SIMONS FOUNDATION

Extrinsic Defects and Possible New Experimental Probes of Topological Order

Maissam Barkeshli

Microsoft Station Q

ESI, Vienna

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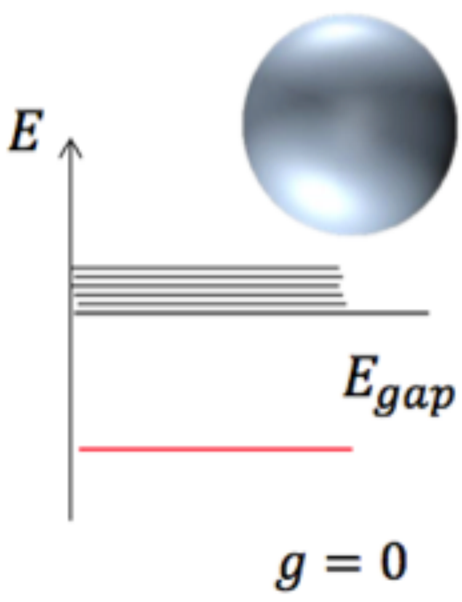
Collaborators:

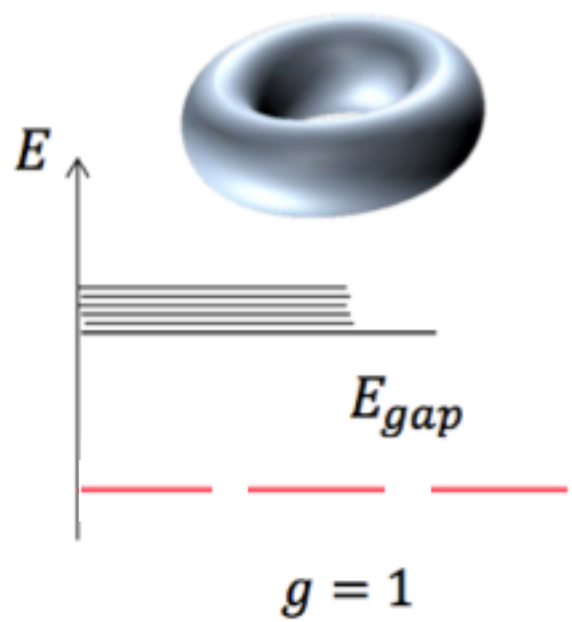
Xiao-Liang Qi (Stanford)
Chao-Ming Jian (Stanford)
Steven Kivelson (Stanford)
Erez Berg (Weizmann)

2+1D Topologically ordered states

- Topology-dependent degeneracies,
- Quasiparticles with fractional charge and statistics,
- Long range entanglement



1 state

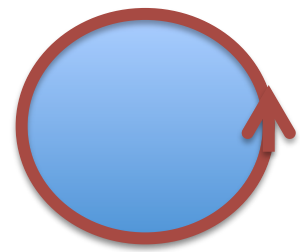


3 states

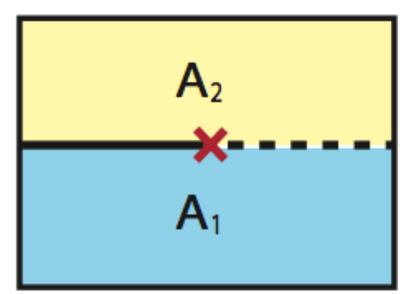
3^g states
(1/3 Laughlin)

An aspect of topological states that has received little attention so far is the physics of extrinsic defects:

- It's well-known that gapless robust edge states can provide a window into the topological phenomena of chiral topological states (eg FQH) Wen 1990

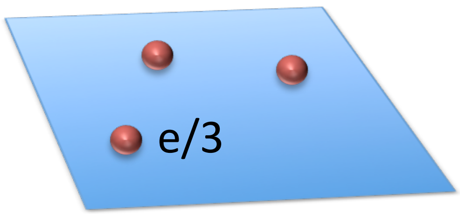


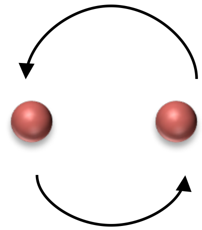
- Similarly, the properties of gapped boundaries, junctions between different gapped boundaries, and other "extrinsic" defects can provide a new window into topological phenomena



1/3 Laughlin FQH state

$$\Psi(\{z_i\}) = \prod_{i<j}(z_i - z_j)^3 e^{-\sum_i |z_i|^2/4l_B^2}$$





$$|\Psi\rangle \rightarrow e^{i\pi/3}|\Psi\rangle$$

3 types of quasiparticles

charge (mod e): 0, e/3, -e/3

exchange statistics: $0,\ \pi/3,\ \pi/3\ \ (\mathrm{mod}\ \pi)$

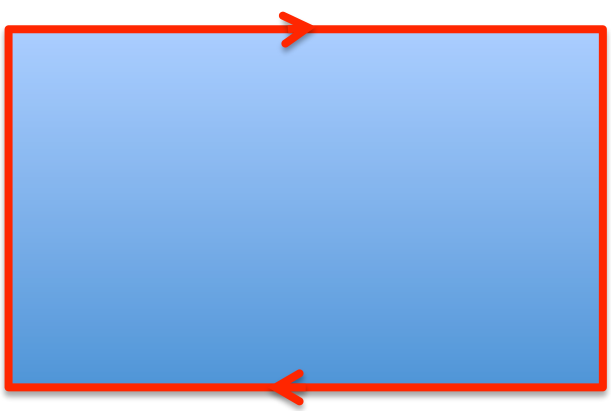
$$\mathcal{L}_{bulk} = -\frac{3}{4\pi} a\partial a + \frac{1}{2\pi} A_E \partial a$$

Quasiparticle loop operator: $W_l(C) = e^{il \oint_C a\cdot dl}$

Chiral edge theory (Wen 1990)

$$\mathcal{L}_{edge} = -\frac{3}{4\pi}\partial_x\phi\partial_t\phi - v(\partial_x\phi)^2$$

$$[\phi(x),\phi(y)] = i\frac{\pi}{3}sgn(x-y) \qquad \phi \sim \phi + 2\pi$$

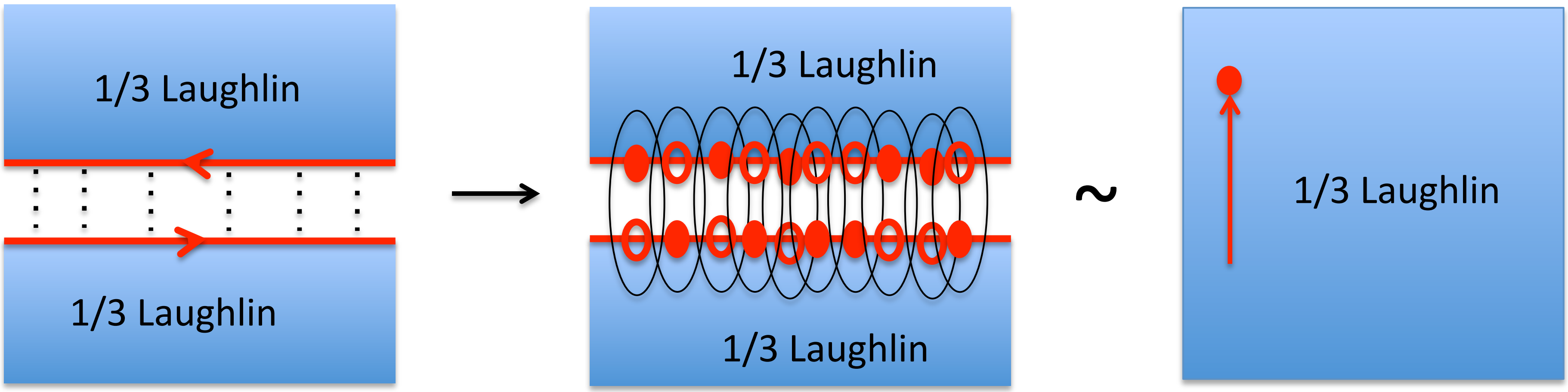


Charge density $\rho = \frac{1}{2\pi}\partial_x\phi$

Charge a/3 qp $V_a = e^{ia\phi}$

Electron operator $\Psi_e = e^{i3\phi}$

Electron tunneling across two 1/3 Laughlin states



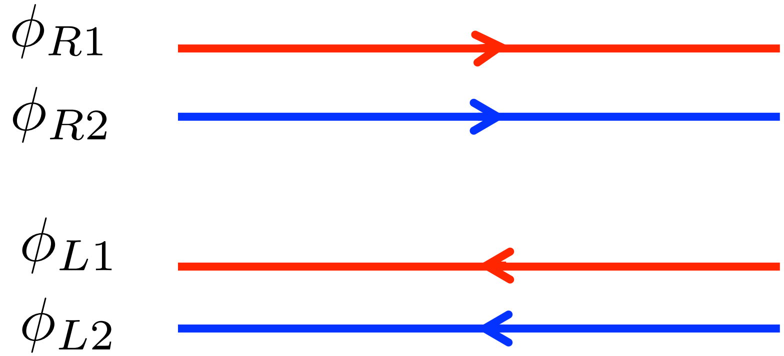
$$\mathcal{L}_{edge} = -\frac{3}{4\pi}\partial_x\phi_1\partial_t\phi_1 + \frac{3}{4\pi}\partial_x\phi_2\partial_t\phi_2 - V_{IJ}\partial_x\phi_I\partial_x\phi_J$$

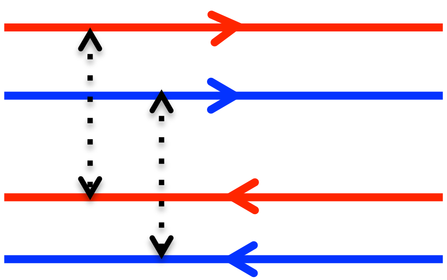
Electron tunneling $\delta\mathcal{L} = -t\cos(3(\phi_1 - \phi_2))$

Large t → Gaps modes $\langle e^{i(\phi_1-\phi_2)}\rangle = e^{2\pi i n/3}$

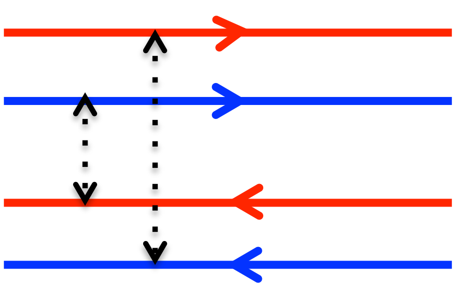
Double layer (1/3 + 1/3)

