Delocalization transitions of semiflexible manifolds

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Semiflexible manifolds such as fluid membranes or semiflexible polymers undergo delocalization transitions if they are subject to attractive interactions. We study manifolds with short-ranged interactions by field-theoretic methods based on the operator product expansion of local interaction fields. We apply this approach to manifolds in a random potential. Randomness is always relevant for fluid membranes, while for semiflexible polymers there is a first-order transition to the strong coupling regime at a finite temperature.

I. INTRODUCTION

Low dimensional manifolds play an important role in a variety of different contexts, e.g., as soft matter objects or as domain boundaries in condensed matter systems. They can perform large shape fluctuations driven by entropy [1]. According to their fluctuations they can be divided into two classes. *Flexible* manifolds, such as interfaces, polymerized membranes, and long polymers, fluctuate under a *tension* controlling their area or length. The other class is governed by *bending energy*; i.e., regions of high *curvature* are penalized. Examples are polymers not much longer than their persistence length, like actin or DNA, and fluid membranes. These objects are stiffer, and we call them *semiflexible* manifolds.

Whenever, a fluctuating manifold is attracted toward some other "defect" manifold, there is a competition between freely fluctuating configurations favored by entropy and configurations bound to the defect, which are preferred by energy. This competition can lead to a phase transition, the so called delocalization or unbinding transition. It is often of second order, that is, the amplitude of the fluctuations diverges continuously as the transition point is approached from within the bound phase. This leads to a scaling regime close to the transition whose universal characteristics can be described by a continuum field theory. Well-known examples of delocalization are wetting phenomena [2]. For interfaces and polymers, these transitions have been widely studied [3]. In the case of polymers, even the generalized problem of N mutually attracting objects can be treated. Analytically continued to $N=0$, this describes a directed polymer in a random medium [4], which in turn is related to theories of stochastic surface growth [5]. The delocalization transition then corresponds to a roughening transition between a smooth and a rough growth mode.

For *flexible* manifolds, there is a well-established field-theoretical framework to understand these phenomena on a unified footing [6–9]. The goal of this communication is to develop a corresponding field-theoretical description for *semiflexible* manifolds with local interactions. For semiflexible manifolds, we find that it is important to take their *orientation* into account in addition to their position in space. This leads us to the introduction of an orientation dependent interaction operator, which can be shown to fulfill an operator algebra. This is a well-known concept in field theory (see, e.g., Ref. [10]), which has been applied extensively in the case of flexible manifolds before [11]. Here, it allows us to write down renormalization group equations for the generic local interactions of semiflexible manifolds. This leads to results for the delocalization of semiflexible polymers and fluid membranes. Most importantly, the bound state of a semiflexible manifold turns out to be maintained by contact interactions at fixed orientation, while the bound state of a flexible manifold involves orientation independent interactions as it is depicted in the case of polymers in Fig. 1.

Our one-loop results for the critical behavior at the unbinding transitions are in agreement with previous results obtained by approximate renormalization methods [12,13] and by approaches specific to polymers [14–16]. They are exact for polymers and can be improved systematically for higher dimensional manifolds.

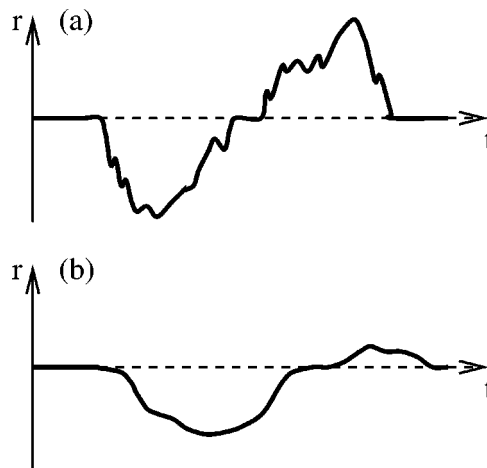Furthermore, they can be applied to a semiflexible mani-

FIG. 1. (a) A flexible and (b) a semiflexible polymer parametrized by their shape functions $r(t)$ bound to an attractive defect indicated by the dashed lines. The unbound segments join the defect at arbitrary orientation and at fixed orientation $dr/dt=0$, respectively.

