopic explanation of black hole entropy! (compares states in flat spacetime without horizon with area of black hole, formed at strong coupling)

5 dimensional black hole

we follow the derivation given in [Kiritsis 07]
SUSY allows us to count string states while sending coupling g_S adiabatically to 0
start with collection of highly coupled strings in 10 dimensions, compactify 5 dimensions, so effective 5 dimensional spacetime $T^5 = T^4 \times S^1$

[Kiritsis 07] Kiritsis, E.: String Theory in a Nutshell, Princton University Press, Princton 2007

5 dimensional black hole

extremal black hole constructed as bound state of three objects of type-IIB string theory; D_1 & D_5 brane along with momentum on common interaction
D_5 brane wrapping $T^5 = T^4 \times S^1$ providing pointlike particle in 5 dimensions $\rightarrow$ charge Q_5, D_1 string wraps $S^1 \rightarrow\ Q_1$, Q_p units of momentum along S^1

5 dimensional black hole

SUSY is preserved by the solution; supercharges (generates space-time SUSY) are reduced during process
BPS condition satisfied; it allows us to write mass at
$M = M_1 + M_2 + M_3$
with horizon (given by S_1 with radius R), 3 radii are associated

$$r_i^2 = \frac{g_S^2 l_S^8}{RV} M_i, \ (V: \text{ volume of torus})$$

5 dimensional black hole

first two masses are due to winding modes

$$M_1 = \frac{Q_1 R}{g_S l_S^2}$$

$$M_2 = \frac{Q_5 R V}{g_S l_S^6}$$

third mass due to Kaluza-Klein excitation of D_5 brane along circular dimension

$$M_3 = \frac{Q_P}{R}$$

5 dimensional black hole

this leads to the radii $r_i = \frac{g_S l_S^4}{\sqrt{RV}} \sqrt{M_i}$

$$r_1 = \frac{\sqrt{g_S} l_S^3 \sqrt{Q_1}}{\sqrt{V}}$$

$$r_2 = \sqrt{g_S} l_S \sqrt{Q_5}$$

$$r_3 = \frac{\sqrt{g_S} l_S^4 \sqrt{Q_p}}{R\sqrt{V}}$$

5 dimensional black hole

5 dimensional area

$$A = 2\pi^2 r_1 r_2 r_3 = 2\pi^2 \frac{g_S^2 l_S^8}{RV} \sqrt{Q_1 Q_5 Q_p}$$

with $G_5 = (2\pi)^2 G_{10}/RV = (2\pi)^5 8\pi^6 g_S^2 l_S^8$ one finds for the entropy

$$S_{BH} = \frac{A}{4G_5} = 2\pi \sqrt{Q_1 Q_5 Q_p}$$

5 dimensional black hole - microscopic

in the low energy limit the dynamics of the $\mathrm{D}_1 - D_5$ bound system is described by a 2 dimensional *superconformal field theory* with *central charge* $c = 6Q_1Q_5$
for large left-moving/right-moving momenta $N_L, \ N_R$ the number of states in unitary conformal field theory is asymptotically determined by the central charge alone

$$\Omega(N_L, N_R) \sim \exp(2\pi\sqrt{c/6}(\sqrt{N_L} + \sqrt{N_R}))$$

entropy

$$S = \ln\Omega = 2\pi(\sqrt{Q_1Q_5N_L} + \sqrt{Q_1Q_5N_R}) + O(\frac{1}{\sqrt{Q_1Q_5}})$$

discoveries

soon after initial breakthrough, agreement between black holes and weakly coupled string and D-branes systems was extended:

- extremal charged black holes with rotation [Horowitz, Roberts 07]
- near extremal black holes with nonvanishing Hawking temperature [see e.g. Kiritsis 07; 12.6]
- radiation rate from black holes and from D-branes agrees [see e.g. Kiritsis 07; 12.8, 12.9]

[Horowitz, Roberts 07] Horowitz, G., Roberts, M. 2007: Counting the Microstates of a Kerr Black Hole, arXiv:0708.1346v1 [hep-th]

neutral black holes

neutral black holes: $S_{BH} \sim A \sim M^2$ and $S_S \sim M$ (cf. above) seems to be a contradiction, but