Barkeshli, Qi PRX 2012





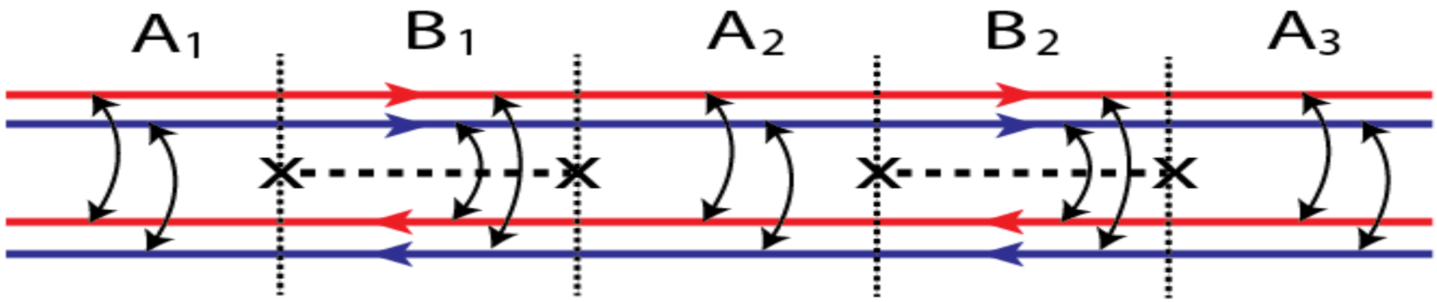
$$\cos(3(\phi_{R1}-\phi_{L1})) + \cos(3(\phi_{R2}-\phi_{L2})) \longrightarrow \begin{array}{l} \langle e^{i(\phi_{R1}-\phi_{L1})}\rangle \neq 0 \\ \langle e^{i(\phi_{R2}-\phi_{L2})}\rangle \neq 0 \end{array}$$

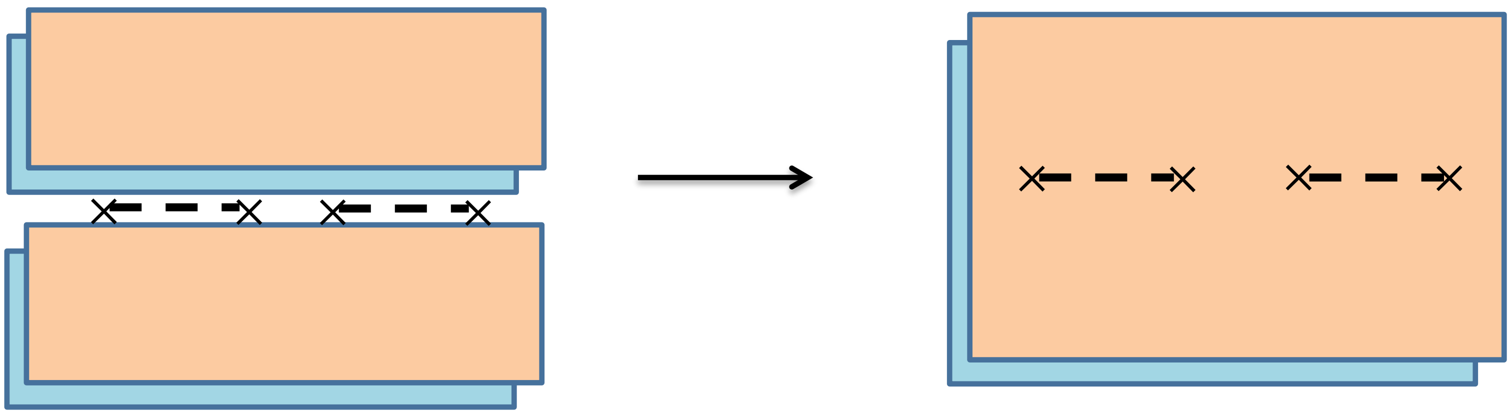


$$\cos(3(\phi_{R1}-\phi_{L2})) + \cos(3(\phi_{R2}-\phi_{L1})) \longrightarrow \begin{array}{l} \langle e^{i(\phi_{R1}-\phi_{L2})}\rangle \neq 0 \\ \langle e^{i(\phi_{R2}-\phi_{L1})}\rangle \neq 0 \end{array}$$

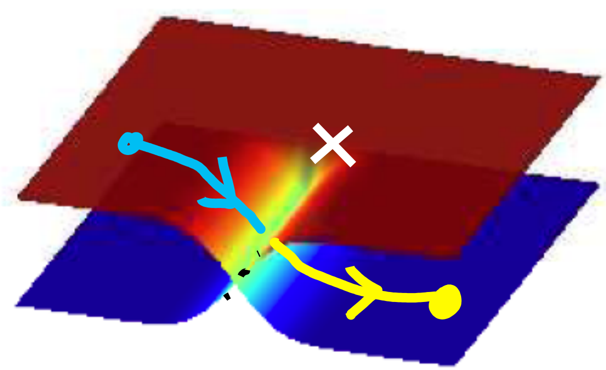
Topologically Distinct Edge Phases!

Domain Walls Between Different Edge Phases



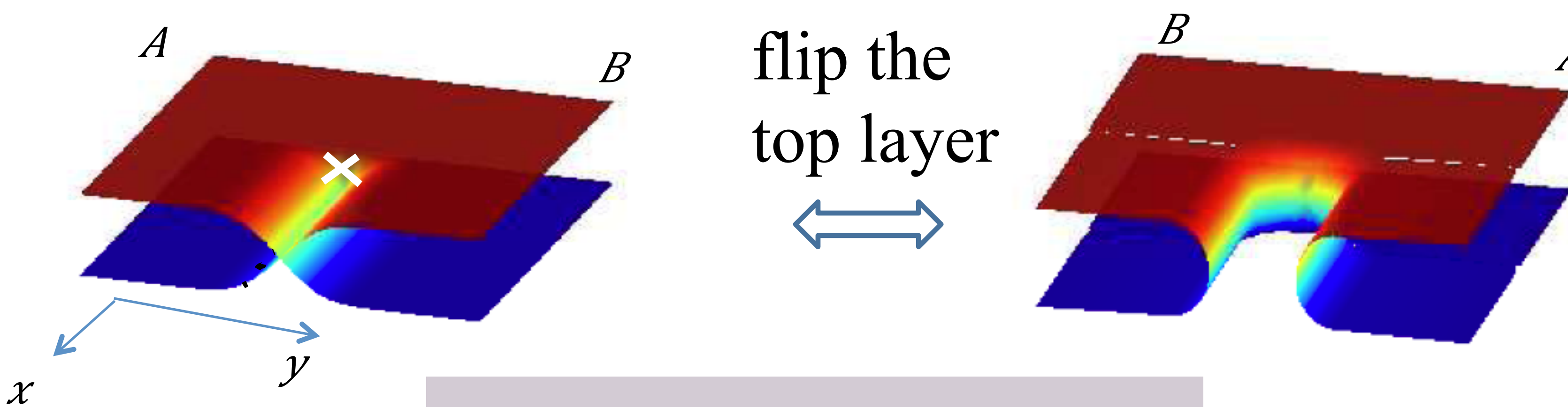


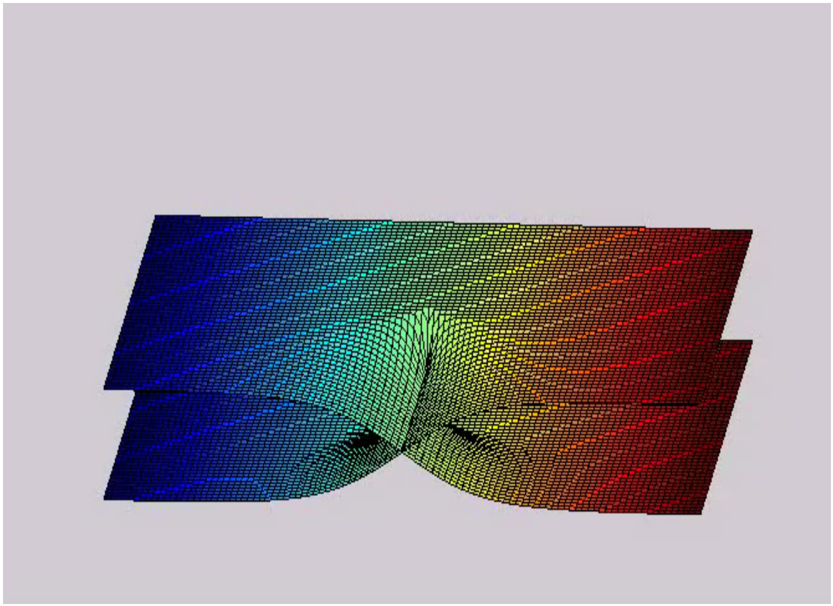
“Twisted” tunneling induces branch cut between layers



Branch cut effectively changes topology

- In bilayers, pair of defects (branch points) creates “worm hole”



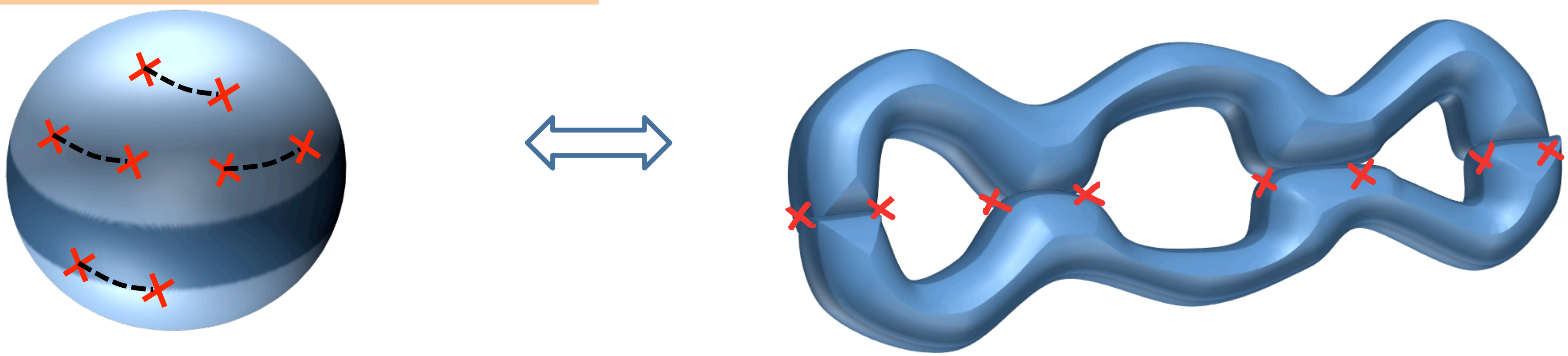


Barkeshli, Wen (2010)
Barkeshli, Qi (2012)

- Every pair of defects add genus 1 to the manifold

$2n$ defects on a sphere

genus $g=n-1$ surface



Defects called genons---genus generators

Barkeshli, Wen (2010)
Barkeshli, Qi (2012)

