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Universal Aspects of Interacting Lines and Surfaces

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Contents

1 Interfaces, strings and membranes

The critical behavior of interfaces is related to their reduced dimensionality. [1] In some cases, the interface can simply be viewed as a planar 2–dimensional system. However, it can also 'escape' into the third dimension and then attain nonplanar morphologies. This *roughening* of the interface can be thermally excited or induced by frozen randomness. In addition, the

interface has a certain depth profile and thus has itself a third dimension. This intrinsic thickness of the interface can become mesoscopic as in *wetting* phenomena [1, 2]: one then has a thin layer which is bounded by two interfaces. The thickening of this layer leads to the *unbinding* of these two interfaces, see fig. 1.

These critical effects are not restricted to three dimensions. Indeed, roughening, wetting and general unbinding phenomena also occur in 2–dimensional systems where they are governed by the behavior of 1–dimensional domain boundaries. [3] Since these domain boundaries are governed by a finite line tension, their statistical mechanics is intimately related to other 1–dimensional lines or strings such as (i) steps or ledges on crystal surfaces, (ii) stretched (or directed) polymers, and (iii) vortex lines in superconductors.

It turns out that these 1–dimensional strings have scaling properties which are very similar to those of 2–dimensional membranes, i.e., thin sheets of molecules. [4] The most prominent examples of such membranes are bilayers of amphiphilic molecules which represent model systems for the rather complex membranes of biological systems. [4, 5] These membranes are also roughened by thermally–excited shape fluctuations. In addition, the adhesion and unbinding of membranes can be understood in close analogy to interfacial wetting, see fig. 1. Adsorption–desorption transitions of polymers [6] are a related unbinding phenomenon.

The unbinding of strings and surfaces is often driven by their shape fluctuations which *renormalize* their direct interaction arising from intermolecular forces. For thermally–excited fluctuations, this renormalization acts to increase the repulsive part of the interaction. At low temperatures, these fluctuations are weak and the renormalized interaction closely resembles the direct interaction. However, as the temperature T is increased, the renormalization becomes more and more effective up to a characteristic unbinding temperature, $T = T_*$, at which the manifolds undergo an transition from a bound to an unbound state.

The critical behavior at these unbinding transitions involves several diverging length scales such as the *mean separation* or the *roughness* of these manifolds . In addition, other quantities such as the *probability for local contacts* are also singular at these transitions. This quantity represents a convenient starting point for a systematic field-theoretic treatment of these transitions [7].

This article reviews recent theoretical work on these critical phenomena

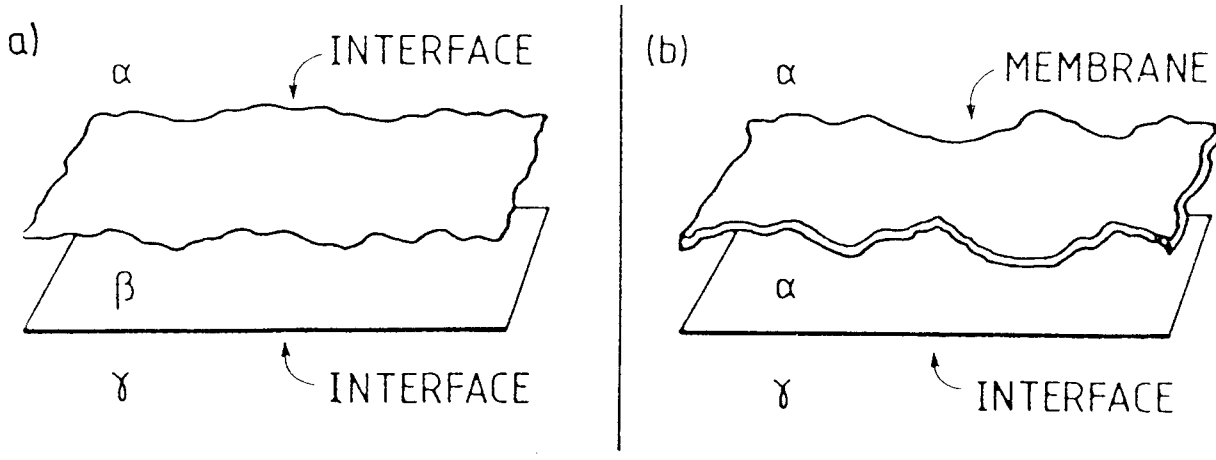

Fig. 1. (a) Wetting layer of phase β between two bulk phases α and γ. The thickening of this layer corresponds to the unbinding of the two interfaces bounding the layer; and (b) Adhesion of a flexible membrane consisting of a thin layer of molecules towards another interface. The shape fluctuations of the membrane act to unbind the two surfaces.

[7-14]. It is organized as follows. The scaling behavior of interacting manifolds and theoretical models for these systems are briefly reviewed in Sect. 2 and Sect. 3, respectively. Then, in Sect. 4, a new scaling picture is described [12] in which the probability of *local contacts* between the interacting manifolds plays a prominent role. This scaling picture is developed in some detail for the case of strings interacting via short–ranged or long–ranged potentials. In this context, we discuss in Sect. 4.7 the unbinding transition of bundles of nonintersecting strings in the transfer matrix approach [8, 9, 10, 13]. Sect. 4.8 contains extensions of the scaling picture to other systems.