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fold in a quenched random potential. In the replica formalism, this is equivalent to N interacting semiflexible manifolds in the limit of vanishing N. For fluid membranes, any amount of disorder is relevant and leads to a strong coupling phase as can be easily checked by power counting. For semiflexible polymers, however, we find that small amounts of disorder are irrelevant (unlike for their flexible counterparts). There is now a first-order transition to the strong coupling phase at a finite amount of disorder. A quantitative description of the disordered strong coupling phase is, however, beyond the means of perturbation theory.

The paper is organized as follows: In Sec. II we shortly review the field-theoretical treatment of flexible manifolds and introduce the necessary notation. Section III treats the case of unbinding transitions in the presence of a hard-wall constraint, which turns out to be the more transparent case for setting up our field-theoretical treatment. In Sec. IV we then discuss the more difficult case of unbinding transitions without a wall. Section V is dedicated to the behavior of a single semiflexible manifold in a random medium and in Sec. VI we summarize our results.

II. SCALING ANALYSIS

In order to prepare for the development of a field-theoretical description of *semiflexible* manifolds with short-ranged interactions we want to first discuss briefly the established field theory for *flexible* manifolds. Here, we also introduce the notation used later and make some simple scaling analysis.

The configurations of manifolds are described by a d-dimensional displacement field $\mathbf{r}(t)$, which depends on a D-dimensional internal variable t. The energy of a configuration $\mathbf{r}(t)$ is given by a continuum Hamiltonian of the form

$$\mathcal{H} = \int \left[\frac{1}{2} (\nabla^k \mathbf{r})^2 + V(\mathbf{r}, \nabla \mathbf{r}) \right] \mathrm{d}^D t, \tag{1}$$

where

$$(\nabla^k \mathbf{r})^2 \equiv \sum_{\alpha=1}^{D} \sum_{i=1}^{d} [\partial^k r_i(t)/\partial t_\alpha^k]^2 \tag{2}$$

is the leading tension ($k=1$) or curvature ($k=2$) energy in a small-gradient expansion. The case $k=1$ corresponds to flexible manifolds and is physically realized, e.g., for directed polymers and flux lines in a type II superconductor ($D=1$, $d=2$) [17,18], steps on a tilted crystal surface ($D=1$, $d=1$), and domain walls in a ferromagnet ($D=2$, $d=1$). The potential V describes the interaction of the manifold with an external object or boundary at $\mathbf{r}=0$, or the mutual interaction between two manifolds with relative displacement $\mathbf{r}(t)$.

Short-ranged interactions are interactions with a range much shorter than typical displacements of the manifold. If they are of definite sign (i.e., repulsive or attractive over their entire range), they can be represented as $V[\mathbf{r}(t)] = g\Phi(t)$ in terms of the local *contact field*

$$\Phi(t) \equiv \delta[\mathbf{r}(t)]. \tag{3}$$

The scaling dimension of this field,

$$x_\Phi = d\chi \tag{4}$$

is given in terms of the roughness exponent χ, which in turn can be determined by simple power counting with the result $\chi = (2-D)/2$ for $k=1$. This interaction has been treated exhaustively in a well-established perturbative framework in $D=1$ [6,7] and in general D [8,9]. This has also been applied extensively to the problem of self-avoiding manifolds [19–23]. More complicated short-ranged [24] and long-ranged [25] interactions have been studied as well.

