black holes

semiclassical considerations

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outline

main references:

- Kiefer, C.: Thermodynamics of Black Holes and Hawking Radiation, avaiable under http://www.thp.uni-koeln.de/gravitation/ → Research → Black holes
- Kiefer, C.: Quantum Gravity, 2^{nd} edition, Oxford University Press, Oxford 2007
- Wald, R.: General Relativity, Chicago University Press, Chicago 1984

in the first part, geometrized units are used; $c \equiv 1 \equiv G$, later just $c \equiv 1$

different solutions

- stationary bh $\rightarrow$ Schwarzschild-solutions
- charged bh $\rightarrow$ Reissner-Nordström-solution
- rotating bh $\rightarrow$ Kerr-solution
- charged rotating bh $\rightarrow$ Kerr-Newman-solution

general proberties

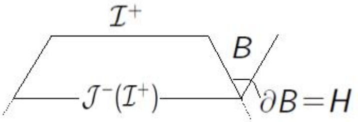
definition: let (M, g_{ab}) be a strongly asymptotically predictible, asymptotically flat spacetime. Let there be a globally hyperbolic region V with $\overline{M \cap \mathcal{J}^-(\mathcal{I}^+)} \subset V$

(M, g_{ab}) contains a *black hole (bh)*, if $M \not\subset \mathcal{J}^-(\mathcal{I}^+)$

black hole region $B := M \backslash \mathcal{J}^-(\mathcal{I}^+)$

event horizon $H := \partial B$

spacetime, that isn't strongly asymptotically predictible, contains *naked singularities*



rotating bh

Consider the *Penrose process* for rotating bhs
event horizon $A = \int_{r=r_+} \sqrt{g_{\Phi\Phi} g_{\Theta\Theta}} d\Theta d\Phi = \int (r_+^2 + a^2) \sin\Theta d\Theta d\Phi = 4\pi(r_+^2 + a^2) =: 16\pi M_{irr}^2$,

$$M_{irr}^2 = \frac{1}{2}(M^2 + \sqrt{M^4 - J^2}) \Leftrightarrow M^2 = M_{irr}^2 + \frac{1}{4}\frac{J^2}{M_{irr}^2} \geq M_{irr}^2$$

as energy can be extracted from the bh, one finds for angular momentum $L < E/\Omega_H$
$\delta M = E,\ \delta J = L \Rightarrow \delta J < \delta M/\Omega_H$

rotating bh

[Christodoulou 70]: $\delta M_{irr}^2 = \frac{Mr_+}{\sqrt{M^2 - a^2}}(\delta M - \Omega_H \delta J) > 0$

with $\Omega_H = a/(r_+^2 + a^2) = a/2Mr_+$ angular velocity of bh

$$\Rightarrow \delta A \sim \delta M_{irr}^2 \geq 0$$

area of the horizon can't decrease

[Christodoulou 70] Christodoulou, D. 1970: Reversible and Irreversible Transformation in Black-Hole Physics, Phys. Rev. Lett., **25**, 1596-1597

general case

[Hawking 71] (cited from [Wald 84], S. 312):
(M, g_{ab}) strongly asymptotically predictible spacetime satisfying $R_{ab}k^a k^b \geq 0 \; \forall$ null k^a. $\Sigma_1, \; \Sigma_2$ spacelike Cauchy surfaces for globally hyperbolic region $\tilde{V}$ with $\Sigma_2 \subset \mathcal{I}^+(\Sigma_1)$,
$H_1 = H \cap \Sigma_1, \;\; H_2 = H \cap \Sigma_2$ with event horizon H
Then area of H_2 is greater than or equal to area of H_1
so, no process decreases future event horizon of bh
$A \geq A_1 + A_2$
future horizon $\Rightarrow$ time asymmetry
2^{nd} law of bh dynamics $dA \geq 0 \; (d_{\mathrm{rev}} A = 0)$

[Hawking 71] Hawking, S. 1971: Gravitational Radiation from Colliding Black Holes, Phys. Rev. Lett., **26**, 1344-1346

0^{th} law of bh dynamics

back to rotating bh
introduce future directed Killing field χ^a, $\chi^a \chi_a = 0$ on horizon
so gradient is normal to horizon, parallel to χ^a

$$\nabla^a(\chi^b \chi_b) = -2\kappa\chi^a$$

κ *surface gravity*
for Kerr bh: $\kappa = \frac{\sqrt{M^2 - a^2}}{2Mr_+}$
Schwarzschild limit: $\kappa \overset{a \to 0}{\to} \frac{1}{2r_+} = \frac{1}{4M}$