The scaling picture can be justified in a systematic way by applying continuum field theory to these systems [7]. The central idea is to treat all interactions as *local operators* which form an operator algebra characterizing their universal short-distance properties. In this article, we do not assume any knowledge of field-theoretic renormalization; Sect. 5 can also be read as a self-contained introduction to some ideas in this subject. In Sect. 6 we apply these methods to a number of more difficult problems in the context of interfaces [7, 11, 14]: strings with long-ranged interactions, systems of many strings (which may be "bosonic" or "fermionic"), and interfaces of general dimensionality. In all cases, we find a number of nontrivial universality classes. We conclude this section with a brief outlook.

2 Scaling behavior of interacting manifolds

In this section, we will introduce the various quantities which are singular at roughening, wetting and unbinding transitions and define the corresponding critical exponents.

2.1 Roughness exponent ζ

As mentioned, low-dimensional manifolds are often rough, i.e., they make large transverse excursions from their mean or average position. More precisely, such a manifold is rough if the typical size, $\xi_\perp$, of its transverse excursions grows with its lateral size, $\xi_\parallel$. This behavior can usually be described by the scaling law

$$\xi_\perp \sim \xi_\parallel{}^{\zeta} \tag{2.1}$$

which defines the *roughness* exponent ζ. For interfaces and domain boundaries, the universality classes for this exponent are primarily determined by the *symmetry* of the two bulk phases adjacent to the interface.

2.2 Roughening exponent $\nu_\perp$

The roughness of the manifold can change as a function of temperature or some other control parameter. For example, the 2-dimensional interface between a periodic crystal and its vapor is smooth at low temperatures T with $\xi_\perp$ confined by the lattice potential. As the temperature is increased, this confining potential becomes less and less effective and the interface becomes delocalized up to a critical temperature T_* at which it undergoes a roughening or delocalization transition. For $T > T_*$, the interface is rough at all scales. Likewise, a domain boundary may be localized at low temperature by a defect line (in $D = 2$) or by a defect plane (in $D = 3$) but may become delocalized at sufficiently high temperatures.

As the critical temperature, T_*, is approached from below, the roughness $\xi_\perp$ typically grows as

$$\xi_\perp \sim 1/(T_* - T)^{\nu_\perp} \tag{2.2}$$

which defines the *roughening exponent* $\nu_\perp$. In general, this exponent depends on the nature of the effective potential confining the interface and one must distinguish several universality classes or scaling regimes of these potentials.

2.3 Unbinding exponent ψ

Now consider two interacting manifolds with local separation l. If these manifolds undergo an unbinding transition, their mean separation $\langle l \rangle$ diverges and grows as

$$\ell \equiv \langle l \rangle \sim 1/(T_* - T)^{\psi} \tag{2.3}$$

which defines the unbinding exponent ψ. This exponent in general also depends on the nature of the interaction potential experienced by the two manifolds.

In most cases of interest, the manifolds become rough or delocalized as they unbind. In fact, we will be primarily interested in situations in which the unbinding is driven by the roughening of the manifolds. One may then consider the roughness of the local separation field l which is defined by $\xi_\perp \equiv \langle [l - \langle l \rangle]^2 \rangle^{1/2}$. If the unbinding is driven by the shape fluctuations, one has $\xi_\perp \sim \ell$, and the unbinding exponent ψ is equal to the roughening exponent $\nu_\perp$ (there is one exceptional case where short–ranged attractive potentials compete with repulsive potentials of 'intermediate range', see Sect. 4.6).

2.4 Contact exponent ζ_0

Another quantity which exhibits singular behavior at roughening and unbinding transitions is the probability $\mathcal{P}_b$ of locally bound segments, i.e., of local contacts between the interacting manifolds. As discussed in some detail in the following sections, this quantity is quite generally given in terms of the one-point function of a local operator Φ and vanishes as

$$\mathcal{P}_b \sim \langle \Phi \rangle \sim {\xi_\parallel}^{-\zeta_0} \sim {\xi_\perp}^{-\zeta_0/\zeta} \quad . \tag{2.4}$$

close to criticality [7] * This defines the contact exponent ζ_0.

The simplest situation is exemplified by an interface characterized by a Gaussian probability distribution $\exp(-l^2/2\xi_\perp^2)/\xi_\perp$ for the fluctuating field l. In this case, we have $\mathcal{P}_b \sim 1/\xi_\perp$, and hence the contact exponent ζ_0 equals ζ.

*A precise definition of "locally bound segments" must distinguish between "bosonic" and "fermionic" systems, see eqns. (5.4) and (6.26) below. The exponent ζ_0 will be called x in Sect. 5.

In general, two cases must be distinguished. If the attractive part of the potential is sufficiently short–ranged, the exponent ζ_0 is related to the roughening exponent $\nu_\perp$ via a scaling relation, see the relations (4.19) and (4.27) below. On the other hand, if the bound state of the manifold is controlled by an attractive potential which is sufficiently long-ranged, the exponent $\nu_\perp$ is determined by this long–ranged potential. The the exponent ζ_0 may still governed by a repulsive short–ranged potential, in which case the two exponents are independent.

3 Effective models for interacting manifolds

In the continuum limit, the position of each fluctuating manifold can be described by a displacement field $l = l(s)$ where s is a $d_\parallel$–dimensional coordinate parallel to a reference plane. For roughening or delocalization phenomena, the field l gives the distance of the manifold from this reference plane; for the unbinding of two interacting manifolds, this field measures the separation of these two manifolds.