The case of *semiflexible* manifolds with local interactions, which is the focus of this paper, is described by the Hamiltonian (1) with $k=2$. Physically interesting cases are again polymers ($D=1$, $d=1,2$) [14–16] and, in particular, fluid membranes ($D=2$, $d=1$) [26]. Since a semiflexible manifold has a locally well-defined orientation, we have to consider interactions $V[\mathbf{r}(t), \nabla \mathbf{r}(t)]$ that depend both on the displacement and on the orientation. Thus, there are now two important scaling fields: Local contacts at arbitrary orientation are still represented by the field $\Phi(t)$ given by Eq. (3), which has dimension

$$x_\Phi = d\chi \tag{5}$$

with

$$\chi = \frac{4-D}{2}. \tag{6}$$

As it will turn out, however, contacts for which the manifold is oriented parallel to the defect are especially important. Such contacts are described by the field

$$\Omega(t) \equiv \delta[\mathbf{r}(t)] \delta[\nabla \mathbf{r}(t)], \tag{7}$$

which has a different scaling dimension. This is again determined by power counting,

$$x_\Omega = d\chi + dD(\chi - 1). \tag{8}$$

In what follows we will argue that the unbinding transition of semiflexible manifolds can be described in terms of these two interaction fields.

III. UNBINDING IN THE PRESENCE OF A WALL

In order to understand the basic mechanism underlying the unbinding transition of semiflexible manifolds we will first study the simpler case in which the manifold is subject to a *hard wall constraint*, i.e., the components of the shape function $\mathbf{r}(t)$ are restricted to positive values. Such a wall constraint is natural in $d=1$ for a fluid membrane at a planar system boundary, or for a pair of membranes that cannot intersect. We generalize this natural hard wall constraint to arbitrary d in such a way that the Hamiltonian (1) remains factorizable.

To this end, we study the manifold displacement field $\mathbf{r}(t)$

in a hypercube of longitudinal extension $0 \le t_\alpha \le T$ ($\alpha = 1, \ldots, D$) and of transversal extension $0 \le r_i \le R$ ($i = 1, \ldots, d$). The hypercube geometry implies that the free system, i.e., the manifold without any short-range interactions, separates into d independent free manifolds for each of the components of $\mathbf{r}$.

By the definition of the roughness exponent χ, regions in t space of size $R^{D/\chi}$ are independent from each other. Thus, for $T \gg R^{1/\chi}$ the free energy becomes extensive and, for dimensional reasons, takes the form

$$F \sim T^D R^{-D/\chi}. \tag{9}$$

In what follows we will always assume that we are in this regime. The perturbation series for the free energy density $f \equiv F/T^D$ then becomes invariant under translations in t.

A. Interaction operator

In the presence of the walls, the probability density $\rho(\mathbf{r}') \equiv \langle \delta[\mathbf{r}(t) - \mathbf{r}'] \rangle$ is forced to vanish at the boundary of the hypercube, in particular along the "edge" $\mathbf{r} = 0$. The density then takes the asymptotic scaling form

$$\rho(\mathbf{r}) \sim (r_1 \cdots r_d)^\theta R^{-d(1+\theta)} \quad \text{for} \quad |\mathbf{r}| \ll R, \tag{10}$$

with an exponent $\theta > 0$ expressing long-ranged suppression of the configurations close to the boundary.

Thus, short-ranged interactions with the manifold $\mathbf{r} = 0$ now have to be described in terms of a local field whose expectation values remain finite even in the presence of a wall. To this end, a direction of approaching the "edge" at $\mathbf{r} = 0$ has to be chosen and the local operator at a point approaching the edge has to be multiplied by the appropriate power of the distance from the edge according to Eq. (10) to compensate for the diminishing density at the wall. While the specific choice of the direction of approach only affects the irrelevant numerical prefactor of the operator, we use the "diagonal" $(r, \ldots, r)$ and define

$$\Omega(t) \equiv \lim_{r \to 0} r^{-d\theta} \prod_{i=1}^{d} \delta[r_i(t) - r]. \tag{11}$$