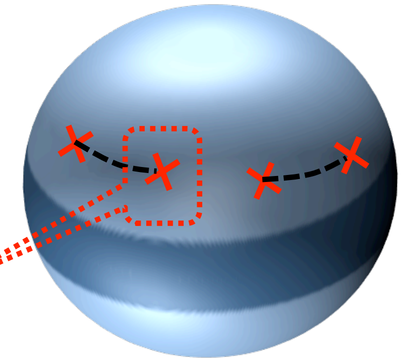
Quantum dimension of genons

- ν = 1/m Laughlin FQH state in each layer →

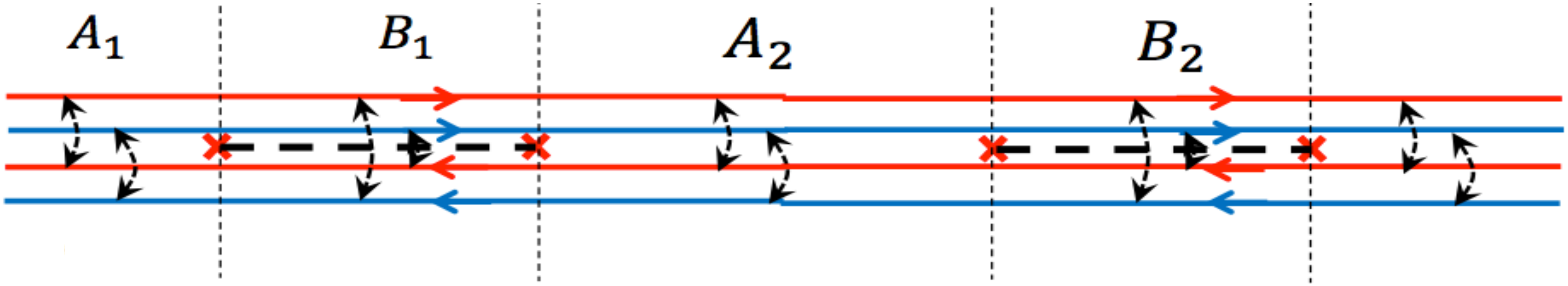
ground state degeneracy m^g,

each pair of defects add ***m*** degrees of freedom

→ Each defect has quantum dimension $\sqrt{m}$

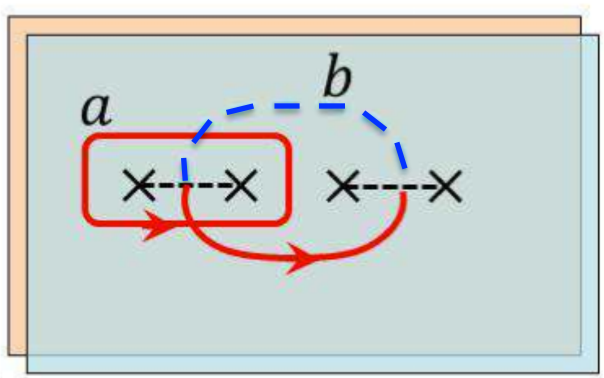


$\sqrt{m}$ states



$$W(a) = e^{i(\phi_{R1}-\phi_{L1})(A_1)}e^{-i(\phi_{R1}-\phi_{L1})(A_2)}$$

$$W(b) = e^{i(\phi_{R1}-\phi_{L2})(B_1)}e^{-i(\phi_{R1}-\phi_{L2})(B_2)}$$



$$[W(a), H] = [W(b), H] = 0$$

$$W(a)W(b) = W(b)W(a)e^{2\pi i/3}$$

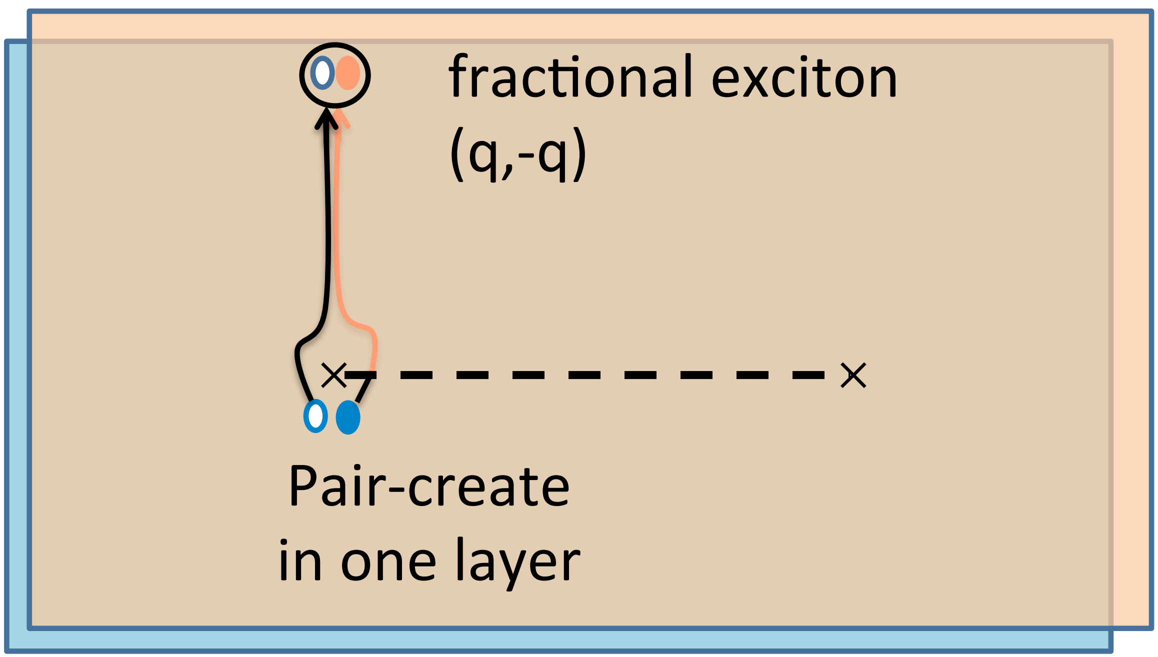
n pairs of genons on sphere → n - 1 copies of loop algebra

→ 3^{n-1} states

→ Quantum dimension = $\sqrt{3}$

Localized "parafermion" zero modes

- Twist defects/genons lead to localized zero energy states for some quasiparticles
- Genons in bilayers can absorb/emit fractional excitons :



Parafermion zero mode operators

- Zero mode = quasiparticle exciton operators at domain walls:

$$\alpha_i = e^{i(\phi_{R1}-\phi_{R2})(x_i)}$$

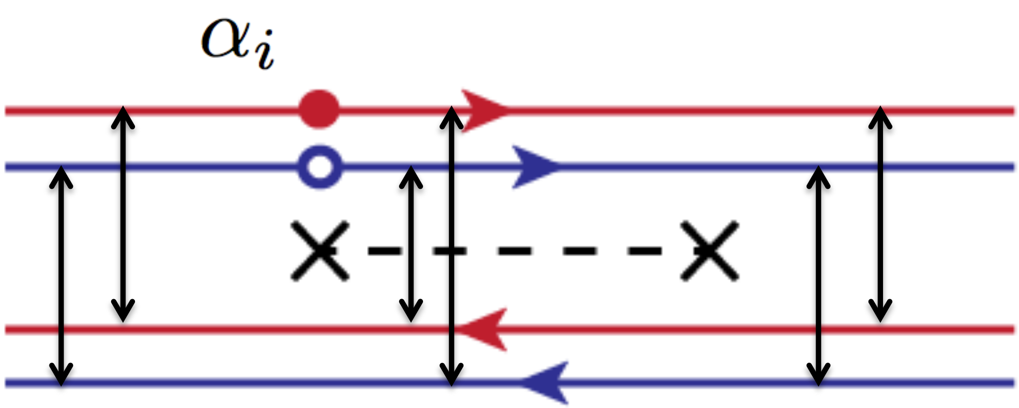
$$\alpha_j \alpha_k = \alpha_k \alpha_j e^{2\pi i sgn(j-k)/3}$$

Z_3 "parafermion" algebra

Beyond Majorana zero modes

[Read-Green 2000
Kitaev 2001]

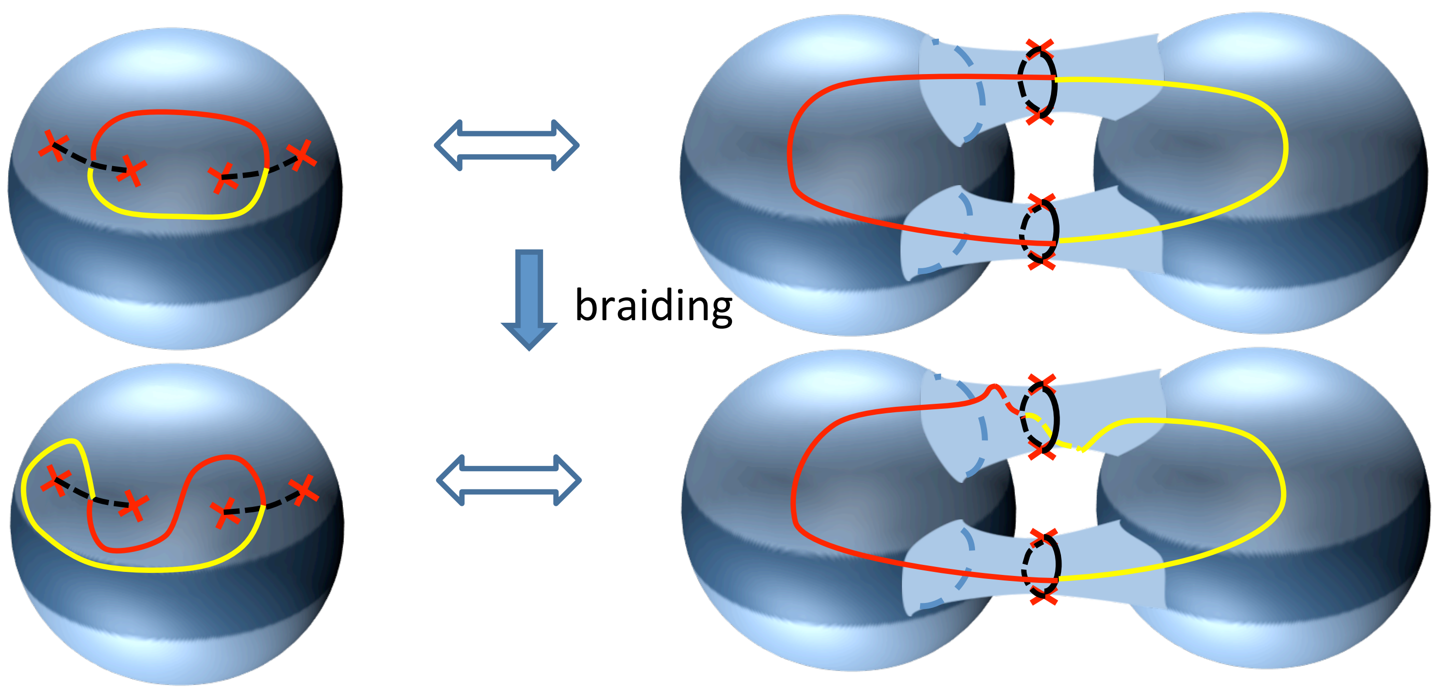
Exponentially localized to defect.