- consider effective Schwarzschild radius $R_S = 2GM$
- increase $g \Rightarrow R_S$ increases and black hole can be formed
- conversaley start with black hole, decrease g
- we have a highly excited string state with mass $M_S^2 \sim N/l_S^2$ (at $G = 0$)

neutral black holes

- if $R_S \leq l_S$, we have a black hole with $M_{bh}^2 \sim l_S^2/G^2 \sim N/l_S^2$
- $l_S^2/G \sim \sqrt{N} \Rightarrow S_{BH} \sim R_S^2/G \sim l_S^2/G \sim \sqrt{N} \sim M_S \sim S_S$ so the entropies agree at the *correspondence point*

$R_S \sim l_S$ does not imply, that the black hole must be small, because since $S_{BH} \sim \sqrt{N} \Rightarrow R_S \sim G^{1/2} N^{1/4}$ and $N \gg 1$

correspondence principle

the considerations above lead to a general relation between black holes and strings, called the *correspondence principle*:

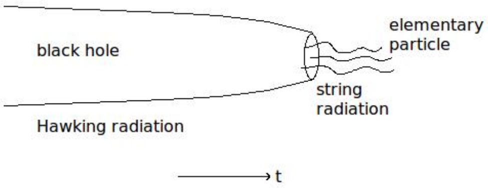
- if curvature at the black hole's horizon becomes bigger than the string scale, the black hole state becomes a string and D-brane state with same charge and angular momentum
- mass changes by at most a factor of order unity during transition

this principle applies to all kinds of black holes including higher dimensional black holes and those which are far from extremality.

string theory and black holes

these leads to a simple picture of the end of a black hole:

- like in Hawking's semiclassical picture it evaporates down to string scale
- turns into highly excited string and radiates until
- it becomes lower excited string, e.g. an elementary particle



information loss problem

in semiclassical picture, black hole evaporation seems to violate thermodynamic principles and therefore be in conflict with unitarity in quantum mechanics

for example for carged, near extremal black hole, the weak coupling limit provides quantum system with equal entropy and radiation - indication, that black holes do not contradict quantum mechanics

this indication becomes much stronger

gravity/gauge correspondence

gravity/gauge correspondence [Maldacena 98]: under certain boundary conditions, string theory (includes gravity) is completly equivalent to a (nongravitational) gauge theory

our intuition about both theories are governed by weak coupling limit

therefore it seems to be a contradiction but the correspondence occurs in the case when one theory is in the weak and the other one in the strong coupling limit

[Maldacena 98] Maldacena, J. 1998: The Large N Limit of Superconformal Field Theories and Supergravity, arXiv:hep-th/9711200v3

solution of information loss problem

consequence: formation and evaporation of black holes in small stadium can be described by ordanary Hamiltonian evolution implicated by gauge theory
violation of quantum mechanics (unitarity) disapears in Hamiltonian evolution

$\Rightarrow$ no information loss!

AdS/CFT correspondence

AdS/CFT correspondence:
conjectured duality between string theory in *Anti de Sitter (AdS) space* and *conformal field theory (CFT)* on boundary of AdS at infinity
CFT manifestly unitary $\Rightarrow$ string theory in AdS information preserving

holographic principle

this correspondence leads to the *holographic principle*, a description of quantum mechanics, where fundamental degrees of freedom live on lower dimensional space

proposed in 1993 by t'Hooft and worked out by Susskind, using two postulates:

- total information in a volume is equivalent to theory that lives only on surface area of region
- boundary of region contains at most single degree of freedom per Planck area

open questions

- precise counting of Schwarzschild black hole's entropy
- how comes information out of the black hole? $\Rightarrow$ seems to violate locallity (reconstruction of string theory from gauge theory $\rightarrow$ physics may not be local on all length scales)
  to this point see [Hawking 05]: black hole described by path integrals $\rightarrow$ information contained in some quantum field, hard to recover
- origin of spacetime? reconstruction from gauge theory? how does black hole know to form itself to have area $A = \frac{4G\hbar}{k_B c^3} S_{BH}$?

[Hawking 05] Hawking, S. 2005: Information Loss in Black Holes, arXiv:hep-th/9711200v3