The *effective Hamiltonian* for the displacement field l has the generic form [1]

$$\mathcal{H}\{l\} = \mathcal{H}_0\{l\} + \int \mathcal{V}[l(s)] d^{d_\parallel} s \tag{3.1}$$

where $\mathcal{H}_0\{l\}$ represents the elastic energy of the shape fluctuations in the completely unbound state and $\mathcal{V}(l)$ is an effective potential which acts to localize these shape fluctuations. $\mathcal{H}$ will also be called the *action* for the field l in order to avoid confusion with the corresponding Hamilton operator $\hat{H}$, the infinitesimal generator of the transfer matrix.

For roughening and delocalization phenomena, the potential $\mathcal{V}(l)$ can describe the effect of an underlying lattice which may be periodic [15] or quasi–periodic [16] Below, we will discuss the influence of a defect line (or defect plane) which acts to localize the interface. In this latter case, the potential $\mathcal{V}(l)$ is taken to be symmetric and to have local minima at the position of the defect and for $l = \pm\infty$. [7]

For wetting and adhesion phenomena, the potential $\mathcal{V}(l)$ describes the interaction energy of the two manifolds at separation l. Usually, the two manifolds cannot intersect one another, and this interaction potential contains a hard wall at $l = 0$ which ensures that the displacement field satisfies

$l \geq 0$.

In Sections 4.7 and 6.2, we will also study the case of many interacting lines and thus of a large number of displacement fields.

4 A refined scaling picture for unbinding phenomena

A localized manifold can be regarded as an ensemble of essentially uncorrelated humps, see fig. 2 below. This view leads to the concept of a fluctuation–induced interaction $\mathcal{V}_{fl}$ between the manifolds. In the case of thermally–excited fluctuations, $\mathcal{V}_{fl}$ represents the loss of entropy arising from the confinement. This fluctuation–induced interaction can be used in a heuristic way in order to understand the critical behavior at unbinding transitions.

It has been previously emphasized that a simple superposition of $\mathcal{V}_{fl}$ and the direct interaction $\mathcal{V}(l)$ does not predict the correct critical behavior unless the interaction $\mathcal{V}(l)$ is sufficiently long–ranged. [17] Here, a refined scaling picture [12] based on a *two–state model* for the interacting segments of the manifolds is described which is appropriate for any type of interaction potential. This scaling picture can be justified in a systematic way in the field-theoretic framework [7] described in Sections 5 and 6 below. A crucial role is played by the probability that two segments of the interacting manifolds form locally bound pairs.

In the following section, the refined scaling picture will be first described for 1–dimensional strings governed by a finite line tension. The case of *wetting in two dimensions*, i.e., of two strings in $D = 1 + 1$ dimensions interacting via general pair potentials is discussed in some detail. For the special case of attractive square–well potentials, similar scaling ideas have been previously formulated for the so–called reflection model in Ref. [18]. The extension of the refined scaling picture to other types of manifolds is briefly described in Sect.4.7 and 4.8. The same picture can be formulated for bundles and bunches of N manifolds where it leads to a N–state model. [12]

Now consider two interacting strings in $1+1$ dimensions with line tensions σ_1 and σ_2, respectively. The action (or effective Hamiltonian) for their local

separation (or relative displacement field) l is given by

$$\mathcal{H}\{l\} = \int \{ \frac{1}{2}\sigma(dl/ds)^2 + \mathcal{V}[l(x)] \} ds \tag{4.1}$$

with the reduced line tension $\sigma = \sigma_1\sigma_2/(\sigma_1+\sigma_2)$. If one string has an infinite stiffness, say $\sigma_2 = \infty$, corresponding to a straight rigid boundary, one has $\sigma = \sigma_1/2$.

This model can be analysed in much detail by transfer matrix methods, and one can obtain the exact critical behavior for many potentials $\mathcal{V}(l)$. From these latter results, one knows that there are several universality classes for the unbinding transition which depend on the long–ranged part of the interaction potentials. As shown below, the refined scaling picture is valid for all of these universality classes.

4.1 Two–state model for interacting strings

In general, the direct interaction potential $\mathcal{V}(l)$ will contain a short–ranged part and a longer–ranged part, which will be denoted by $\mathcal{V}_b(l)$ and $\mathcal{V}_{ub}(l)$, respectively (the indices b und ub will become clear in a moment). It will be convenient to introduce a microscopic length scale l_b and to define these two parts of the potential via

$$\mathcal{V}_b(l) \equiv \mathcal{V}(l) \qquad \text{for} \quad l < l_b \tag{4.2}$$

and

$$\mathcal{V}_{ub}(l) \equiv \mathcal{V}(l) \qquad \text{for} \quad l > l_b \quad , \tag{4.3}$$

respectively.

If the interaction potential contains an *attractive* short–ranged part, the scale l_b is given by the potential range of this attractive part. If the short–ranged potential is purely repulsive, the choice of l_b is somewhat arbitrary but it should be small compared to the length scales which enter the long–ranged part. In any case, the short–ranged part will contain the hard wall interaction which ensures that the two manifolds cannot intersect.

