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www.thp.uni-koeln.de/gravitation/courses/rcii19.html
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2^{nd} exercise sheet on Relativity and Cosmology II

Summer term 2019

Release: Mon, Apr. 8th **Submit**: Mon, Apr. 15th in lecture **Discuss**: Thu, Apr. 18th

Exercise 42 (14 points): *Redshift in the Schwarzschild spacetime*

Consider a stationary* observer $\mathcal{A}$ at $r = R$, $R \geqslant 2GM$ in the Schwarzschild spacetime of mass M and an observer $\mathcal{B}$ at infinity. The timelike Killing vector shall be denoted by $\xi^\mu = (1, 0, 0, 0)$. Furthermore, we define the quantity $V^2 := -\xi_\mu \xi^\mu$. Observer $\mathcal{A}$ emits energy with frequency ω_R (measured in her/his rest frame) which is measured by observer $\mathcal{B}$ as being ω_∞.

42.1 Express the four-velocity u^μ of observer $\mathcal{A}$ in terms of ξ^μ and V and use this to derive the relation between the frequencies ω_R and ω_∞.

42.2 What does observer $\mathcal{B}$ measure when observer $\mathcal{A}$ reaches the Schwarzschild radius $r = 2GM$? What does this mean for the redshift?

Exercise 43 (6 points): *Time dilation in the Schwarzschild spacetime*

Show that the proper time $\mathrm{d}\tau$ on a circular geodesic in the Schwarzschild geometry of mass M obeys the relation:

$$\mathrm{d}\tau = \sqrt{1 - \frac{3GM}{r}}\,\mathrm{d}t \,.$$

Use this to give an estimate for the time dilation of a satellite flying in a low orbit around the Earth.

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*A *stationary* observer is an observer in a stationary spacetime whose 4-velocity u^μ is proportional to the given timelike Killing vector.