0^{th} law of bh dynamics

as shown in [Wald 84, p.331ff], taking the Lie derivative w.r.t. Killing field χ^a gives

$$\mathcal{L}_\chi \kappa = 0$$

i.e. $\kappa =$ const on orbits of χ^a
so $\kappa =$ const on horizon
0^{th} law of bh dynamics

1^{st} law of bh dynamics

[Bardeen, Carter, Hawking 73]:

$$dM = \frac{\kappa}{8\pi} dA + \Omega_H dJ$$

1^{st} law of bh dynamics relationship

- $E \leftrightarrow M$
- $\kappa \leftrightarrow T$
- $A \leftrightarrow S$

[Bardeen, Carter, Hawking 73] Bardeen, J.M., Carter, B., Hawking, S. 1973: The Four Laws of Black Hole Mechanics, Commun. Math. Phys., **31**, 161-170

3^{rd} law of bh dynamics

surface gravity for charged rotating bh ([Wald 84, p 331])

$$\kappa = \frac{\sqrt{M^2 - a^2 - q^2}}{2M(M + \sqrt{M^2 - a^2 - q^2}) - q^2}$$

κ vanishes for *extreme* bh, $M^2 = a^2 + q^2$
[Wald 74]: the closer one gets to extreme bh, the harder it is, to go a step further
impossible to achive $\kappa = 0$ in a finite number of steps
3^{rd} law of bh dynamics

[Wald 74] Wald, R. 1973: Gedanken Experiments to Destroy a Black Hole, Ann. Phys., **82**, 548-556

summary

	thermodynamics	black hole
0^{th}	thermal equilibrium: $T = \text{const}$ throughout body	stationary bh: $\kappa = \text{const}$ over horizon
1^{st}	$dE = TdS - pdV + \mu dN$	$dM = \frac{\kappa}{8\pi} dA + \Omega_H dJ + \Phi dq$
2^{nd}	$dS \geq 0$	$dA \geq 0$
3^{rd}	$T = 0$ impossible in physical process	$\kappa = 0$ impossible in physical process

bh entropy & temperature

$$dE = dM = T_{bh} dS_{bh} = \frac{\kappa}{8\pi G} dA$$

$$\Rightarrow \frac{dS_{bh}}{dA} = \frac{\kappa}{8\pi G} \frac{1}{T_{bh}} \Rightarrow S_{bh} = \frac{\kappa}{8\pi G} \frac{A}{T_{bh}}$$

$$T_{bh} = \frac{\kappa}{G\varsigma}, \;\; S_{bh} = \frac{\varsigma A}{8\pi}$$

$$[S_{bh}] = [k_B] \Rightarrow [\frac{k_B}{\varsigma}] = [A] = \text{lenght}^2$$

bh entropy & temperature

fundamental lenght: $l_{Pl} \stackrel{c\equiv 1}{=} \sqrt{G\hbar}$ $(\approx 10^{-33}\text{cm})$

$$\varsigma \sim \frac{k_B}{G\hbar}$$

$$T_{bh} \sim \frac{\hbar\kappa}{k_B}, \;\; S_{bh} \sim \frac{k_B A}{G\hbar}$$

factors of proportionality ⇒ QFT

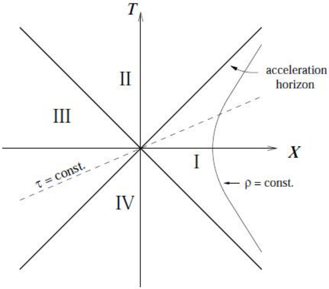
to determin the proportonality factors, one need to consider quantum fields in the neighborhood of the bh horizon. The space is highly curved ⇒ QFT in curved space time [Hawking 75]
no uniquely defined vacuum ⇒ particle creation/annihilation

[Hawking 75] Hawking, S. 1975: Particle Creation by Black Hole, Comm. Math. Phys., **43**, 199-220

accelerated observer

first, uniformely accelerated observer in (1+1)-Minkowski space time in Rindler coordinates $(\tau,\ \rho)$

$$\begin{pmatrix} T \\ X \end{pmatrix} = \begin{pmatrix} \rho \sinh(a\tau) \\ \rho \cosh(a\tau) \end{pmatrix}$$