It is important to notice that due to the constraint, the manifold always has the fixed orientation $\nabla \mathbf{r} = 0$ in the vicinity of $\mathbf{r} = 0$. Hence, we have used the same symbol Ω as for the field Eq. (7) of the unconstrained system. The correlation functions

$$\langle \Omega(t) \rangle \sim R^{-x_\Omega/\chi}, \tag{12}$$

$$\langle \Omega(t)\Omega(t') \rangle \sim |t - t'|^{-x_\Omega} \langle \Omega(t) \rangle + \cdots (|t - t'|^\chi \ll R) \tag{13}$$

define the scaling dimension x_Ω, which will be different from its value given in Eq. (8) in the absence of a wall. In the constrained system, we cannot do simple power counting any more in order to obtain this scaling dimension. However, we can obtain its value by noting that the density Eq. (12) is

linked to the pressure of the system by a wall theorem. This applies independently to the d components of $\mathbf{r}$ and yields together with Eq. (9),

$$\langle \Omega \rangle \sim (\partial f/\partial R)^d \sim R^{-d(1+D/\chi)}. \tag{14}$$

This immediately determines the exponent value [27]

$$x_\Omega = d(\chi + D), \tag{15}$$

which is in agreement with a conjecture from functional renormalization [28] for general D and a direct calculation [29] for $D = 1$.

B. Perturbation theory

The interaction part $\delta f(h, R) \equiv f(h, R) - f(0, R)$ of the free energy density of the system with $V[\mathbf{r}(t), \nabla \mathbf{r}(t)] = h\Omega(t)$ can be formally expanded as a power series

$$\delta f = h\langle \Omega \rangle + \frac{h^2}{2} \int d^D t \langle \Omega(0)\Omega(t) \rangle_c + O(h^3) \tag{16}$$

containing the connected correlation functions

$$\langle \Omega(0)\Omega(t) \rangle_c \equiv \langle \Omega(0)\Omega(t) \rangle - \langle \Omega \rangle^2, \tag{17}$$

etc., taken at $h = 0$. Since all the integrals in this perturbation series diverge, it has to be regularized. We will choose for this purpose a dimensional regularization scheme. To this end, we have to identify the dimension in which the interaction Ω is marginal, i.e., where $x_\Omega = D$. By Eq. (15), this happens on a whole *line* in the (D, d) plane. This line is given by

$$d^*(D) = \frac{2D}{4 + D}. \tag{18}$$

Thus, the divergences in the perturbation series (16) can be written as poles in the distance

$$\epsilon \equiv D - x_\Omega = D - d(2 + D/2) \tag{19}$$

from the line of marginality (18). As discussed in Ref. [19] in the case of flexible manifolds, these poles can be regularized around *any point* of this line.

The singularity of the two-point function (17) can be determined on physical grounds. The presence of the wall and the stiffness of the manifold implies that the configurations of the manifold that contribute to an expectation value containing a product $\Omega(t)\Omega(t')$ are those that are close *and parallel* to the wall at positions t and t'. As t and t' approach each other being close and parallel to the wall at position t implies being close and parallel to the wall at position t' as well. Thus, the product of the two operators can be replaced by simply one of them times a characteristic divergence if t and t' are close together. That suggests the operator product expansion

$$\Omega(t)\Omega(t') \sim |t - t'|^{-x_\Omega} \Omega(t) + \cdots. \tag{20}$$

The exponent of the divergence has to be the scaling dimension x_Ω of the operator Ω for dimensional reasons. The unknown numerical prefactor of this operator product expansion can always be absorbed in the definition of the operator Ω.

The singularity in the operator product expansion determines in a standard way [11] the one-loop renormalization group equation of the dimensionless coupling constant

$$v \equiv h R^{\epsilon/\chi} \tag{21}$$

associated with the operator Ω. In an appropriate scheme, this takes the form

$$\dot{v} = \epsilon v - v^2 + O(v^3). \tag{22}$$

The unstable fixed point $v^* = \epsilon + O(\epsilon^2)$ represents the unbinding transition. Linearizing Eq. (22) around the fix point value v^* yields $\dot{v} = \epsilon^*(v - v^*) + \cdots$ with

$$\epsilon^* = -\epsilon + O(\epsilon^2). \tag{23}$$

This immediately determines the scaling of the transversal localization length $\xi \equiv \langle \mathbf{r}^2 \rangle^{1/2}$,

$$\xi \sim (v^* - v)^{-\chi/\epsilon^*} \quad (v < v^*), \tag{24}$$

within the bound phase and the scaling dimension

$$x_\Omega^* = D - \epsilon^* = 2D - x_\Omega + O(\epsilon^2), \tag{25}$$

which takes the place of x_Ω in the correlations (12) and (13) at the transition point. These relations describe the scaling of a bound state maintained by contact forces at fixed orientation. Typical configurations look similar to those of Fig. 1(b) but are confined to the region $r_i > 0$.

