Negative index of refraction, perfect lenses and transformation optics – some words of caution.

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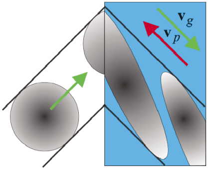
August 18, 2010

Overview: 'Negative refractive index $\neq$ Folding of space '.



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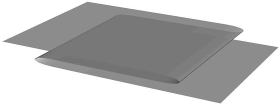

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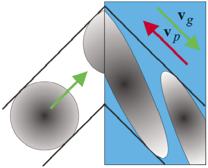

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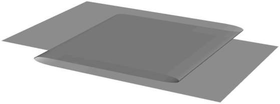

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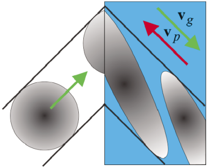

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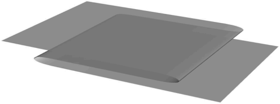

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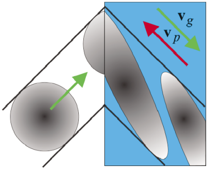

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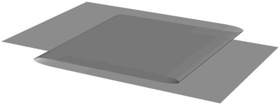

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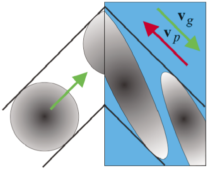

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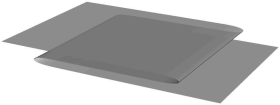

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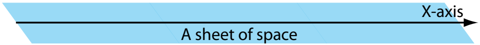
- Folding gives no perfect lensing, as it introduces an extra source, rather than amplifying evanescent fields.
- Other ways to get a negative index do work, but is it really worth it?

Often negative index is (wrongly) linked to Folding. Why?



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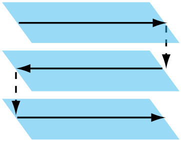

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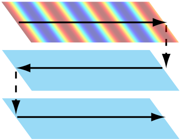

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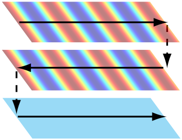

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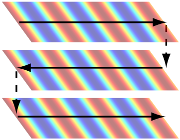

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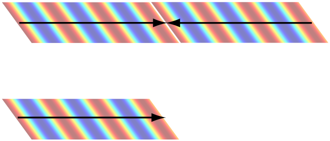

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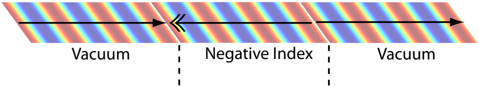

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Impression: a negative index slab in vacuum...

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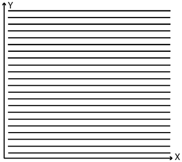

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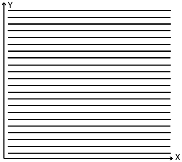

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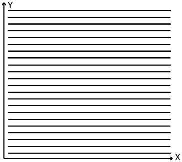

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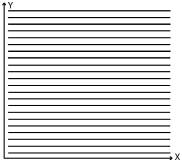

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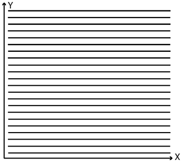

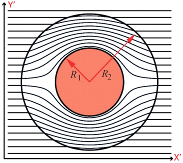

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Figure: J.B. Pendry et al., Science **312** (5781), 2006.

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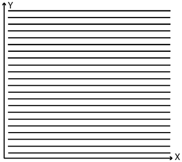

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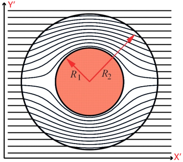

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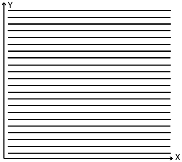

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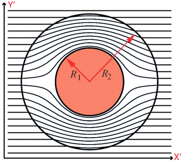

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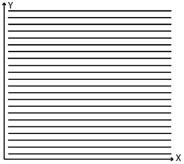

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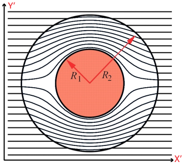

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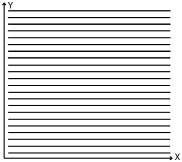

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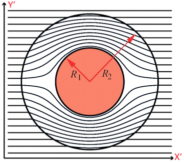

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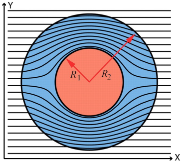

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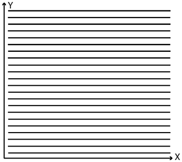

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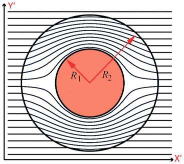

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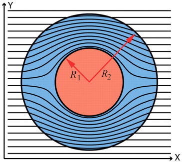

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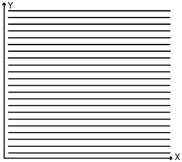

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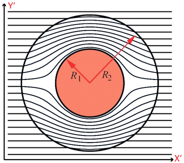

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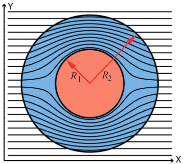

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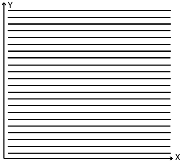

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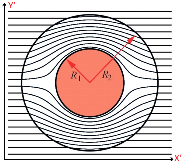

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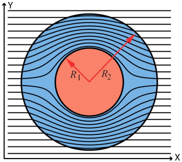

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Stage 1: γ^{ij}

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$\mathrm{Diag}(1, 1, 1)$	$\mathrm{Diag}((-1)^2, 1, 1)$

So, let's fold space…but get no negative index!

