

11th exercise sheet on Relativity and Cosmology I

Winter term 2012/13

Deadline for delivery: Thursday, 10th January 2013 during the exercise class.

Exercise 27 (6 credit points): *Summary*

Summarize in your own words the main features of general relativity as discussed in the lecture course so far (max. 3 pages).

Exercise 28 (6 credit points + 5 bonus points): *Polarization*

Consider two coordinate systems (t, x, y, z) and (t, x', y', z) that can be transformed into each other by a rotation with the angle θ around the z -axis.

28.1 Let $\hat{e}_x, \hat{e}_y, \hat{e}_{x'},$ and $\hat{e}_{y'}$ be the unit polarization vectors in both coordinate systems for an electromagnetic wave that propagates in the z -direction. Show that

$$\hat{e}_{x'} = \hat{e}_x \cos(\theta) + \hat{e}_y \sin(\theta), \quad \hat{e}_{y'} = -\hat{e}_x \sin(\theta) + \hat{e}_y \cos(\theta).$$

28.2 Analogously, let $e_+, e_-, e_{+'}, e_{-}'$ be the polarization tensors for a gravitational wave in the linearized theory. Show that

$$e_{+'} = e_+ \cos(2\theta) + e_- \sin(2\theta), \quad e_{-}' = -e_+ \sin(2\theta) + e_- \cos(2\theta).$$

28.3 Bonus exercise

Let $|\uparrow\rangle$ and $|\downarrow\rangle$ be the quantum-mechanical states of a neutrino whose spin is aligned parallelly or anti-parallelly with respect to the x -direction, respectively, and analogously $|\uparrow'\rangle$ and $|\downarrow'\rangle$ with respect to the x' -direction. Show that

$$|\uparrow'\rangle = |\uparrow\rangle \cos\left(\frac{\theta}{2}\right) + i|\downarrow\rangle \sin\left(\frac{\theta}{2}\right), \quad |\downarrow'\rangle = i|\uparrow\rangle \sin\left(\frac{\theta}{2}\right) + |\downarrow\rangle \cos\left(\frac{\theta}{2}\right).$$

28.4 Bonus exercise

Determine the generalization for the basis states of linear polarization for a radiation field of arbitrary spin s .

Exercise 29 (8 credit points): *Gauge transformation*

In the linear approximation to general relativity, we make the ansatz

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\psi_{\mu\nu}(x),$$

where $\psi_{\mu\nu}$ is 'small'.

29.1 Show that under the infinitesimal transformation

$$x'^{\mu} = x^{\mu} - 2f^{\mu}(x^{\nu})$$

one arrives at the following 'gauge' transformation law for $\psi_{\mu\nu}$:

$$\psi'_{\mu\nu}(x') = \psi_{\mu\nu}(x) + f_{\mu,\nu}(x) + f_{\nu,\mu}(x).$$

29.2 Show that the Riemann tensor at the linearized level is invariant under this gauge transformation.