C. Special cases

The most interesting application of Eqs. (24) and (25) is the delocalization transition of a fluid membrane from a hard wall ($D = 2$, $d = 1$), where the one-loop result leads to $\xi \sim (T_c - T)^{-1}$ (since the effective coupling is temperature dependent) and $x_\Omega^* = 1$. These predictions are in agreement with those from functional renormalization [12]. They also fit very well the numerical values of Ref. [13], which implies that higher-order corrections must be small. The system with wall constraint at $h = 0$ can be regarded as an unconstrained system in the limit of a large repulsive interaction. Conversely, the scaling at the transition point of the constrained system may be related to that of the free unconstrained system. Indeed, the one-loop value x_Ω^* from Eq. (25) equals the dimension $x_\Omega = 1$ of the free field Eq. (7), indicating that the sum of the higher-order corrections in Eqs. (22) and (25) may vanish altogether at the specific point ($D = 2$, $d = 1$).

For the case of polymers ($D = 1$) it is easy to show that the multipoint correlations entering Eq. (16) factorize after "time ordering" the interaction points,

$$\langle \Omega(t_1) \cdots \Omega(t_n) \rangle = \langle \Omega(t_1) \rangle \mathcal{R}(t_2 - t_1) \cdots \mathcal{R}(t_n - t_{n-1}), \tag{26}$$

for $t_1 < \cdots < t_n$. The factors $\mathcal{R}(t) \equiv \langle \Omega(0) \Omega(t) \rangle / \langle \Omega \rangle$ can be interpreted as "return" probabilities to the wall. Equation (26) provides the same factorization structure for the higher-order correlation functions as in the case $k = 1$. Thus, the perturbation expansion (16) becomes formally identical to the case $k = 1$. Therefore, studies of the perturbation expansion in the latter case [8,11] imply that also the polymer perturbation series (16) studied here is *one-loop renormalizable*; i.e., the connected two-point function $\langle \Omega(0) \Omega(t) \rangle_c$ generates the only primitive singularity, and there are no higher-order terms in Eq. (22). The implications of Eqs. (24) and (25) on general semiflexible polymers are discussed and verified numerically in Ref. [30]. Here we have given a *unified derivation* of these relations, stressing the theoretical analogies with their known counterparts for $k = 1$ [31,11].

IV. UNBINDING WITHOUT A WALL

We now turn to systems without the wall constraint and restrict ourselves to $D = 1$, namely mutually interacting semiflexible polymers. In the absence of a wall constraint, we have to study generic contact interactions

$$V(t) = g\Phi(t) + h\Omega(t) \tag{27}$$

involving the fields Φ and Ω as defined in Eqs. (3) and (7). The perturbation series then contains connected correlations $\langle \Phi(t_1) \cdots \Phi(t_n) \Omega(t_1') \cdots \Omega(t_m') \rangle_c$ in the free theory ($g = h = 0$). In $D = 1$ the configurations of the polymer before t and after t become independent from each other as long as the position $\mathbf{r}$ and slope v at t are fixed. Thus, in $D = 1$ all the correlation functions can be expressed in terms of the propagator $G_{\Delta t}(\mathbf{r}', v' | \mathbf{r}, v)$ of the free theory, i.e., in terms of the probability to find the semiflexible polymer at position $\mathbf{r}'$ with slope v' if it was at position $\mathbf{r}$ with slope v a longitudinal distance Δt before. This is a particularity of the one-dimensional case. For $D \neq 1$ (e.g., for membranes with $D = 2$) it is not possible any more to express the correlation functions in terms of the two-point function alone. Thus, the derivation of an operator algebra analogous to our result Eqs. (30)–(32) remains a difficult outstanding problem in the case $D \neq 1$.

