Splitting Kramers degeneracy with superconducting phase difference

Bernard van Heck, Shuo Mi (Leiden),
Anton Akhmerov (Delft)
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Plan

Using phase difference in a Josephson junction as a means of breaking time reversal symmetry.

- What does ‘breaking time reversal’ mean?
- Why it won’t work.
- How to make it work (and why 3 is much better than 2)?
Several manifestations:

- Splitting of Kramer’s degeneracy
  (Chtchelkatchev & Nazarov, Béri & Bardarson & Beenakker)
- Closing of the induced gap
- Protected zero energy level crossings (switches in the ground state fermion parity)

\[ P = \text{Pf}(iH) \]

- Spectral peak in the DOS (Ivanov, Altland & Bagrets)

\[ \rho(E) = \rho_0 \left( 1 + \frac{\sin(2\pi E/\delta)}{2\pi E/\delta} \right) \]
Setup and formalism

Scattering matrices of electrons and holes:

\[ S_h(-E) = S_e^*(E) \]

Andreev reflection matrix:

\[ r_A = ie^{i\phi_i} \]

Bound state condition:

\[ S_e(E)r_A S_h(E)r_A^* \psi = e^{-2i \arccos(E/\Delta)} \psi \]
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Short junction limit

\[ S(E) \approx S(0) \]

Lowest density of Andreev states, strongest effect phase difference on a single state.
Due to unitarity and time reversal symmetry of \( S \) the energies are given by 
\[ E_n = \pm \Delta \sqrt{1 - T_n \sin^2(\phi/2)} \] (Beenakker)
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A big improvement

All the special properties of the spectrum originate from the small number of leads!
Kramers degeneracy splitting

Take a Rashba quantum dot with $E \sim E_{SO}$, $R \gtrsim l_{SO}$, and $\lambda \lesssim R$. 

The diagram illustrates the Kramers degeneracy splitting. 

$\square$ Kramers degeneracy is strongly broken.
Kramers degeneracy splitting

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Protected level crossings

Once again, try a random quantum dot:
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✓ Level crossings are allowed.
Protected level crossings

Are level crossings allowed for any \((\phi_1, \phi_2)\)?
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No: the gap may only close when all the clockwise phase differences are smaller (or larger) than \(\pi\). (Note that this result holds for any junction)
Proof

1. The expression for Andreev spectrum:

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4. This means \( S \psi \equiv \psi', \quad S(r_A \psi) = \frac{2|E|e^{i \alpha}}{\Delta} \psi - (r_A \psi') \)
5. The necessary and sufficient condition for existence of a unitary \( S \):
   \[ \exists \psi, \psi': \langle \psi | r_A | \psi \rangle + \langle \psi' | r_A | \psi' \rangle = \frac{2|E|}{\Delta} e^{i \chi} \langle \psi' | \psi \rangle . \]
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2. Graphical solution:

3. The lower bound on the gap:

\[ E \geq \Delta \min_{i,j} \cos \frac{\phi_i - \phi_j}{2} \]
Gap closing and the spectral peak

Both phenomena are visible

✓ In the ensemble
(averaging over random antisymmetric $S$)

![Graph showing the spectral peak and gap closing](inclusion)
Gap closing and the spectral peak

Both phenomena are visible

✓ In the ensemble
  (averaging over random antisymmetric $S$)

✓ In a single realization
  (averaging over chemical potential)

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![Graph 1](image1.png)

![Graph 2](image2.png)
Conclusions

- Superconducting phase difference can strongly break time reversal symmetry in a Josephson junction.
- This requires more than two superconducting leads.
- Spin degeneracy is split by a large fraction of $\Delta$.
- The induced superconducting gap only closes in a finite subregion of the phase space.
Thank you all.
The end.