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www.thp.uni-koeln.de/gravitation/courses/rci12.html

4th exercise sheet on Relativity and Cosmology I

Winter term 2012/13

Deadline for delivery: Thursday, 8th November 2012 during the exercise class
together with exercises **6**, **8.2** and **8.3** from the **3rd** exercise sheet.

Exercise 10 (6 credit points): *Motion in the gravitational field*

The equation of motion for a test particle in a gravitational field is given by

$$\ddot{x}^\mu + \Gamma^\mu{}_{\nu\kappa}\, \dot{x}^\nu \dot{x}^\kappa = 0\,, \tag{1}$$

where $\dot{x}^\mu = \mathrm{d}x^\mu/\mathrm{d}\tau$, τ is the proper time and $\Gamma^\mu{}_{\nu\kappa} = \frac{1}{2}\, g^{\mu\sigma}\,(\partial_\kappa\, g_{\sigma\nu} + \partial_\nu\, g_{\sigma\kappa} - \partial_\sigma\, g_{\nu\kappa})$.

10.1 Repeat briefly the derivation of (1) from the variational principle $\delta \int \mathrm{d}\tau = 0$ as presented in the lecture. Why can the derivation not be used for photons?

10.2 Derive (1) from the alternative variational principle

$$\delta \int g_{\mu\nu}\, \dot{x}^\mu \dot{x}^\nu\, \mathrm{d}\lambda \equiv \delta \int \mathcal{K}\, \mathrm{d}\lambda = 0\,,$$

where λ is an affine parameter and $\dot{x}^\mu = \mathrm{d}x^\mu/\mathrm{d}\lambda$.
Show that this derivation also holds for photons and determine $\mathcal{K}$ for the solution of (1).

Exercise 11 (6 credit points): *Christoffel symbols*

Derive the transformation properties of the Christoffel symbols

$$\Gamma_{\mu\nu\lambda} = \frac{1}{2}\,(g_{\mu\nu,\lambda} + g_{\lambda\mu,\nu} - g_{\nu\lambda,\mu})$$

under a coordinate transformation $x^\mu \to x'^\mu(x^\alpha)$.

(The result shows that the Christoffel symbols do not form a tensor.)