Useful things:

- 3 stages: Vacuum, Transformation and Interpretation.
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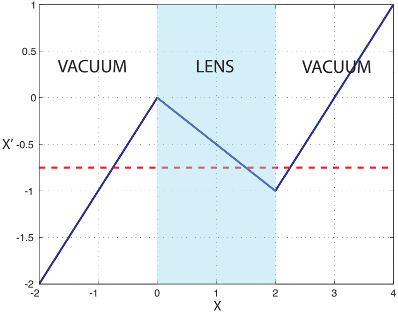
- ◇ Cf. Cartan's "twist": F.W. Hehl, Birkhäuser, 2003.

'Folding' argument gives no perfect lens (preamble).



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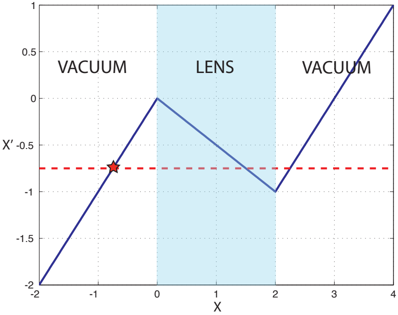

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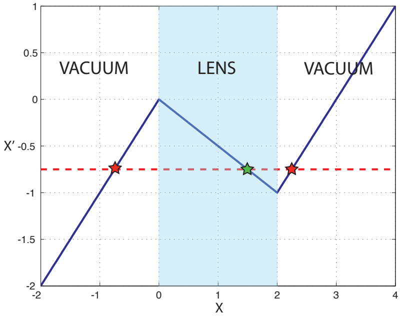

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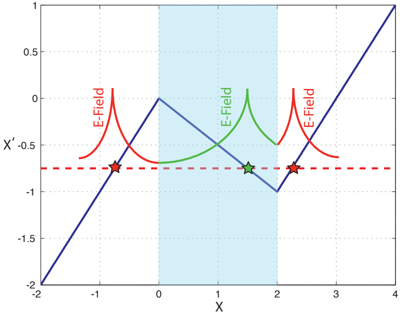

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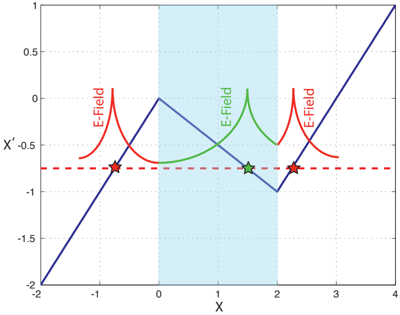

- Fold X-axis into a slab (allegedly, a perfect lens).
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- Contrary common belief: the answer is NO...

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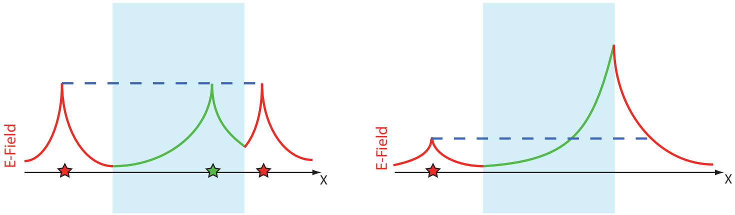

◇ Compare: 'Fold' lens (left) with 'Pendry' lens (right).

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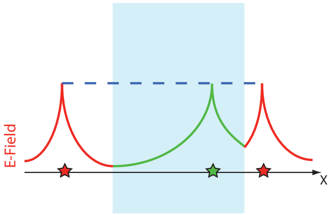

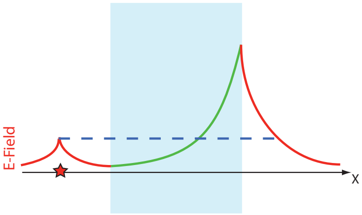

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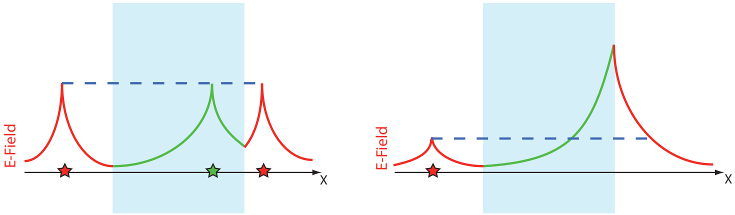

