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Second exercise sheet on Relativity and Cosmology I

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Exercise 3 (9 points): *Inertial frames*

A rocket with a rest length L_0 moves with constant velocity radially away from Earth. From Earth a light pulse is emitted, which is then reflected by mirrors at the front as well as at the rear of the rocket. The first signal is received after the time t_A, the second after the time t_B.

3.1 Calculate the velocity at which the rocket moves in terms of L_0, t_A and t_B.

3.2 Determine at which distance from Earth the rocket is located when the first signal reaches Earth.

Exercise 4 (9 points): *Addition of velocities*

Consider a mass point moving with velocity $\vec{w}'$ with respect to the inertial system $\mathcal{I}'$.

4.1 What is its velocity $\vec{u}$ with respect to an inertial system $\mathcal{I}$ if $\mathcal{I}'$ moves with velocity $\vec{v}$ against $\mathcal{I}$? Show that the result can be written as (in units where $c = 1$)

$$\vec{u} = \frac{\vec{v} + \vec{w}'_{\parallel} + \frac{\vec{w}'_{\perp}}{\gamma(v)}}{1 + \vec{v} \cdot \vec{w}'} ,$$

where $\vec{w}'_{\parallel}$ and $\vec{w}'_{\perp}$ denote the parallel and orthogonal components of $\vec{w}'$ with respect to $\vec{v}$, respectively. Discuss the special cases $\vec{v} \parallel \vec{w}'$ and $\vec{v} \perp \vec{w}'$.

4.2 Show that

$$\vec{u}^2 = 1 - \frac{\left(1 - \vec{w}'^2\right)\left(1 - \vec{v}^2\right)}{\left(1 + \vec{v} \cdot \vec{w}'\right)^2} \leqslant 1 .$$

When does the equality hold? Discuss the limiting case $\left|\vec{w}'\right| \to 1^-$.

Exercise 5 (6 points): *Aberration*

Consider an inertial system $\mathcal{I}'$ that moves with velocity $\vec{v}$ against an inertial system $\mathcal{I}$. Consider a ray of light which arrives in $\mathcal{I}$ at an angle θ with respect to $\vec{v}$ (in units where $c = 1$).
Under which angle θ' does this light ray arrive in $\mathcal{I}'$? Show that this relation can be written in the form

$$\tan\frac{\theta}{2} = \sqrt{\frac{1+v}{1-v}}\,\tan\frac{\theta'}{2} .$$

Hint: Draw a picture of the angle. Use the law for the addition of velocities from exercise **4**.