The propagator $G_{\Delta t}(\mathbf{r}', v' | \mathbf{r}, v)$ can be calculated exactly [16]. It is given by

$$G_{\Delta t}(\mathbf{r}', v' | \mathbf{r}, v) = \left(\frac{\sqrt{3}}{\pi \Delta t^2} \right)^d \exp\left[-\frac{6}{\Delta t} S_{\Delta t}(\mathbf{r}', v' | \mathbf{r}, v) \right], \tag{28}$$

where

$$S_{\Delta t}(\mathbf{r}', v' | \mathbf{r}, v) = (\mathbf{r}' - \mathbf{r} - v\Delta t)^2 + \frac{\Delta t^2}{3}(v' - v)^2$$
$$- \Delta t(\mathbf{r}' - \mathbf{r} - v\Delta t)(v' - v) \tag{29}$$

is the action of the minimal energy path.

From this propagator, a closed operator expansion algebra for the two operators Φ and Ω can be derived. To this end, we have to calculate correlations involving the two operators

in question. Let us start with the expectation value $\langle \cdots \Phi(t)\Phi(t') \cdots \rangle$ where we always assume some suitably chosen infrared cutoff. We are interested in the limit $t' \to t$ from above. Due to the Markov property governing $D=1$, we can subsume the contributions of all operators with an argument less than t into the expectation value $G_t^<(\mathbf{r},v)$ subject to the condition that the polymer is at position $\mathbf{r}$ with slope v at "time" t and the contributions of all operators with an argument larger than t' into the corresponding expectation value $G_{t'}^>(\mathbf{r}',v')$. Then, we get

$$\langle \cdots \Phi(t)\Phi(t') \cdots \rangle = \int d^D\mathbf{r} \int d^D v \int d^D\mathbf{r}' \int d^D v',$$

$$G_t^<(\mathbf{r},v)\,\delta(\mathbf{r})G_{t'-t}(\mathbf{r}',v'|\mathbf{r},v)\,\delta(\mathbf{r}')G_{t'}^>(\mathbf{r}',v')$$

$$= \int d^D v \int d^D v' \, G_t^<(0,v)G_{t'-t}(0,v'|0,v)G_{t'}^>(0,v').$$

Using the explicit form (28) of the propagator, we find

$$G_{t'-t}(0,v'|0,v) = \left(\frac{\sqrt{3}}{\pi(t'-t)^2}\right)^d \exp\left[-\frac{2}{t'-t}(v'-v)^2 \right.$$

$$\left. -\frac{6}{t'-t}vv' \right]$$

$$= \frac{1}{(t'-t)^d}\left(\frac{\sqrt{3}}{\pi(t'-t)}\right)^d \exp\left[-\frac{1}{t'-t}(vv')^T \right.$$

$$\left. \times \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} v \\ v' \end{pmatrix} \right]$$

$$\approx \frac{1}{(t'-t)^d}\,\delta(v)\,\delta(v')$$

in the limit $t' \to t$. This finally yields

$$\langle \cdots \Phi(t)\Phi(t') \cdots \rangle \approx G_t^<(0,0)(t'-t)^{-d}G_t^>(0,0)$$

$$= (t'-t)^{-d}\langle \cdots \Omega(t) \cdots \rangle.$$

Similar calculations using

$$G_{t'-t}(0,0|0,v)$$

$$= \left(\frac{3}{2\pi}\right)^{d/2}\left(\frac{1}{t'-t}\right)^{(3/2)d}\left(\frac{2}{\pi(t'-t)}\right)^{d/2}e^{[-(2/t'-t)v^2]}$$

and

$$G_{t'-t}(0,0|0,0) = \left(\frac{\sqrt{3}}{\pi}\right)^d\left(\frac{1}{t'-t}\right)^{2d}$$

yield the complete operator product expansion

$$\Phi(t)\Phi(t') = |t-t'|^{-d}\Omega(t) + \cdots, \tag{30}$$

$$\Phi(t)\Omega(t') = \left(\frac{3}{2\pi}\right)^{d/2}|t-t'|^{-(3/2)d}\Omega(t) + \cdots, \tag{31}$$

$$\Omega(t)\Omega(t') = \left(\frac{\sqrt{3}}{\pi}\right)^d|t-t'|^{-2d}\Omega(t) + \cdots. \tag{32}$$