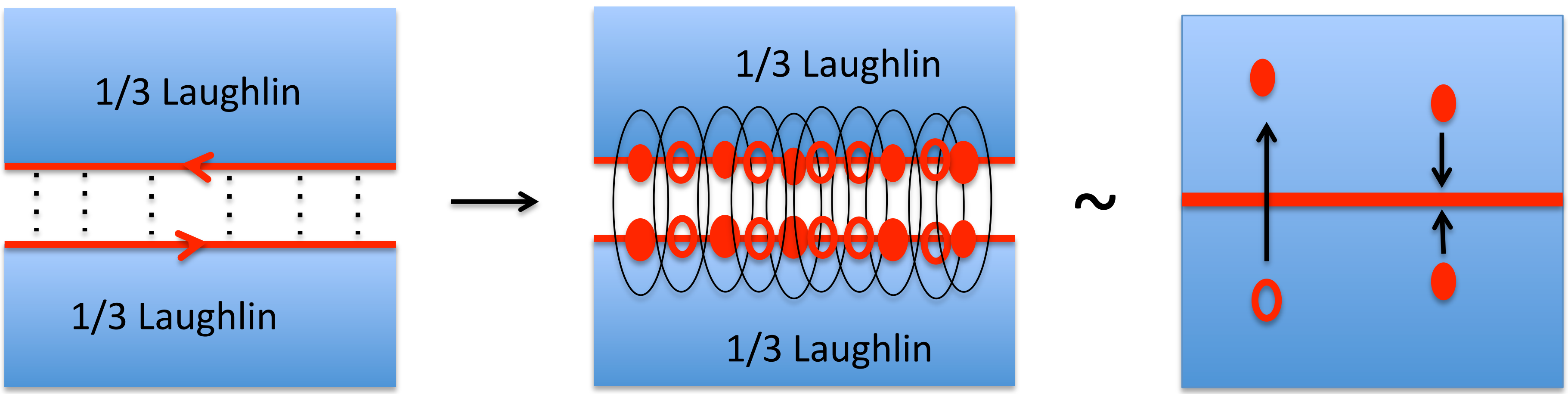
Projective braiding statistics of genons

- Braiding two genons = "Dehn twist" on the high genus surface

Overall phase not topological → **Projective non-Abelian statistics**



Cooper pair tunneling in 1/3 Laughlin state

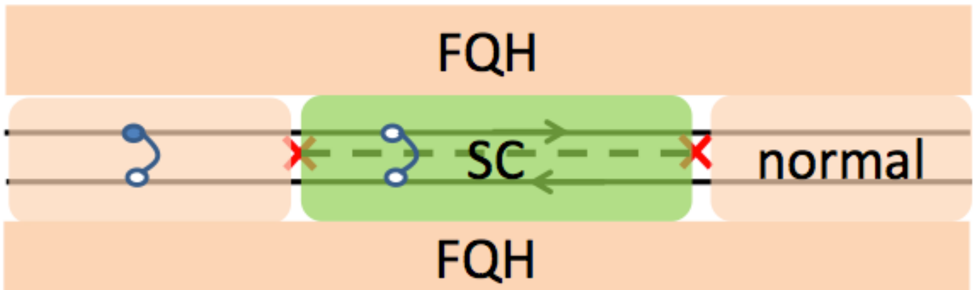


$$\delta\mathcal{L} = -\frac{t}{2}(\Psi^\dagger_{eR}\Psi^\dagger_{eL} + H.c.) = -t\cos(3(\phi_R + \phi_L))$$

Large t → Gaps modes $\langle e^{i(\phi_R+\phi_L)}\rangle = e^{2\pi i n/3}$

Topologically distinct way to gap out modes (cf. normal tunneling)

Normal – Superconducting Domain Walls



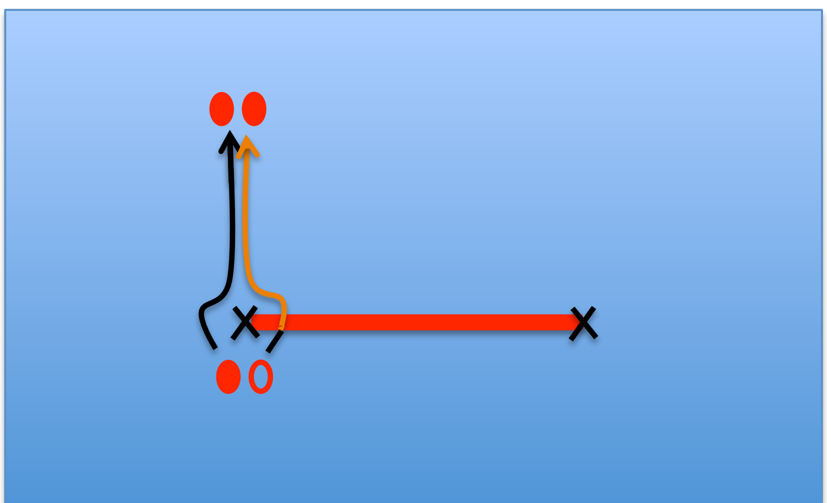
Lindner, Berg, Refael, Stern 2012;
Clarke, Alicea, Shtengel 2012;
Cheng 2012; Vaezi 2012

IQSH: Fu-Kane 2008

Quantum Dimension of domain walls:

$\sqrt{2}\sqrt{m}$ m odd

$\sqrt{m/2}$ m even



Parafermion zero modes

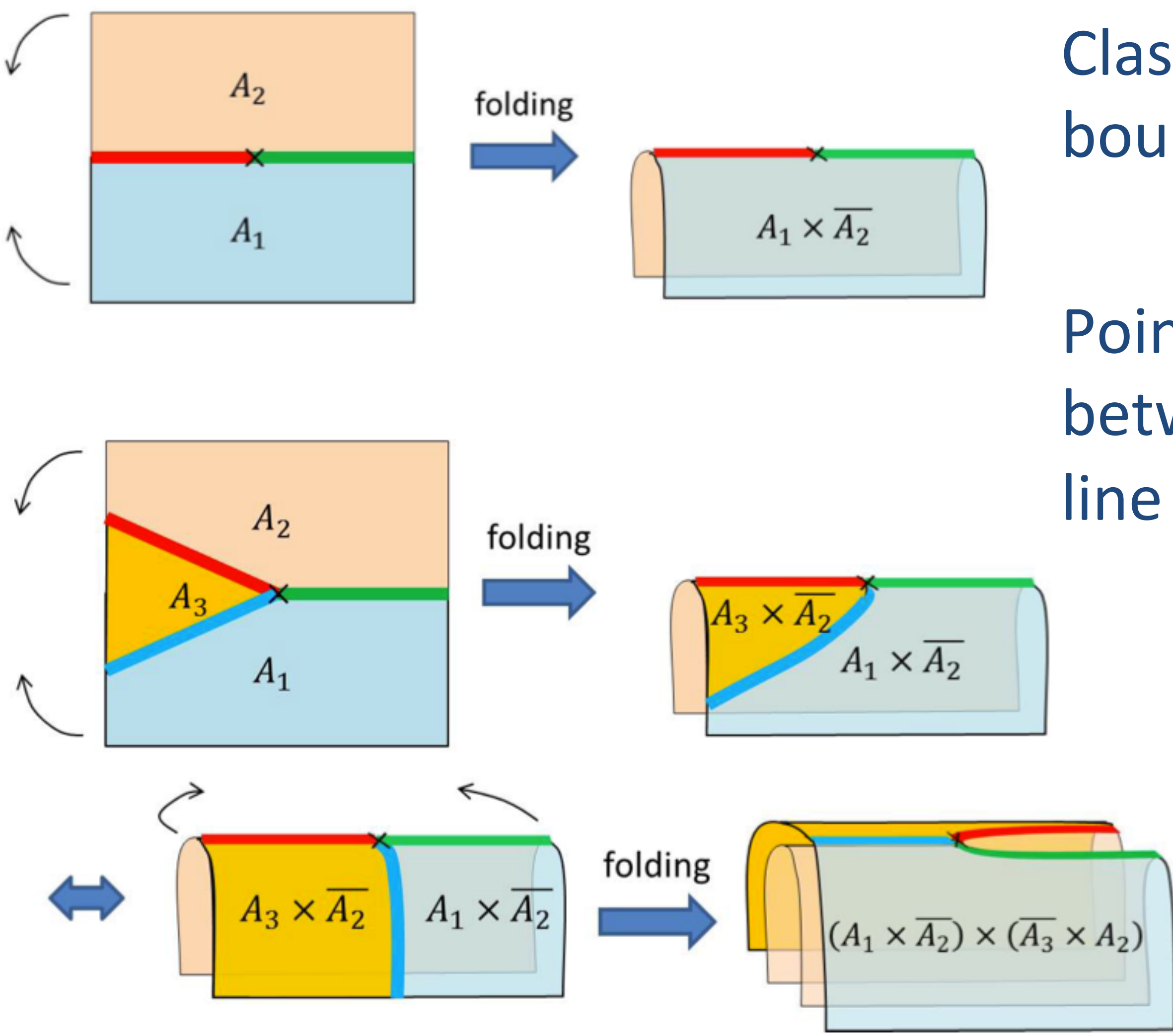
General theory of topologically distinct gapped edges? Domain walls and junctions?

