University of Cologne
Institute for Theoretical Physics
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www.thp.uni-koeln.de/gravitation/courses/rcii16.html

6th exercise sheet on Relativity and Cosmology II

Summer term 2016

Deadline for delivery: Thursday, 2nd June 2016 during the exercise class.

Exercise 11 (16 credit points + 2 bonus points): *Kerr metric I*

11.1 Calculate the surface area of the event horizon of a rotating black hole ($|a| \leq M$) for $t = \text{const.}$

Furthermore, show that the circumference around the poles is always smaller than the circumference around the equator, i.e. that the geometry of the event horizon is not spherical. Give an estimate for the ratio of the polar and equatorial circumference for the extremal case $|a| = M$.

11.2 Consider the extremal case $r \gg M$ and $r \gg |a|$ for the Kerr metric.

Show that the non-diagonal term $\propto \mathrm{d}\phi\,\mathrm{d}t$ is equal to the non-diagonal term of a slowly and rigidly rotating spherical mass distribution, which has been given in the lecture as

$$\frac{4\,I}{r^3}\,\varepsilon^{\alpha\beta\gamma}\,\mathrm{d}x^\alpha\,\omega^\beta\,x^\gamma\,\mathrm{d}t\,, \qquad I: \text{ moment of inertia,}$$

if one chooses the z-axis to be parallel to the direction of rotation.

11.3 Consider the Kerr metric in the limit $M \to 0\,,\ a = \text{const.}$

Show – preferably by means of an explicit coordinate transformation to Cartesian coordinates – that this limit describes flat spacetime. What is the geometric interpretation of the coordinate r in this limit?

Hint: Try the ansatz $x = f(r)\sin(\theta)\cos(\phi)\,,\ y = f(r)\sin(\theta)\sin(\phi)\,,\ z = g(r)\cos(\theta)$ to transform the metric to Cartesian coordinates.

11.4 ***Bonus exercise*** Try to bring the Kerr metric in a diagonal form.

Exercise 12 (4 credit points): *Kerr metric II*

Give a rough estimate for the parameter a/M – as given in the Kerr metric – for Sun and Earth. Take the required physical parameters – like the radius of the Sun etc. – from the literature.

What do you notice?