These relations can be understood very intuitively. They say that *any* pair of close-by contacts with the defect looks from larger distances like a single *tangential* contact, multiplied by a singular prefactor. This is rather obvious for Eqs. (31) and (32) for which at least one of the original contacts is already forced to be tangential on the left-hand side. The more surprising relation (30) on the other hand is a consequence of the stiffness of the polymer: configurations that cross the defect line at two very close positions t and t' without being tangential to the defect necessarily are strongly bent and, therefore, strongly suppressed energetically. In the end, only the tangential contacts with the defect line keep an appreciable weight.

Within this intuitive picture it is also clear why the case of membranes $D=2$ is more complicated. If the membrane is forced to have a contact at two close-by positions this at most can force the membrane to be tangential to the defect in the direction of the line connecting the two points. In the direction perpendicular to this line the membrane is free to take any slope without any energetic penalty. Thus, the operator algebra in $D=2$ must be more complicated than Eqs. (30)–(32) in order to distinguish between longitudinal and transversal components of the slope. It cannot even be derived by some power counting approach since in $D=2$ the slope v [and thus also the operator $\delta(v)$] is dimensionless in the free theory and thus any function of the slope v can appear on the right-hand side of the operator algebra. We note, however, that this operator degeneracy is a somewhat technical problem of the Gaussian theory. In all physically interesting cases of self-avoiding or mutually avoiding membranes (which may be difficult for other reasons) this degeneracy is lifted again.

Returning to $D=1$, an important feature of Eqs. (30)–(32) is that the operator $\Phi(t)$ never appears on the right-hand sides. Thus its dimensionless coupling

$$u \equiv gR^{(1-3d/2)/\chi} \tag{33}$$

will not be renormalized and obeys the trivial renormalization flow equation

$$\dot{u} = (1-3d/2)u. \tag{34}$$

On the other hand, the dimensionless coupling constant

$$v \equiv hR^{\epsilon/\chi} \tag{35}$$

of $\Omega(t)$ with

$$\epsilon \equiv 1 - x_\Omega = 1 - 2d \tag{36}$$

has to be renormalized in order to absorb the short distance singularities in the operator algebra Eqs. (30)–(32). The three ways to obtain the operator $\Omega(t)$ in Eqs. (30)–(32)
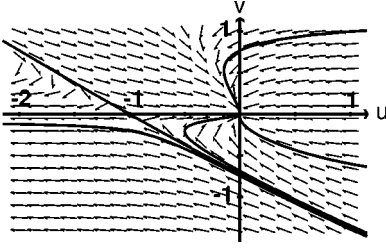
FIG. 2. Renormalization group flow of contact interactions for a semiflexible polymer in $(1+1)$ dimensions. The flow equations (34),(37) have the single unstable fixed point $(u,v)=(-1,0)$ marking the delocalization transition.

then lead in the usual way [11] to the three quadratic terms in the one-loop renormalization group equation

$$\dot{v} = \epsilon v - v^2 - u^2 - cuv \qquad (37)$$

for the dimensionless coupling v where c is an undetermined constant.

The corresponding flow diagram for $d=1$ is shown in Fig. 2. The unique delocalization fixed point $(u^*=0,v^* =\epsilon)$ is on the line $u=0$. This property ensures that the constant c in Eq. (37) drops out of the critical exponents. It is a simple consequence of the fact that the field $\Phi(t)$ is not renormalized at all and is irrelevant in any dimension $d >2/3$. Thus, this feature will be preserved also at higher orders. As long as $d>2/3$, u will be driven toward zero by the renormalization group flow and the operator $\Phi(t)$ does not play any role in identifying the critical behavior of the system.