Previous Work:

1. Beigi, Shor, Whalen (2011) — Gapped edges of Kitaev quantum double models
2. Kitaev, Kong (2012) — Gapped edges of Levin-Wen models
3. Kapustin-Saulina (2011) — Conjectured classification of “topological boundary conditions” in Abelian CS theory
4. Fuchs, Schweigert, Valentino (2013) — Mathematical theory of “topological boundary conditions” for general Modular Tensor Category
5. M. Levin (2013) — General condition for possibility of a gapped edge in Abelian states

Classification of general defects

Use folding process to map all defects to boundary defects:



Classify different kinds of boundaries → Line defects

Point defects = domain walls between different boundary line defects

(Barkeshli, Jian, Qi 2013)

Effective theory of Abelian states

$$\mathcal{L}_{bulk} = \frac{1}{4\pi} K_{IJ} a^I \partial a^J$$

K = N x N symmetric integer matrix

K even (odd) → Bosonic (fermionic) system

Distinct quasiparticles labelled by $l \in \mathbb{Z}^N \quad l \sim l + K\mathbb{Z}^N$

Self Statistics $\theta_l = \pi l^T K^{-1} l$

Mutual Statistics $\theta_{ll'} = 2\pi l^T K^{-1} l'$

General edge theory

$$\mathcal{L}_{edge} = \frac{1}{4\pi} K_{IJ} \partial_x \phi_I \partial_t \phi_J - V_{IJ} \partial_x \phi_I \partial_x \phi_J$$

N_L (N_R) = No. of positive (negative) eigenvalues of K

Edge can only be fully gapped if $N_L = N_R$

Local tunneling terms: $\delta\mathcal{L} = -\sum_I t_I \cos(\Lambda_I^T K \phi)$

$$\Lambda_I \in \mathbb{Z}^N$$

Null vector condition (Haldane 1995):

$$\text{If } \Lambda_I^T K \Lambda_J = 0, \quad I, J = 1, \cdots, n$$

→ 2n modes are gapped

Lagrangian Subgroups

"Maximal" subgroup of bosonic quasiparticles:

M is a Lagrangian subgroup if:

1. $e^{i\theta_m} = 1, \;\; \forall m \in M$
2. $e^{i\theta_{mm'}} = 1, \;\; \forall m, m' \in M,$
3. $\forall l \notin M, \exists m \in M \text{ such that } e^{i\theta_{lm}} \neq 1$

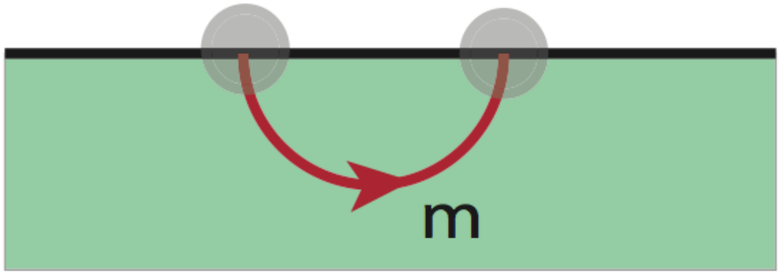
Condensation of M → All quasiparticles are either condensed or confined. Resulting state is trivial.

Lagrangian Subgroups and gapped edges

M. Levin (2013):

- An Abelian state can support a gapped edge if and only if $N_L = N_R$ and it has a Lagrangian subgroup

- A Lagrangian subgroup must be condensed at a gapped edge



Is there a one-to-one correspondence between Lagrangian subgroups and topologically distinct gapped edges?

One-to-One Correspondence (Classification)

Every Lagrangian subgroup M corresponds to a possible gapped edge where M is condensed

Barkeshli, Jian, Qi (2013)
Levin v2 (2013)

Proof (sketch):

1. Expand K in a trivial way: $K \to K' = K \oplus P$

   $$|\mathrm{Det}\ P| = 1 \quad \mathrm{Tr}\ P = 0$$

2. There is a choice of generators $\{m_i'\}$ of M such that

   $$m_i'^T K'^{-1} m_j' = 0$$

3. Set $\Lambda_i \equiv c_i K'^{-1} m_i'$ and pick tunneling terms

$$\delta\mathcal{L} = -t \sum_i \cos(\Lambda_i^T K' \phi) = -t \sum_i \cos(c_i m_i'^T \phi)$$

→ Large t fully gaps edge, $\langle e^{i m_i'^T \phi} \rangle \neq 0$

General Examples

1. $K = \begin{pmatrix} 0 & N \\ N & 0 \end{pmatrix}$ is equivalent to Z_N gauge theory

Lagrangian subgroups $\longleftrightarrow$ subgroups of Z_N

2. $K = \begin{pmatrix} A & 0 \\ 0 & -A \end{pmatrix}$

Lagrangian subgroups $\longleftrightarrow$ Automorphisms of A-theory

Point defects in Abelian states

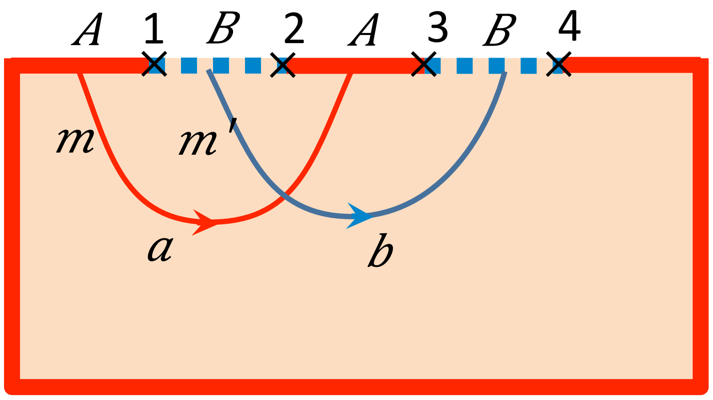
Barkeshli, Jian, Qi (2013)

- Point defects = domain wall between edges with different Lagrangian subgroups M, M'

- Carry non-trivial topological degeneracy due to Wilson line algebra. $m \in M, \ m' \in M'$

$$W_m(a)W_{m'}(b) = W_{m'}(b)W_m(a)e^{2\pi i m^T K^{-1} m'}$$

- This can always be mapped to Wilson loop algebra of some Abelian CS theory on high genus surface → **Genons**



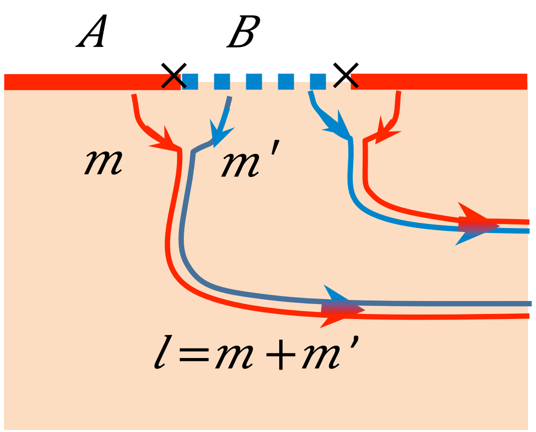
Parafermion zero modes/Non-Abelian statistics

Barkeshli, Jian, Qi, 2013

- Quasiparticles l = m + m' can be emitted/absorbed at defects
- l has fractional statistics
- Generalized "parafermion" zero modes

$$\chi_m \equiv e^{im^T \phi}$$

$$\gamma_l \equiv \lim_{\epsilon \to 0^+} \chi_m(-\epsilon)\chi_{m'}(\epsilon)$$



- Non-abelian braiding defined by coupling the defects in various patterns.

$$H_{ab} = \sum_{l \in L} t_l \gamma_l^\dagger(x_a)\gamma_l(x_b) + H.c.$$

Alicea et al. 2010; Clarke et al, Lindner et al 2012; BJQ 2013

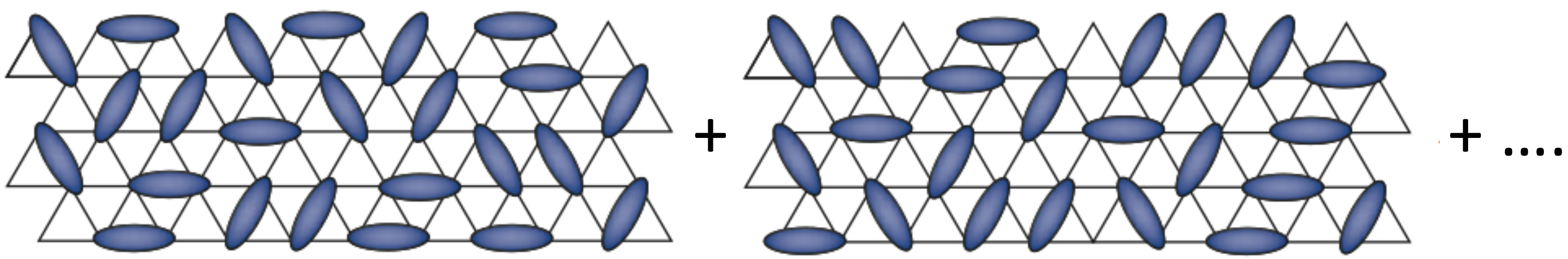
Braiding → Dehn twist of effective high genus surface

Part II:
Possible New Experimental Probes

Quantum Spin Liquids

Electrically insulating, quantum-disordered states, not adiabatically connected to band insulator

Physical picture: Resonating Valence Bonds (RVB)



Anderson 1973, 1987;
Kivelson, Rokhsar, Sethna; Read, Chakraborty; Read, Sachdev; Wen; Many more.....

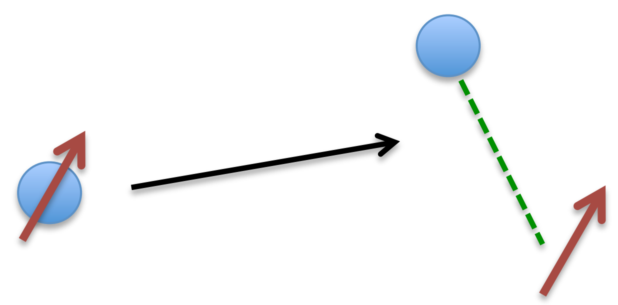
Quantum Spin Liquids

- Experimental signature:

  Cool material to lowest temperatures, and look for evidence of Nothing.

No electrical conductivity
No spin order
No neutron scattering peaks
No specific heat, no thermal conductivity (gapped spin liquids)

- Spin liquids have rich and profound internal structure
  - Fractionalization



  - Emergent gauge fields
  - Long-range Quantum Entanglement
- Gapped Spin Liquids: **Topological Order**

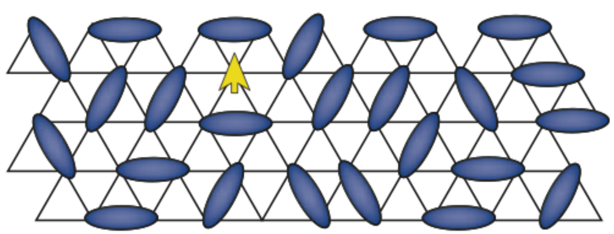
Example: Z_2 spin liquid (Z_2 short-ranged RVB)

$$|Z_2\ \mathrm{RVB}\rangle = \mathcal{P}_G|\mathrm{BCS}\rangle$$

Described by Z_2 lattice gauge theory

4 topological classes:

1. Local excitations (e.g. spin 1)
2. Spinons (spin-1/2, charge 0), holons (spin 0, charge 1):

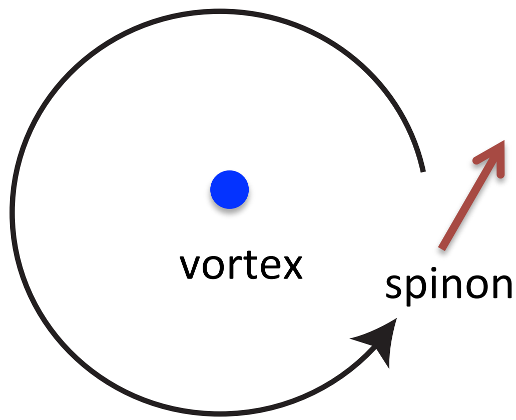


"e" particles

3. Vortices (Z_2 flux) "m" particles
4. em particles

Fractional Statistics

- Spinons and holons can be bosonic or fermionic
- Mutual statistics between e and m particles



= -1

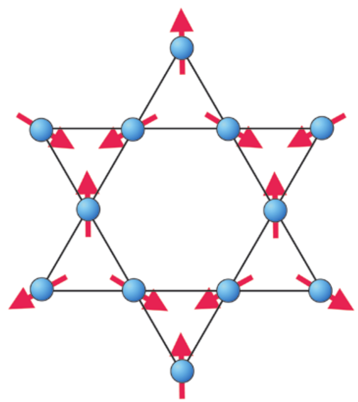
$$\mathcal{L} = \frac{1}{\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu b_\lambda + j_v \cdot b + j_s \cdot a + j_h \cdot a$$

$$K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

QSLs realized in frustrated magnets

- Numerics: Gapped QSL in frustrated Heisenberg models.

$$H = J_1 \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$



Yan et al 2011, Depenbrock et al 2012
Jiang et al 2012, Wang et al 2011

Topological order = Z_2 sRVB or doubled semion ?