Two string segments which interact via such a potential can attain two different local states, see fig. 2: (i) They are locally unbound if their separation exceeds the length scale l_b; and (ii) They form a locally bound pair if

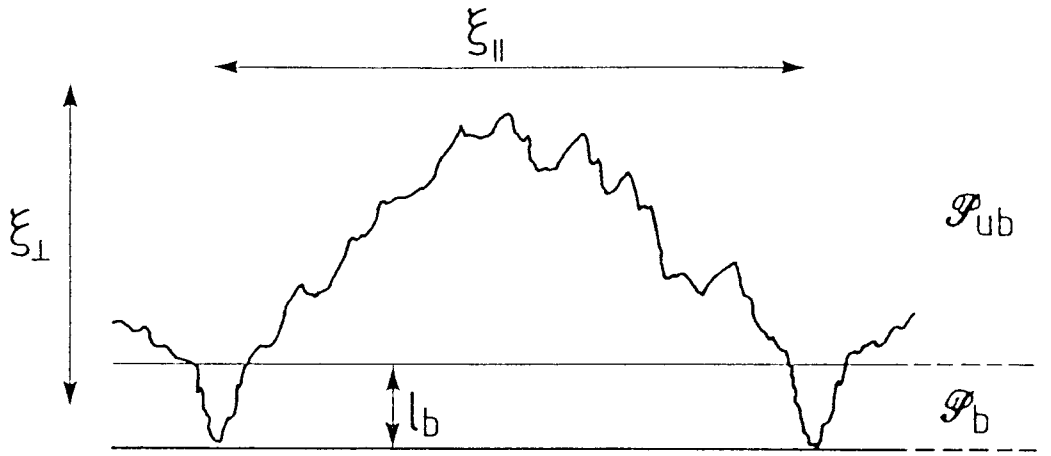

Fig. 2. The bound state of two strings consists of humps which have lateral and transverse extension $\xi_{\|}$ and $\xi_{\perp}$, respectively. The microscopic scale l_b represents the range of the short-ranged part of the interaction potential. Two adjacent segments of the two strings are locally bound and unbound with probability $\mathcal{P}_b$ and $\mathcal{P}_{ub}$, respectively.

their separation is smaller than l_b. The probabilities for these two different local configurations will be denoted by $\mathcal{P}_{ub}$ and $\mathcal{P}_b$, respectively.

Note that even if the string segments are locally unbound, they still have a finite separation. Therefore, compared to the situation in which the strings are completely separated, locally unbound segments have an excess free energy ΔF_{ub} per unit length. Likewise, the excess free energy per unit area of a locally bound pair will be denoted by ΔF_b. Thus, the excess free energy per unit length of the two strings can be estimated as

$$\Delta F = \Delta F_{ub}\, \mathcal{P}_{ub} + \Delta F_b\, \mathcal{P}_b \quad . \tag{4.4}$$

If the unbinding transition is continuous, both the excess free energy ΔF_{ub} of the locally unbound segments and the probability $\mathcal{P}_b$ for locally bound pairs must vanish in a continuous way whereas $\mathcal{P}_{ub} = 1 - \mathcal{P}_b \approx 1$ as the transition is approached. In addition, all critical quantities should scale with a single length scale which is here taken to be the roughness $\xi_{\perp}$ of the string separation. Since $\xi_{\perp}$ diverges at the transition, one anticipates that both ΔF_{ub} and $\mathcal{P}_b$ scale as inverse powers of $\xi_{\perp}$.

The excess free energy ΔF_b for bound segments, on the other hand, arises from configurations which have a separation of the order of the microscopic length scale l_b and, thus, will not depend on the diverging scale $\xi_{\perp}$.

4.2 Excess free energy of locally unbound segments

As indicated in fig. 2, the fluctuations in the string separation l can be regarded as an ensemble of humps which have the typical height $\xi_\perp$ and the longitudinal extension $\xi_\parallel$. Since the string separation is governed by a line tension, it diffuses like a (directed) random walk. Therefore, the two length scales $\xi_\perp$ and $\xi_\parallel$ satisfy the scaling relation

$$\xi_\perp \sim (T/\sigma)^{1/2}\, {\xi_\parallel}^{1/2} \,, \tag{4.5}$$

i.e., the roughness exponent has the value $\zeta = 1/2$.

Each hump of longitudinal and perpendicular extension $\xi_\parallel$ and $\xi_\perp$ has the volume $\mathcal{V} \simeq \xi_\parallel \xi_\perp$. Assuming that these humps are essentially uncorrelated and using the ideal gas law $PV = T$ for a single degree of freedom together with the relation (4.5), one arrives at the pressure $P \sim T^2/\sigma {\xi_\perp}^3$. Alternatively, one may allude to the equipartition theorem and postulate that each such hump has a free energy $\sim T$. This implies that the hump free energy per unit (projected) area behaves as

$$\mathcal{V}_{fl}(\xi_\perp) \sim T/\xi_\parallel \sim T^2/\sigma\xi_\perp^2 \qquad \text{for large} \quad \xi_\perp \quad . \tag{4.6}$$

The disjoining pressure is now obtained from $P = \partial \mathcal{V}_{fl}/\partial \xi_\perp$. This estimate of the excess free energy of interacting strings is implicit in the work of Gruber and Mullins on steps or ledges on crystal surfaces [19] and has been explicitly derived by Prokovsky and Talapov for commensurate–incommensurate transitions in two dimensions [20].