Unruh effect

$$ds^2 = dT^2 - dX^2 = a^2\rho^2 d\tau^2 - d\rho^2$$

vacuum *global* state correlating regions I and III, whereas observer restricted to I
vacuum for massless scalar field in Minkowski space time

$$|0\rangle_M = \prod_\omega \sqrt{1 - e^{-2\pi\omega/a}} \sum_n e^{-n\pi\omega/a} |n_\omega^{\mathrm{I}}\rangle \otimes |n_\omega^{\mathrm{III}}\rangle$$

[Unruh 76] Unruh, W. 1976: Notes on Black-hole evaporation, Phys. Rev. Lett., **14**, 870-892

derivation in e.g. [Crispino, Higuchi, Matsas 08] Crispino, L., Higuchi, A., Matsas, G, 2008: The Unruh effect and its applications, avaiable at arXiv:0710.5373v1 [gr-qc]

Unruh effect

oberserver in region I, trace out all degrees of freedom in region III in density matrix

$$\rho_{\mathrm{I}} = \mathrm{tr}_{\mathrm{III}} |0\rangle_M \langle 0|_M = \prod_{\omega} (1 - e^{-2\pi\omega/a}) \sum_n e^{-n\pi\omega/a} |n_\omega^{\mathrm{I}}\rangle \langle n_\omega^{\mathrm{I}}|$$

ρ_{I} corresponding to thermal canonical ensemble with Davis-Unruh temperature

$$T_{DU} = \frac{\hbar a}{2\pi k_B}$$

uniformely accelerated observer sees *thermal* distribution of particles

Hawking temperature

equivalence principle $\Rightarrow$ bh radiates with T_{DU} where a is replaced by κ

$$T_H = \frac{\hbar\kappa}{2\pi k_B} \text{ ([Hawking 75])}$$

[Hawking 75] Hawking, S. 1975: Particle Creation by Black Hole, Comm. Math. Phys., **43**, 199-220

special case: Schwarzschild

$$\kappa = \frac{1}{4GM} \Rightarrow$$

$$T_H = \frac{\hbar}{8\pi G k_B M} \approx 10^{-6} \frac{M_\odot}{M} K$$

lifetime

assumption: decrease in mass corresponds to energy radiated to infinity

Stefan-Boltzmann:

$$\frac{dM}{dt} \sim -AT_H^4 \sim -M^2 M^{-4} = -M^{-2}$$

integration $t(M) \sim (M_0^3 - M^3) \overset{M \ll M_0}{\approx} M_0^3$

$$\text{lifetime } \tau_{bh} \sim \left(\frac{M_0}{m_{Pl}}\right)^3 t_{Pl} \approx 10^{65} \left(\frac{M_0}{m_\odot}\right)^3 \text{years}$$

Bekenstein-Hawking entropy

$$S_{bh} = \frac{\varsigma A}{8\pi}$$

Bekenstein-Hawking entropy

$$S_{BH} = \frac{k_B A}{4G\hbar}(\cdot c^3) = S_{bh}$$

entropy is measure for lack of information about global quantum state

Schwarzschild case

$A = 4\pi R_S^2$

$$\Rightarrow S_{BH} = \frac{k_B \pi R_S^2}{G\hbar} \approx 1 \cdot 10^{77} k_B \left(\frac{M}{M_\odot}\right)^2$$

comparisson to statistical mechanics ($S = k_B \ln(\#(\text{states}))$):
$\Rightarrow S_\odot \sim 10^{57} k_B$
so entropy of resulting bh is much larger than the one of the colliding star

statistical explanation of bh entropy?

microscopic description of entropy like

$$S_{bh} \stackrel{?}{=} -k_B \mathrm{tr}(\rho \ln \rho)$$

with density matrix ρ given from statistical mechanics?
are states hidden behind the horizon?
what are/where are microscopic degrees of freedom?
what happens with entropy after bh evaporation (no hair theorem)?

information loss problem

if bh radiation purely thermal
$\Rightarrow$ any initial (in particular pure) state evolves into mixed state as final stage of bh evolution
then it would be a contradiction to unitary evolution in quantum mechanics
in unitaryly evolutioning systems, there is no increase of entropy
if bh mechanics violates this principle, it means, that information gets destroyed
$\rightarrow$ *information loss problem*

loopholes

- information indeed lost
  $\Rightarrow$ replace quantum mechanical Liouville equation by
  $\rho \to \$\rho \neq S\rho S^\dagger$
- evolution completely unitary; semiclassical approximations doesn't handle bh radiation
- bh doesn't evaporate completely; some core is left (e.g. of order of the Planck units) where all the information is hidden