- Experiments: suggestive evidence of spin liquids.

  No observable magnetic ordering
  No well-defined spin-1 excitations (neutron scattering)
  Large low-T thermal conductivity, specific heat

Materials: Herbertsmithite, kappa-ET, dmit, …

Major Challenge:

How can we directly probe topological order and fractionalization in an experimentally accessible setting?

Use new insights about extrinsic defects: gapped edges, domain walls, etc

Edge Luttinger Liquid Theory

$$\mathcal{L}_{bulk} = \frac{1}{\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu b_\lambda$$

$$\mathcal{L}_{edge} = \frac{1}{\pi}\partial_t\phi\partial_x\theta - v_1(\partial_x\phi)^2 - v_2(\partial_x\theta)^2 - v_{12}\partial_x\phi\partial_x\theta$$

$$[\phi(x), \theta(y)] = \frac{i\pi}{2}sgn(x-y)$$

Describes either spin or charge fluctuations

(a) Spin fluctuations: $S^z = \frac{1}{2\pi}\partial_x\phi$ $S^\pm = e^{\pm i2\theta}$

$e^{\pm i\theta + in\phi}$ creates spinon, $e^{i\phi}$ creates vortex,

(b) Charge fluctuations: $\rho_c = \frac{1}{\pi}\partial_x\phi$

$e^{\pm i\theta + in\phi}$ creates holon, $e^{i\phi}$ creates vortex,

Local operators: $e^{\pm i2\theta}, e^{\pm i2\phi}$

Gapped Edges

Edge modes gapped by backscattering:

$$\delta\mathcal{L}_{Z_2} = \lambda_m \cos(2\phi) + \lambda_e \cos(2\theta)$$

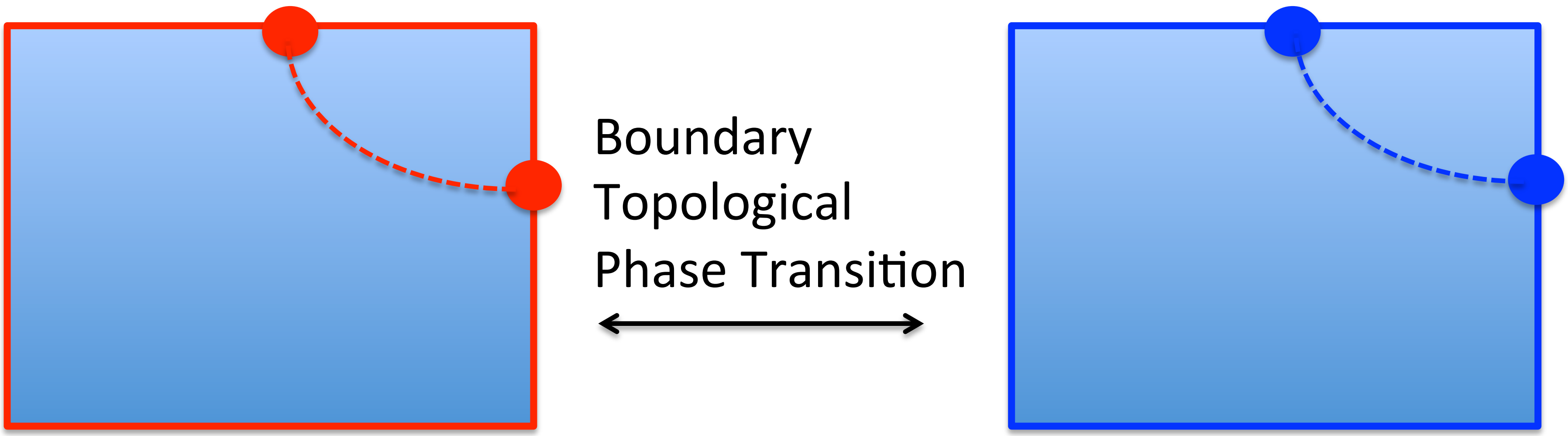
Only local operators can be added to edge.

Two gapped edge phases:

1. $\langle e^{i\phi}\rangle \neq 0$ $\langle e^{i\theta}\rangle = 0$ Vortex condensed (m-edge)
2. $\langle e^{i\phi}\rangle = 0$ $\langle e^{i\theta}\rangle \neq 0$ Spinons or holons condensed (e-edge)