Thus, the locally unbound segments suffer a loss of entropy per unit length which is given by $\mathcal{V}_{fl}(\xi_\perp)$ and which represents one contribution to the excess free energy ΔF_{ub}. In addition, these segments also have an interaction energy $\mathcal{V}_{ub}(l)$. It is plausible to assume and it can be checked a posteriori that the mean separation $\ell \equiv \langle l \rangle$ is proportional to $\xi_\perp$ close to the transition. In such a situation, the excess free energy of the locally unbound segments can be estimated as

$$\Delta F_{ub} = c_1 T^2/\sigma\xi_\perp^2 + \mathcal{V}_{ub}(c_2\xi_\perp) \quad . \tag{4.7}$$

Thus, for all long–ranged interactions which decay faster than $\sim 1/l^2$ for large l, one has $\Delta F_{ub} \sim 1/\xi_\perp^2$.

4.3 Scaling form for the probability distribution

Next, let us address the dependence of the probability $\mathcal{P}_b$ for locally bound segments on the roughness scale $\xi_\perp$. In general, the probability distribution $\mathcal{P}(l)$ for the string separation l should have the scaling form

$$\mathcal{P}(l) \approx \Omega(l/\xi_\perp)/\xi_\perp \qquad \text{for} \quad l \gg l_b \tag{4.8}$$

where the explicit factor $1/\xi_\perp$ arises from normalization. The probability $\mathcal{P}_b$ can then be estimated as

$$\mathcal{P}_b \simeq l_b \mathcal{P}(l_b) = (l_b/\xi_\perp)\Omega(l_b/\xi_\perp) \quad . \tag{4.9}$$

Therefore, the probability $\mathcal{P}_b$ is determined by the behavior of $\Omega(s)$ for small s.

For strings in two dimensions, the probability distribution $\mathcal{P}(l)$ is given in terms of the ground state wavefunction of the transfer matrix operator which can be explicitly calculated for many potentials. The results of these calculations will be described in the following subsections 4.5 and 4.6. In all cases, one finds the scaling behavior

$$\Omega(s) \sim s^{2\zeta_0 - 1} \qquad \text{for small } s \qquad \text{with } \zeta_0 > 0 \tag{4.10}$$

provided (i) the transition is continuous and (ii) the mean separation $\ell \sim \xi_\perp$ as assumed here (exceptions occur for interaction potentials with a long–ranged *repulsive* part which decays to zero not faster than $\sim 1/\xi_\perp^2$). The relation (4.10) implies that the probability for locally bound segments behaves as

$$\mathcal{P}_b \sim 1/{\xi_\perp}^{2\zeta_0} \sim 1/{\xi_\parallel}^{\zeta_0} \quad . \tag{4.11}$$

4.4 Competition between locally bound and unbound segments

Now, we can insert the two relations (4.11) and (4.7) into the expression (4.4) for the excess free energy in order to arrive at the estimate

$$\Delta F \approx c_1 T^2/\sigma\xi_\perp^2 + \mathcal{V}_{ub}(c_2\xi_\perp) + c_3 \Delta F_b/{\xi_\perp}^{2\zeta_0} \tag{4.12}$$

in the limit of large $\xi_\perp$ where c_1, c_2 and c_3 are dimensionless and positive coefficients.

The continuous unbinding of the two strings now correponds to a minimum of ΔF with respect to $\xi_\perp$ which goes continuously to infinity as the temperature or some interaction parameters are varied and the unbinding transition is approached.

Since the first term in (4.12) is positive, either the second or the third term must be negative in order to have a minimum of ΔF at a finite value of $\xi_\perp$, i.e., in order to have a bound state of the two strings. The third term, $\Delta F_b \, \mathcal{P}_b$, for locally bound segments, involves the exponent ζ_0 which has not been specified so far. A balance of this term with the two other terms in (4.12) shows that two cases must be distinguished: (i) For $0 < \zeta_0 < 1$, the third term dominates provided the long–ranged part $\mathcal{V}_{ub}(l)$ does not decay more slowly than $\sim 1/l^2$. The unbinding transition then occurs as ΔF_b goes to zero from below, and the critical behavior of $\xi_\perp$ is directly related to the probability $\mathcal{P}_b$; and (ii) For $1 < \zeta_0 < \infty$, a bound state is only possible for an attractive interaction $\mathcal{V}_{ub}(l) < 0$ which grows for large l or decays more slowly than $\sim 1/l^2$. In this case, unbinding occurs as this long–ranged attractive part goes to zero, and the corresponding critical behavior of $\xi_\perp$ is not affected by the probability $\mathcal{P}_b$.

4.5 Strong–fluctuation regime

The so–called strong–fluctuation regime consists of all interaction potentials which decay faster to zero than $\sim 1/l^2$ for large l. First, consider the case of an *attractive* square–well potential of depth $|U|$ and range l_b. In this case the longer–ranged part $\mathcal{V}_{ub}$ is identically zero. The excess free energy for bound pairs can be estimated as $\Delta F_b \simeq -|U| + cT^2/\sigma {l_b}^2$ where the first and the second term represent the interaction energy and the entropy loss within the square well, respectively. In addition, transfer matrix calculations show that the probability distribution $\mathcal{P}(l)$ exhibits the scaling form (4.8) with $\Omega(s) \sim e^{-s}$. Since $\Omega(s) \sim$ const for small s, one has

$$\zeta_0 = 1/2 \qquad \text{and} \quad \mathcal{P}_b \sim 1/\xi_\perp \quad . \tag{4.1}$$