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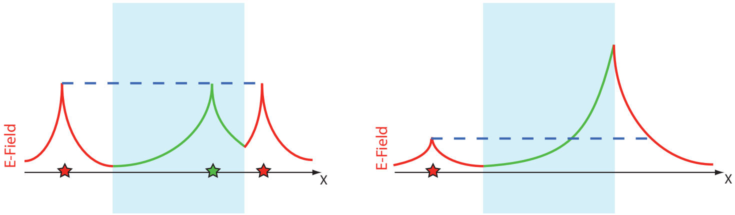

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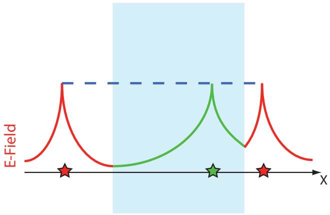

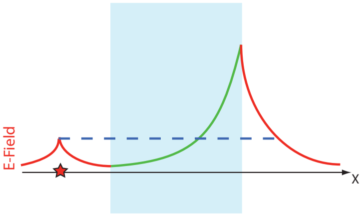

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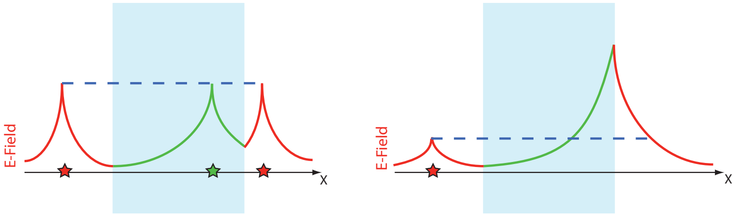

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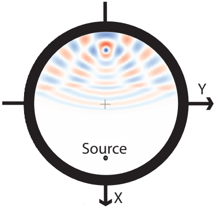
- ◇ The middle "active sink"? A carefully phased source...

The fish-eye lens needs an active sink... Physical? Useful?



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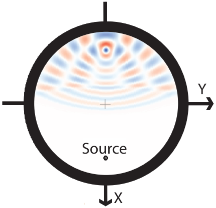

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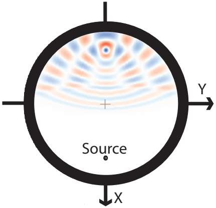

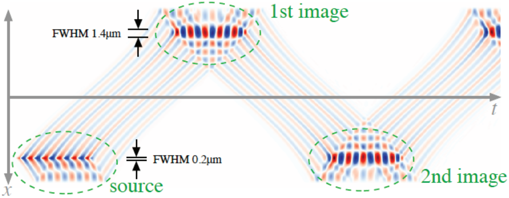

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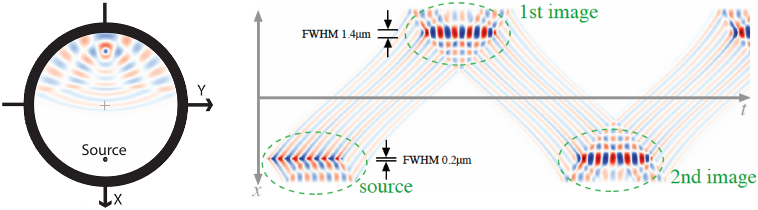

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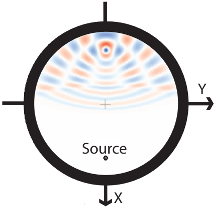

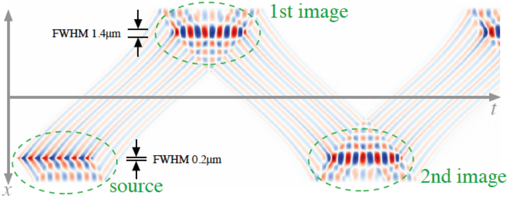

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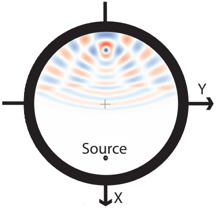

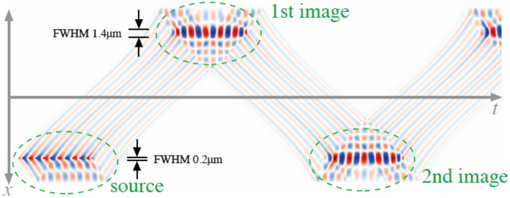

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- Hotly debated: active sinks are useful? physical?

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Folding fails? Other ways to get a negative index

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Fundamental (non transf-based):

- Start: vacuum using space+time metric $g^{\alpha\beta}$:
$$\chi_0^{\mu\nu\alpha\beta} = \quad (\mu_0/\varepsilon_0)^{-\frac{1}{2}} \left(g^{\mu\alpha} g^{\nu\beta} - g^{\mu\beta} g^{\nu\alpha} \right)$$

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Conclusions.

- Negative index often thought as a folding of space.
- But with this approach:
  - Rigorously, $\epsilon < 0$ and $\mu < 0$ are not obtained.
  - Perfect lensing does not occur, rather...
  - Carelessness generates extra sources/sinks.
- So...do not argue in terms of 'folding'!
- Other transformations work: but no real advantage.
- Further information:
  - Luzi Bergamin and Alberto Favaro, arXiv:1001.4655
  - And, of course, the EMTS proceedings!

Thank-you!