Boundary of Z_2 sRVB

- Z_2 spin liquid: two topologically distinct types of gapped boundaries



e - type

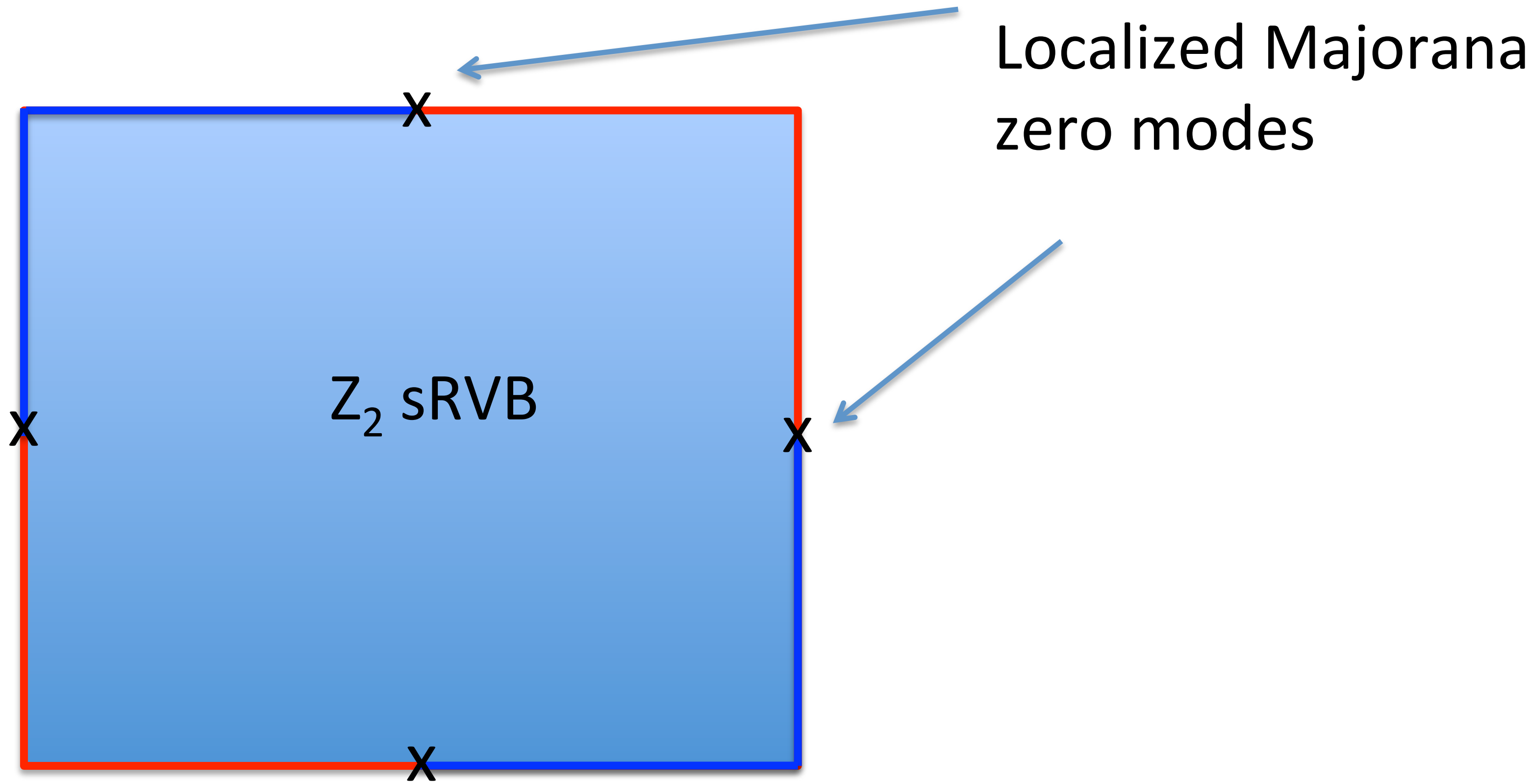
m - type

Z_2 gauge symmetry **broken**
e particles condensed

Z_2 gauge symmetry **unbroken**
m particles condensed

- Related to rough/smooth edges of toric code

Topological Zero Modes and Non-Abelian Defects



Fermionize edge theory → fermions form p-wave SC

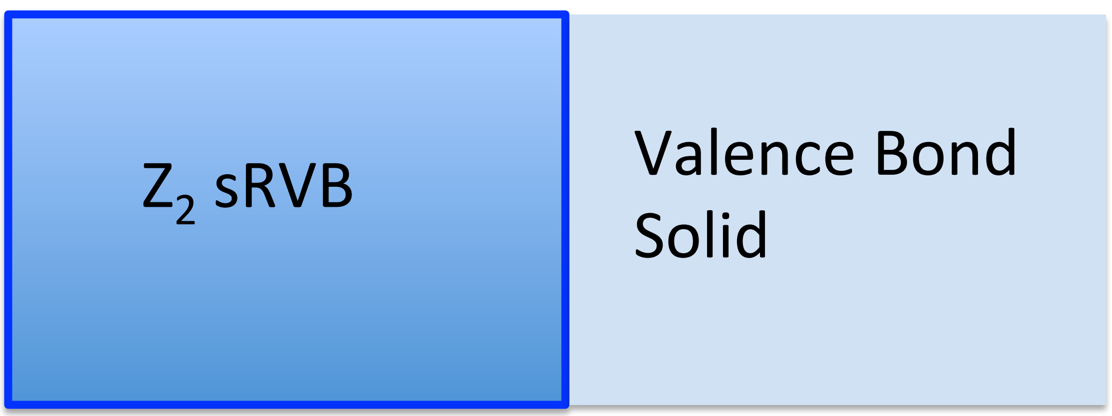
Realizing e, m-type boundaries

Barkeshli, Berg, Kivelson, 2014

- e-edge requires breaking spin or charge conservation

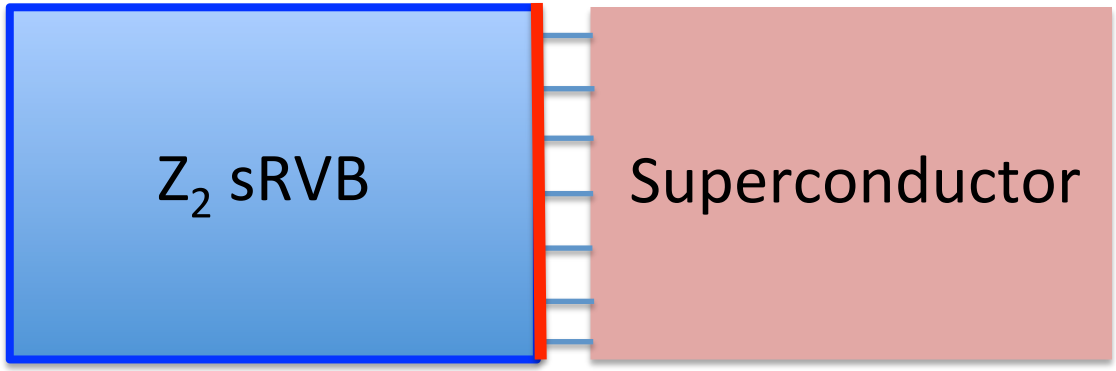
  $\cos(2\theta)$ breaks spin or charge

- Charge and spin conservation → m-edge



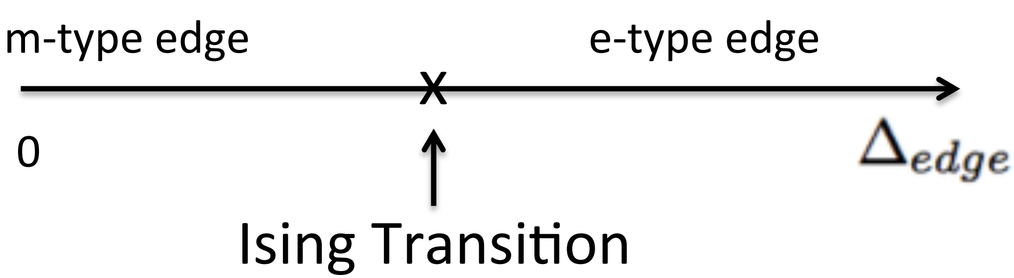
Vacuum

e-edge from superconductivity



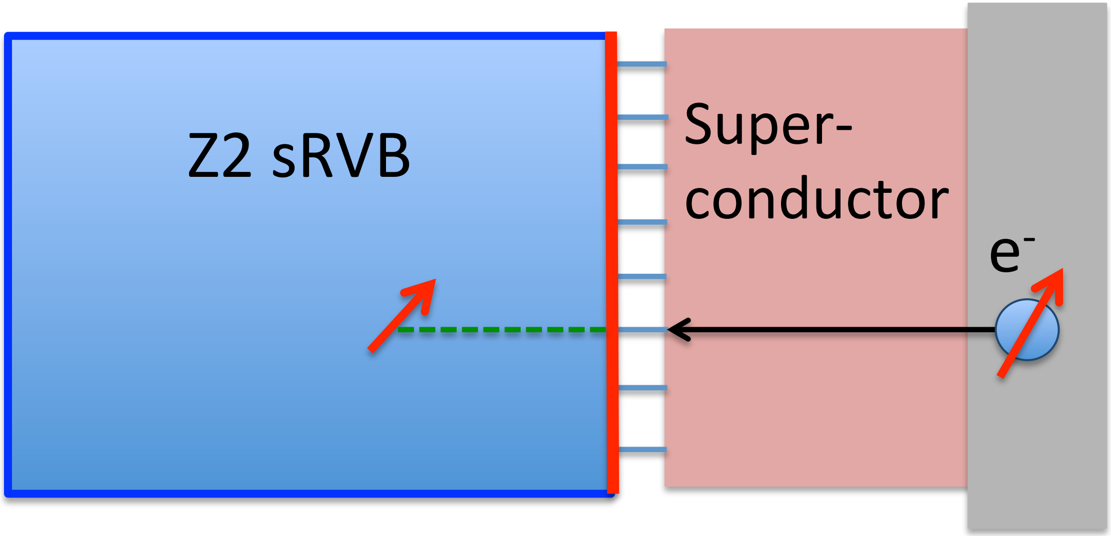
$$H_{edge} = \Delta_{edge} \sum_i c^\dagger_{Li\uparrow} c^\dagger_{Li\downarrow} c_{Ri\uparrow} c_{Ri\downarrow} + J_{edge} \sum_i \vec{S}_{Li} \cdot \vec{S}_{Ri} + H.c.$$

$$\delta \mathcal{L}_{edge} \sim \Delta_{edge} \cos(2\theta) - \mu \partial_x \phi$$



Critical Δ_{edge}
can be made small
by gating superconductor

Coherent spinon injection



$$c_\alpha = b f_\alpha$$

b = bosonic holon → Condensed on edge

f_α = fermionic spinon

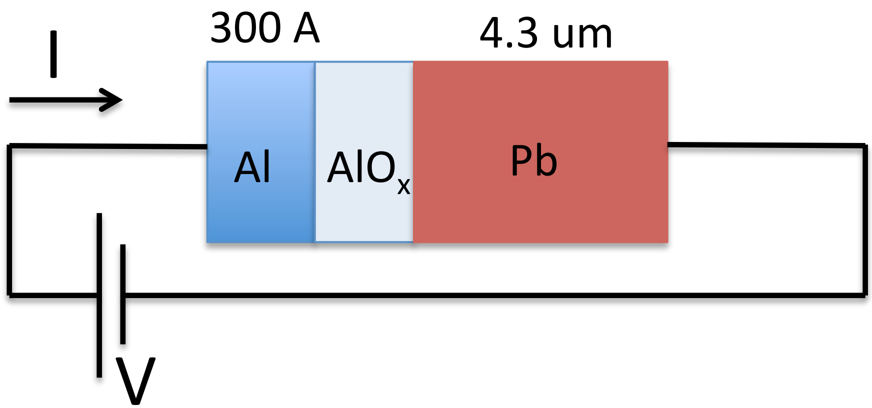
$$\delta H_{edge} = t_{edge} \sum_i c^\dagger_{Li} c_{Ri} + H.c. = t_{edge} \langle b_L \rangle \sum_i f^\dagger_{Li} c_{Ri} + H.c.$$

Electron can coherently pass through the superconductor and into the spin liquid as a fermionic spinon

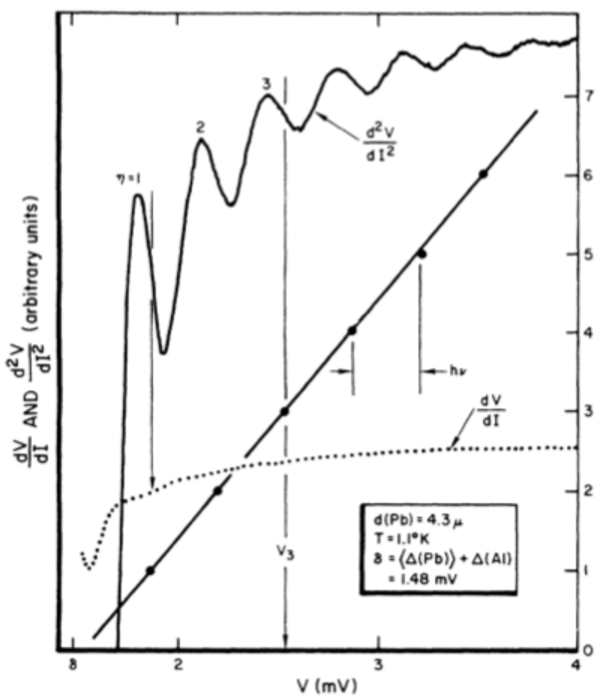
Measurable consequences

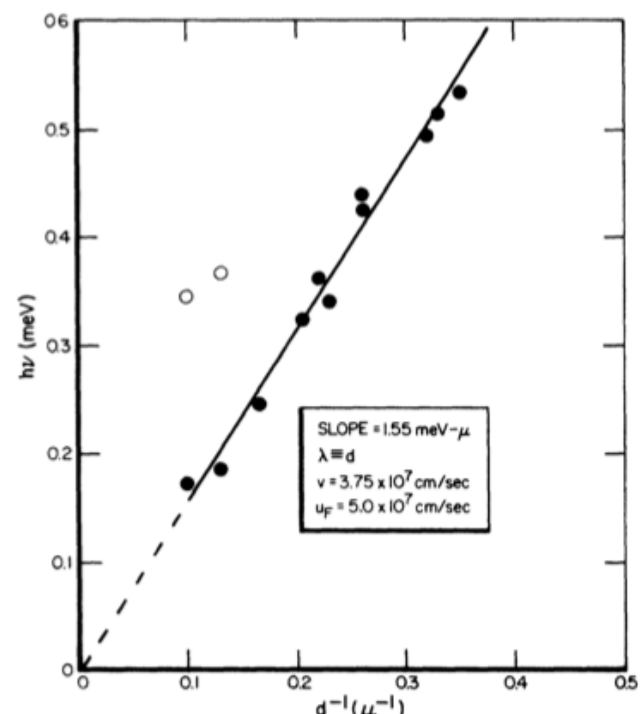
Tomasch Oscillations

In 1965, Tomasch studied superconducting film diodes:



- Oscillations in I-V curve
- Period set by thickness of Pb film. Independent of Al, AlO$_x$ thickness