The remaining perturbation series at $u=0$, however, is factorizable according to Eq. (26) and one-loop renormalizable in exactly the same way as with the wall constraint. Hence, the (in $D=1$) exact relations Eqs. (24) and (25) still hold [with ϵ given by Eq. (36) and $x_\Omega=2d$], resulting in $\xi \sim (T_c-T)^{3/(2-4d)}$ for $2/3<d<1$ and $x_\Omega^*=2-2d$. This scaling dimension turning negative for $d>1$ indicates that the transition becomes of first order; see the discussion and extensive numerics in Ref. [30]. An analogous first-order regime is known for flexible polymers [31].

V. SEMIFLEXIBLE MANIFOLDS IN A RANDOM MEDIUM

We will now turn our attention to semiflexible manifolds in a random medium. We describe this medium by an uncorrelated Gaussian potential $V(\mathbf{r},\nabla\mathbf{r})$ characterized by its first two moments as

$$\overline{V(\mathbf{r},\nabla\mathbf{r})} = 0, \qquad (38)$$

$$\overline{V(\mathbf{r},\nabla\mathbf{r})V(\mathbf{r}',\nabla\mathbf{r}')} = \sigma\delta(\mathbf{r}-\mathbf{r}'), \qquad (39)$$

where the bar denotes the average over the ensemble of disorder configurations. We will apply the results of the last section to this problem in the case $D=1$ of a semiflexible polymer. The latter system has a possible biostatistical application in the theory of sequence alignment [32].

The critical behavior of a semiflexible polymer in such a random potential can be obtained using the replica trick. To this end, a system of N replicas in the same potential has to be studied and finally the limit $N\to 0$ has to be considered. The (annealed) disorder average of the N replica system is easy to perform and yields a system of N semiflexible polymers with an attractive contact interaction between each pair of them. This contact interaction is given by the correlation Eq. (39) of the random potential. For the uncorrelated potential of interest here, this amounts to pairwise interactions as described by the short-range interaction operator $\Phi(t)$.

If the pairwise interaction operator is given by the orientation dependent operator $\Omega(t)$, the arguments from the last section can be immediately generalized to the case of N semiflexible polymers. The time-ordered perturbation series of this N polymer problem can be mapped term by term onto the perturbation series of *flexible* polymers with Φ interactions. Its leading divergences are known to be due to "ladder" diagrams with the same pair of polymers interacting at subsequent points t_i [33,34].

The presence of Φ interactions for semiflexible polymers does not change these singularities by the same argument as for $N=2$. Due to their stiffness, any two semiflexible polymers interacting twice in a short interval have to be parallel to each other, or, in other words, the leading divergent diagrams behave like diagrams involving only Ω operators. For the Ω system, however, the results of [33,34] immediately carry over and imply that the critical behavior at the delocalization transition does not depend on N. In particular, the random limit of vanishing N becomes trivial. We conclude that a $(1+d)$-dimensional semiflexible polymer in a random potential has a phase transition between a weak and a strong coupling phase at a critical strength of the randomness for $d>2/3$. This phase transition corresponds to the roughening transition of the Kardar-Parisi-Zhang equation in $4d$ dimensions. For fluid membranes, on the other hand, simple power counting shows that an arbitrarily small amount of disorder is relevant and leads to a strong coupling phase.

VI. CONCLUSIONS

In this paper we have established a unified conceptual framework for the description of the unbinding transitions of semiflexible manifolds. The main new ingredient is the identification of a short-ranged interaction field, which depends on the orientation of the manifold. The one-loop renormalization group treatment of this interaction field is exact in the case of the unbinding transition of semiflexible polymers. For fluid membranes our one-loop results are no longer exact but they coincide very well with previous numerical estimates. The origin of this remarkably good agreement further deserves a more detailed investigation. This requires a systematic study of the higher-order corrections, which is possible in the field-theoretic formalism developed here. The nonperturbative aspects of fluid membranes with quenched randomness also deserve future attention.

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