If these expressions are inserted into the excess free energy ΔF, minimization with respect to $\xi_\perp$ leads to

$$\xi_\perp \sim 1/|\Delta F_b|^{\nu_\perp} \qquad \text{with} \quad \nu_\perp = 1 \tag{4.}$$

which is indeed the correct critical behavior for the unbinding transition within a square–well potential. [21, 22]

The same critical behavior applies to all potentials within the strong–fluctuation regime, i.e., to all potentials with a tail $\mathcal{V}_{ub}(l)$ which decays faster than $\sim 1/l^2$. [23] This can be understood by inspection of the expression (4.12) for the excess free energy: the tail $\mathcal{V}_{ub}(c\xi_\perp)$ is irrelevant compared to the short–ranged part $\Delta F_b\, \mathcal{P}_b \sim \Delta F_b/{\xi_\perp}^{2\zeta_0}$ provided the exponent ζ_0 still has the value $\zeta_0 = 1/2$. Thus, these interaction potentials are short–ranged even though there tails decay as inverse powers.

On the other hand, if the short–ranged potential is *repulsive*, field–theoretic renormalization group calculations [7, 11], described in Sect.5 below, and exact transfer matrix calculations [13] yield

$$\zeta_0 = 3/2 \qquad \text{and} \quad \mathcal{P}_b \sim 1/{\xi_\perp}^3 \quad . \tag{4.15}$$

Since this probability decays faster than the entropy loss $\sim 1/\xi_\perp^2$ and since ΔF_b is always positive, the term $\Delta F_b\, \mathcal{P}_b$ does not affect the minimum of ΔF and thus does not affect the critical behavior of $\xi_\perp$.

For example, one may confine the strings by a potential $\mathcal{V}_{ub}(l) \sim l$ or $\sim l^2$ and consider the limit in which the amplitude of such a potential goes to zero. In fact, the same value $\zeta_0 = 3/2$ applies to all attractive long–ranged potentials which decay more slowly than $\sim 1/l^2$. The marginal case $\mathcal{V}_{ub} \sim 1/l^2$, on the other hand, is more complex since it leads to a nonuniversal value for ζ_0 as discussed in the next subsection.

4.6 Intermediate fluctuation regime

Now, consider interaction potentials which behave as

$$\mathcal{V}(l) \approx W/l^2 \text{ for large } l \tag{4.16}$$

which defines the so–called intermediate fluctuation regime. [24] In this case, the behavior of the probability distribution $\mathcal{P}(l) = \Omega(l/\xi_\perp)/\xi_\perp$ for small l depends explicitly on the dimensionless parameter

$$w = 2\sigma W/T^2 \tag{4.17}$$

where σ and T are the string tension and the temperature, as before.

In fact, when considered as a function of w, the exponent ζ_0 has two branches depending on the sign of the short-ranged part of the interaction potential. For *attractive* short-ranged potentials, the probability distribution can be calculated using the results of Ref. [24]. As a result, one finds the singular behavior

$$\Omega(s) \sim s^{2\zeta_0 - 1} \quad \text{with} \quad \zeta_0 = 1 - \sqrt{w + 1/4} \tag{4.18}$$

for small s. If these values for ζ_0 are used in the expression for ΔF, minimization of this excess free energy leads to

$$\xi_\perp \sim 1/|\Delta F_b|^{\nu_\perp} \quad \text{with} \quad \nu_\perp = 1/(2 - 2\zeta_0) \tag{4.19}$$

which is again the correct critical behavior at the unbinding transitions. The case of a short-ranged potential is recovered for $w = 0$ with $\zeta_0 = 1/2$ and $\nu_\perp = 1$. Thus, the roughening exponent $\nu_\perp$ and the contact exponent ζ_0 are not independent but satisfy a scaling relation in this case.

The critical behavior as given by (4.19) applies to $-1/4 < w < 3/4$. For $w < -1/4$, the attractive potential $\mathcal{V}_{ub}(l) \sim 1/l^2$ is so strong, that the two strings cannot unbind. For $w > 3/4$, on the other hand, one has a relatively large potential barrier, and one then enters the so-called subregime (C) in which the probability distribution $\mathcal{P}(l)$ has the scaling from $\mathcal{P}(l) = (l_b/\xi_\perp)^{2\mu-1}\, \Omega(l/\xi_\perp)$ with $\mu \equiv \sqrt{w + 1/4}$ and $\Omega(s) \sim s^{1-2\mu}$ for small s. [25] This implies that the probability distribution $\mathcal{P}(l)$ attains the limiting form $\mathcal{P}(l) \sim l^{1-2\mu}$ and $\mathcal{P}_b = \mathcal{P}(l_b) \approx$ const at the unbinding transition.

The interactions within the intermediate fluctuation regime have also been studied by functional renormalization. [26, 27, 28] As a result, one finds a parabolic renormalization group flow with a whole line of fixed points. The fixed point line has two branches corresponding (i) to critical wetting transitions in the presence of attractive short-ranged potentials, and (ii) to completely wet states for *repulsive* short-ranged potentials. The latter branch is governed by completely repulsive fixed points at which the short-ranged potentials represent irrelevant perturbations. The corresponding scaling index depends on w. In two dimensions, one may set up an exact functional renormalization group (RG) in which the transfer matrix is diagonalized in an iterative manner. The scaling indices obtained from numerical iterations of this RG transformation [27] imply the contact exponent

$$\zeta_0 = 1 + \sqrt{w + 1/4} \quad . \tag{4.20}$$

The case of short–ranged potentials is again recovered for $w = 0$ with $\zeta_0 = 3/2$. Thus, if the strings experience an effectively repulsive short–ranged potential, the probability for local contacts is more and more suppressed with increasing w or W.

