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www.thp.uni-koeln.de/gravitation/courses/rcii13.html

9th exercise sheet on Relativity and Cosmology II

Summer term 2013

Deadline for delivery: Thursday, 27th June 2013 during the exercise class.

Exercise 20 (6 credit points): *Friedmann I*

Consider a Friedmann model with $k \neq 0$ and present density parameters $\Omega_{\mathrm{m},0}$, $\Omega_{\mathrm{r},0}$ and $\Omega_{\mathrm{v},0}$ as well as $\Omega := \Omega_{\mathrm{m},0} + \Omega_{\mathrm{r},0} + \Omega_{\mathrm{v},0}$. Furthermore let

$$\rho_{\mathrm{c}}(a) = \frac{3\,\dot{a}^2}{8\pi G\, a^2}$$

be the critical density at a time when the scale parameter had the value a, and let $\Omega_{\mathrm{m}}(a) = \rho_{\mathrm{m}}(a)/\rho_{\mathrm{c}}(a)$ etc. be the corresponding relative densities.

Determine the quantity $\Omega(a) - 1$ as a function of $\Omega_{\mathrm{m},0}$, $\Omega_{\mathrm{r},0}$, $\Omega_{\mathrm{v},0}$ and a. This quantity indicates how much the considered model "deviated" from a flat model at a certain time. What kind of "aesthetic problem" arises for a Friedmann model whose density parameter Ω differs only slightly from unity today?

Exercise 21 (8 credit points): *Friedmann II*

Solve the Friedmann equation for a universe that contains radiation as well as non-relativistic matter (dust). For this purpose, rewrite the Friedmann equation as a differential equation with respect to the conformal time η, solve this equation for the three possible values of k and write out the result in the form $(a(\eta), t(\eta))$.

Show further how one can obtain the result for $k = 0$ from the results for $k \neq 0$ and analyze the implicitly defined functions $a(t)$ for large and small values of t.

Exercise 22 (6 credit points): *Friedmann II*

Current observations indicate that we live in a flat ($k = 0$) universe with positive cosmological constant, in which the contribution of radiation to the total energy density can be neglected.

Solve the Friedmann equation for this model. (*Hint:* The substitution $x^2 = (1/\Omega_{\mathrm{m},0} - 1)\, a^3$ could be helpful.) Determine the age of the universe as a function of H_0 and $\Omega_{\mathrm{m},0}$. How does $a(t)$ behave for large and small values of t?

According to well-established models for stellar evolution, several globular clusters in our galaxy are at least 12 billion years old. Draw a $(h - \Omega_{\mathrm{m}})$-diagram ($h$ is the parameter in the definition of H_0) and sketch the contour lines for a constant age of the universe. Determine which parameter range is compatible with the above-mentioned observation. In doing so, only consider values $0.4 < h < 1$.