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www.thp.uni-koeln.de/gravitation/courses/rci1516.html

4th exercise sheet on Relativity and Cosmology I

Winter term 2015/16

Deadline for delivery: Thursday, 19th November 2015 during the exercise class.

Exercise 9 (5 credit points): *Accelerated frame of reference*

Show that the equations

$$\begin{aligned}
t &= \frac{c}{g}\,\sinh\!\left(\frac{g\,t'}{c}\right) + \frac{x'}{c}\,\sinh\!\left(\frac{g\,t'}{c}\right)\,,\\
x &= \frac{c^2}{g}\,\left[\cosh\!\left(\frac{g\,t'}{c}\right) - 1\right] + x'\,\cosh\!\left(\frac{g\,t'}{c}\right)\,,\\
y &= y'\,,\\
z &= z'\,,
\end{aligned}$$

describe a transformation from an inertial frame to an accelerated frame of reference ($g = \text{const.}$). Calculate the components of the metric with respect to the frame (t', x', y', z').

Exercise 10 (10 credit points): *Rindler coordinates*

Consider the two-dimensional metric

$$\mathrm{d}s^2 = -\,v^2\,\mathrm{d}u^2 + \mathrm{d}v^2\,.$$

At which point in space do the components of the metric tensor exhibit a singularity?

Find a coordinate transformation which shows that this so-called Rindler space is only a part of the two-dimensional Minkowski space, which is usually represented by $\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \mathrm{d}x^2$.

Compare the Rindler coordinates with the coordinates from exercise **7**.

Give an illustrative interpretation of the Rindler coordinates (consider $u = \text{const.}$ and $v = \text{const.}$).

Determine the proper acceleration along the curve $v = \text{const.}$

Exercise 11 (5 credit points): *Clocks*

Two atomic clocks are transported in two airplanes once around the Earth in either eastern or western direction. Calculate the respective time dilations the clocks exhibit after the airplanes have landed compared to a clock which stayed on the ground. For this purpose, assume for reasons of simplicity that the airplanes fly directly above the equator, where the rotation velocity of the Earth is about $v_\mathrm{E} \approx 1667$ km/h. Furthermore, use an average cruising speed of $v_\mathrm{F} \approx 800$ km/h and a mean flying altitude of 10 km.

Hint: Take into account the separate contributions due to the gravitational and the special relativistic velocity effect, respectively.