It is interesting to note that the intermediate fluctuation regime characterized by the contact exponents as given in (4.19) and (4.20) also applies to several other string systems if one makes an appropriate identification of the parameter w. First of all, the same critical behavior is found if the interaction potential $\mathcal{V}(l)$ is symmetric and does not contain a hard wall. [29] Such a potential would arise, e.g., for a domain boundary with stiffness σ_1 which interact with a plane of defects. In this case, w is still given by (4.17) with $\sigma = \sigma_1/2$.

Secondly, two strings in $D = 1 + d_\perp$ dimensions interacting with *short–ranged* interaction potentials belong to this intermediate regime. In this case, one has $w = (d_\perp - 3)(d_\perp - 1)/4$ [25] and thus

$$\zeta_0(d_\perp) = 1 \pm |d_\perp - 2|/2 \quad . \tag{4.21}$$

Thus, there are two branches for the contact exponent with $\zeta_0 = d_\perp/2$ and $\zeta_0 = 2 - d_\perp/2$, respectively, which cross at $d_\perp = 2$. The branch with $\zeta_0 = d_\perp/2$ represents effectively Gaussian fluctuations with $\mathcal{P}(l) \sim \exp[-l^2/2{\xi_\perp}^2]/{\xi_\perp}^{d_\perp}$.

For $1 \leq d_\perp \leq 2$, the critical unbinding transition and the completely unbound state are characterized by $\zeta_0 = d_\perp/2$ and by $\zeta_0 = 2 - d_\perp/2$, respectively. For $d_\perp \geq 2$, the two branches have exchanged and the critical unbinding transition now corresponds to $\zeta_0 = 2 - d_\perp/2$. The latter value is valid up to $d_\perp = 4$; for $d_\perp > 4$, one has $w > 3/4$ and thus enters the so–called subregime (C) as explained above.

Thirdly, the necklace models that we discuss in Sect. 4.7 can also be mapped onto the intermediate fluctuation regime.

4.7 Necklace models and nonintersecting strings

Consider the necklace model for three strings with line tensions σ_1, σ_2 and σ_3 [30, 8]: the strings interact via a hard-wall pair potential, which ensures that they cannot intersect, and via a short-ranged attractive 3–body force. Thus, the two outer strings experience the 3–body force and an effective repulsion arising from the confinement of the interior string. The entropy loss of the

interior string behaves as $\sim 1/l^2$ which implies that the effective repulsion between the two outer strings scales in the same way.

This necklace model is characterized by the parameter $w = (\pi/\theta)^2 - 1/4$ with $\tan(\theta) = \sqrt{(\sigma_2/\sigma_1) + (\sigma_2/\sigma_3) + (\sigma_2{}^2/\sigma_1\sigma_3)}$ and $0 \leq \theta \leq \pi/2$. [8] It then follows from (4.18) and (4.19) that $\zeta_0(3) = 1 \pm \pi/\theta$. The minus sign corresponds to unbinding transitions in the presence of an effectively attractive 3–body force. However, since the minus sign leads to $\zeta_0 < 0$ corresponding to $w > 3/4$, these transitions belong to subregime (C) for which $\zeta_0 = 0$. On the other hand, if the short–ranged 3–body force between the three strings is effectively repulsive, this system is characterized by the contact exponent

$$\zeta_0(3) = 1 + \pi/\theta \quad . \tag{4.22}$$

Thus, if one keeps the three strings together by an external pressure or by some other long–ranged potential, the probability $\mathcal{P}_{3b}$ that all 3 strings form a local bound state behaves as $\mathcal{P}_{3b} \sim 1/\xi_\parallel{}^{\zeta_0(3)}$. As the line tension σ_2 of the interior string decreases, the angle θ decreases and the contact exponent ζ_0 increases. This is rather intuitive: as the interior string fluctuates more strongly, local contacts between the two outer strings become less likely.

The necklace model for m identical strings as described in Refs. [31, 32] also belongs to the intermediate fluctuation regime [33]. In this latter model, the strings again experience hard wall potentials between nearest neighbors and thus do not intersect whereas their attractive interaction is restricted to a short–ranged m–body potential. In this case, the parameter w has the value $w = [(m^2-3)^2 - 1]/4$ [33] and the two relations (4.18) and (4.19) for ζ_0 lead to $\zeta_0(m) = 1 \pm |m^2 - 3|/2$. For $m = 2$, one recovers the contact exponents for two strings and $w = 0$: there are no interior strings in this case and, therefore, there is no effective repulsion $\sim 1/l^2$.

For $m \geq 3$, on the other hand, the branch of $\zeta_0(m) = 1 \pm |m^2 - 3|/2$ with the minus sign corresponds to critical transitions in the presence of attractive m–body forces which again belong to subregime (C). Likewise, the branch with the plus sign again correponds to effectively repulsive m–body forces for which one has

$$\zeta_0(m) = (m^2 - 1)/2 \quad . \tag{4.23}$$

Now, this exponent governs the probability $\mathcal{P}_{mb}$ that all m identical strings form a local bound state, i.e., $\mathcal{P}_{mb} \sim 1/\xi_\parallel{}^{\zeta_0(m)}$ where a finite value of $\xi_\parallel$ is enforced by an external pressure or by another long–ranged potential.

