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Intensive Week "Tools of Topology for Quantum Matter" – Day 1
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Hints

Exercise 1.1. *Chern Insulator*

a. Use the basis $|k_x, k_y, \alpha\rangle := \sum_{x,y} e^{i(k_x x + k_y y)} |x, y, \alpha\rangle$ and apply H to it.

b. A traceless Hamiltonian $H = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $a, b, c, d \in \mathbb{C}$ has to satisfy $H^\dagger = H$ and $\mathrm{Tr}(H) = 0$.

c. ...

d. ...

e. Investigate the eigenstate at points (k_x, k_y) for which $\sin(k_x) = \sin(k_y) = 0$.

f. Eigenstates of a Hamiltonian are always orthogonal. Can you reconstruct $\tilde{H}$ from its negative eigenspace?

Exercise 1.2. *Boundaries*

a. Use $|k_x, y, \alpha\rangle := \sum_x e^{ik_x x} |x, y, \alpha\rangle$ and apply H to it.

Exercise 1.3. *Quantum Spin Hall Effect*

a. $\mathcal{T}(H(k)) = H(k)$ for $H(k) = H_\uparrow(k) \oplus H_\downarrow(k)$

b. $\sigma_l \sigma_m \sigma_n = i\varepsilon_{lmn}\mathbb{1}$.