The values (4.23) for the contact exponents have previously been derived by field-theoretic renormalization [11]; they are the scaling dimensions of the m-string contact operators (6.26) at the free Fermi fixed point (see (6.25) below). They govern the contact probabilities $\mathcal{P}_{mb}$ as long as all attractive interactions are sufficiently weak

In systems of several *nonintersecting* strings, there may be two-body and many-body interactions of either sign. Numerical transfer matrix results [8] as well as Monte Carlo simulations of bundles of strings or membranes [10] indicate in general a second order unbinding transition not in the universality class of the necklace model with a transition temperature that is independent of the number N of strings. This is understandable from the scaling picture [12] since with attractive pair forces the unbinding should be governed by $\mathcal{P}_{2b}$ alone. However, the effective critical exponents were found to depend on N over the numerically accessible range of scales [8, 10].

On the other hand, if only pair interactions are taken into account, the transfer matrix can be mapped onto that of a spin 1/2 xxz quantum spin chain; this model is soluble by a Bethe ansatz and yields the N-independent "Gaussian" exponents $\nu_\perp = 1$ and $\nu_\parallel = 2$ [9]. (Bethe ansatz methods may be extended to treat the unbinding of a system of such strings from a wall [13].)

The renormalization group discussed in Sect. 6.2 [11] reconciles these two results: if the unbinding is driven by pair forces, it is in the Gaussian universality class for an arbitrary number of strings, but the three-particle interactions contribute large corrections to scaling that may account for the N-dependence of the effective exponents. Moreover, there is a discrete sequence of new universality classes characterized by $\zeta_0 = 0$.

4.8 Extension to interfaces and membranes

The scaling picture for interacting strings as described above can be easily extended in the following way. First of all, the fluctuating humps of the manifolds will in general be governed by $\xi_\perp \sim \xi_\parallel{}^\zeta$ with $\zeta \neq 1/2$. As in the case of interacting strings, the roughness exponent ζ determines the fluctuation–induced interaction $\mathcal{V}_{fl}$ between the manifolds which can represent a loss of entropy or an increase in energy. The latter situation arises in systems with quenched or frozen randomness for which the manifolds are subject to

a random potential. For thermally–excited fluctuations, one has

$$\mathcal{V}_{fl} \sim 1/\xi_\perp^\tau \qquad \text{with} \quad \tau = d_\parallel/\zeta \quad . \tag{4.24}$$

The case of 1–dimensional strings corresponds to $d_\parallel = 1$, $\zeta = 1/2$ and $\tau = 2$.

For fluctuations induced by quenched or frozen randomness, one has [34]

$$\mathcal{V}_{fl} \sim 1/\xi_\perp^\tau \qquad \text{with} \quad \tau = (2-2\zeta)/\zeta \quad . \tag{4.25}$$

Thus, the excess free energy of two interacting manifolds can now be written in the form

$$\Delta F \approx c_1 A/\xi_\perp^\tau + \mathcal{V}_{ub}(c_2\xi_\perp) + \Delta F_b/{\xi_\perp}^{\zeta_0/\zeta} \tag{4.26}$$

Minimization of this expression with respect to $\xi_\perp$ now leads to $\xi_\perp \sim 1/|\Delta F_b|^{\nu_\perp}$ with the roughening exponent

$$\nu_\perp = 1/(\tau - \zeta_0/\zeta) \quad . \tag{4.27}$$

One nontrivial check of this prediction can be obtained for wetting in 2–dimensional random bond systems. In this case, one has two interacting strings which feel a random potential with short–ranged correlations, and the roughness exponent has the value $\zeta = 2/3$ which implies the decay exponent $\tau = 1$. For a square–well potential, transfer matrix calculations using the replica trick lead to the scaling form $\mathcal{P}(l) \approx \Omega(l/\xi_\perp)/\xi_\perp$ for the probability distribution $\mathcal{P}(l)$ with the singular behavior $\Omega(s) \sim 1/s^{1/2}$ for small s, see [1], p.317. This implies $\zeta_0/\zeta = 1 - 1/2 = 1/2$. Thus, the excess free energy becomes

$$\Delta F \approx c_1 A/\xi_\perp + \Delta F_b/{\xi_\perp}^{1/2} \quad . \tag{4.28}$$

Minimization of this expression with respect to $\xi_\perp$ leads to the critical behavior

$$\xi_\perp \sim 1/|\Delta F_b|^{\nu_\perp} \qquad \text{with} \quad \nu_\perp = 2 \tag{4.29}$$

at the wetting transition. This agrees with the critical behavior as obtained via transfer matrix methods. [35, 36]

On the other hand, *repulsive* short–ranged potentials should also be characterized, in general, by a nontrivial value for the contact exponent ζ_0. The expression (4.26) for the excess free energy implies that, in this latter case, the exponent ζ_0 should satisfy the inequality $\zeta_0/\zeta \geq \tau$ which implies

$$\zeta_0 \geq d_\parallel \qquad \text{and} \quad \zeta_0 \geq 2(1-\zeta) \tag{4.30}$$

for thermally–excited fluctuations and for fluctuations excited by frozen randomness, respectively.

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