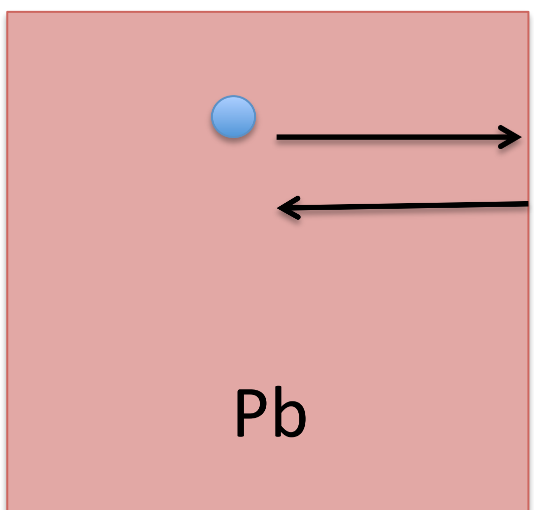
Tomasch Oscillations

VOLUME 16, NUMBER 3 PHYSICAL REVIEW LETTERS 17 JANUARY 1966

THEORY OF GEOMETRICAL RESONANCES
IN THE TUNNELING CHARACTERISTICS OF THICK FILMS OF SUPERCONDUCTORS

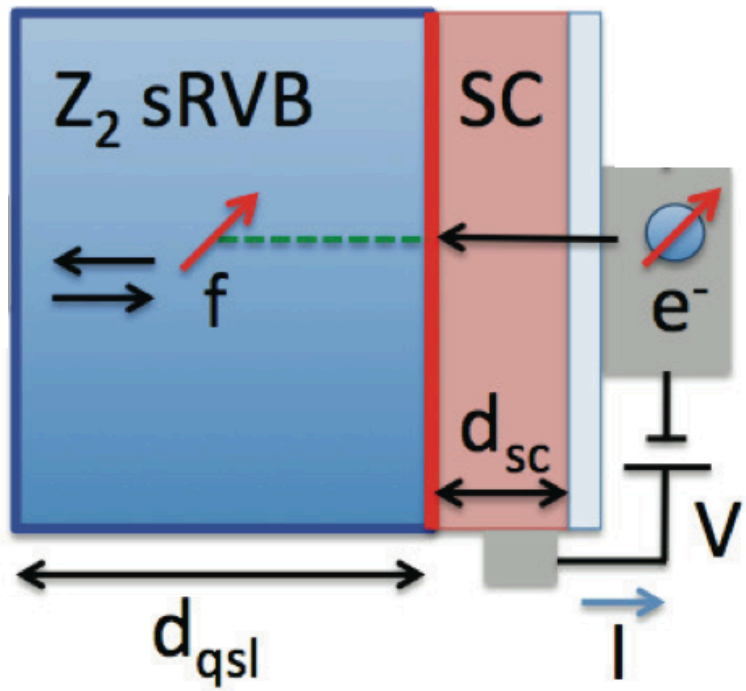
W. L. McMillan and P. W. Anderson

Explained by McMillan and Anderson as due to quasiparticle interference from scattering off of boundary of sample:



Tomasch Oscillations in Spin Liquids

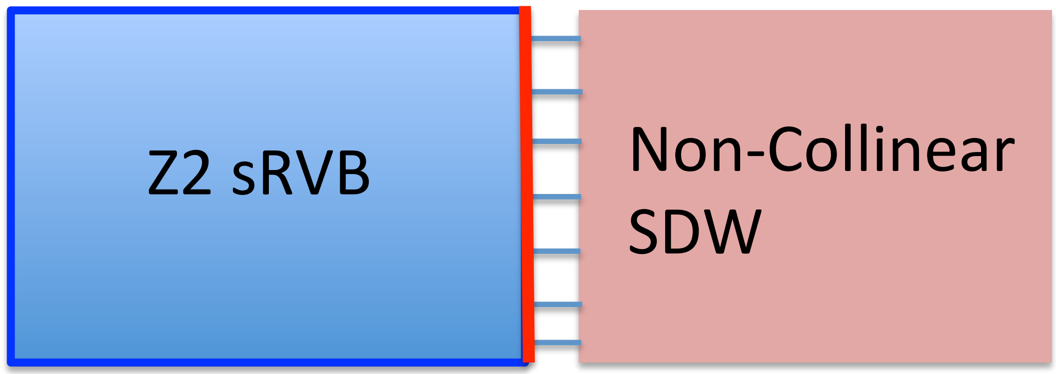
- If the fermionic spinon is a stable quasiparticle excitation in the Z2 spin liquid:



Expect oscillations in dI/dV with period set by d_{qsl}

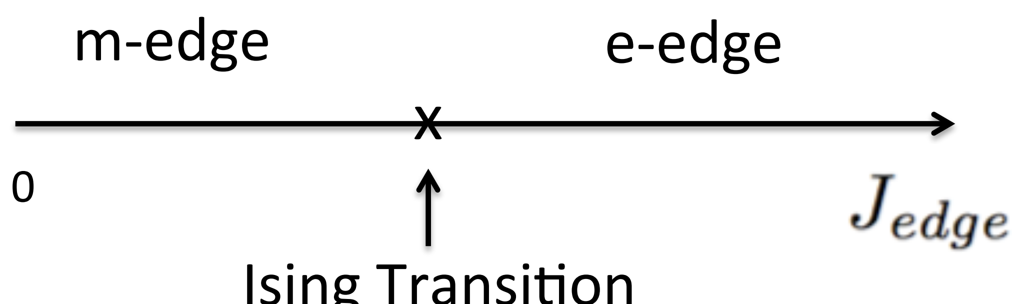
Only possible with e-type edge

e-type boundaries from magnetism

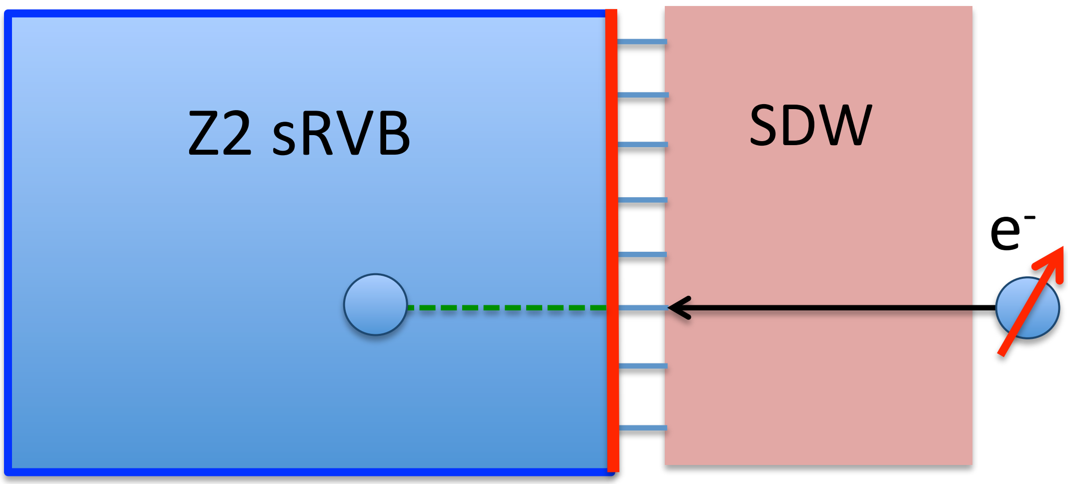


$$H_{edge} = J_{edge} \sum_i \vec{S}_{Li} \cdot \vec{S}_{Ri}$$

$$\delta \mathcal{L}_{edge} \sim J_{edge}[h_1 \cos(2\theta - \alpha) + h_2 \partial_x \phi]$$



Coherent holon injection



$$c_\alpha = d z_\alpha$$

d = fermionic holon

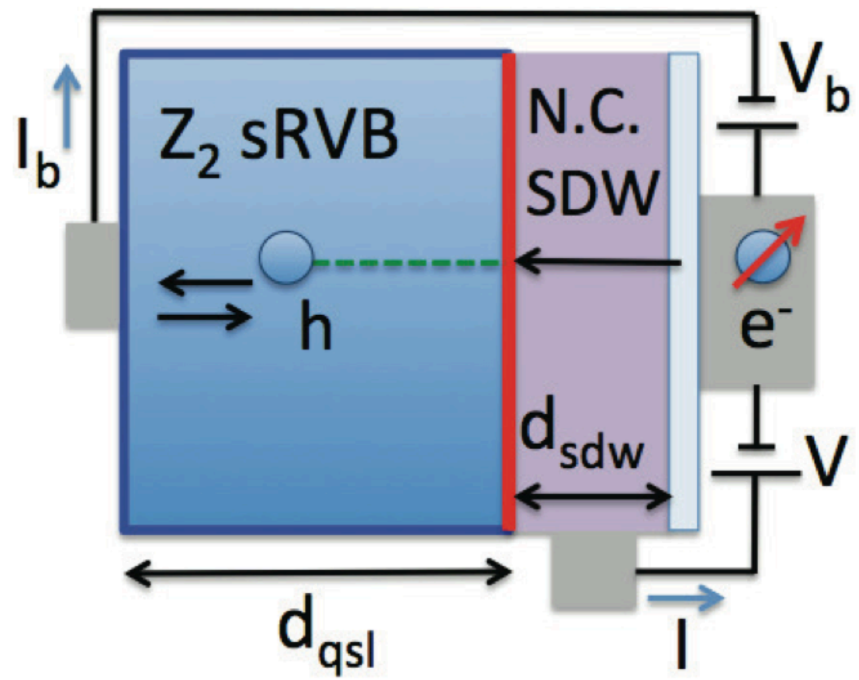
z_α = bosonic spinon

$$\delta H_{edge} = t_{edge} \sum_i c^\dagger_{Li} c_{Ri} + H.c. = t_{edge} \langle z_{L\alpha} \rangle \sum_i d^\dagger_{Li} c_{Ri} + H.c.$$

Electron can coherently pass through the SDW and into the spin liquid as a fermionic holon

Tomasch Oscillations in Spin Liquids

- If the bosonic holon is a stable quasiparticle excitation in the Z2 spin liquid:

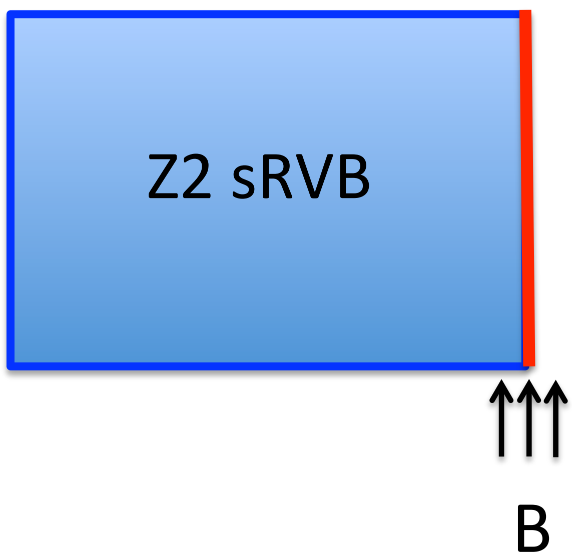


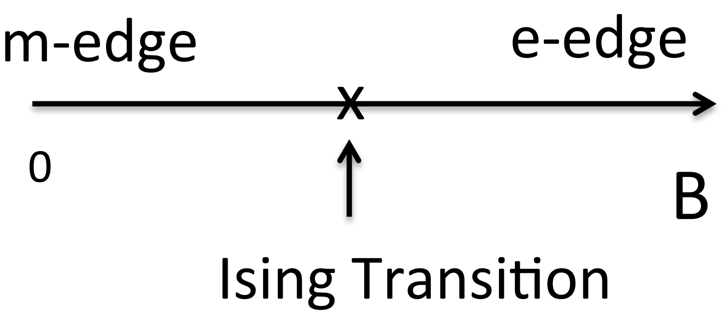
Expect oscillations in dI/dV with period set by d

Only possible with e-type edge

e-type boundaries from magnetism

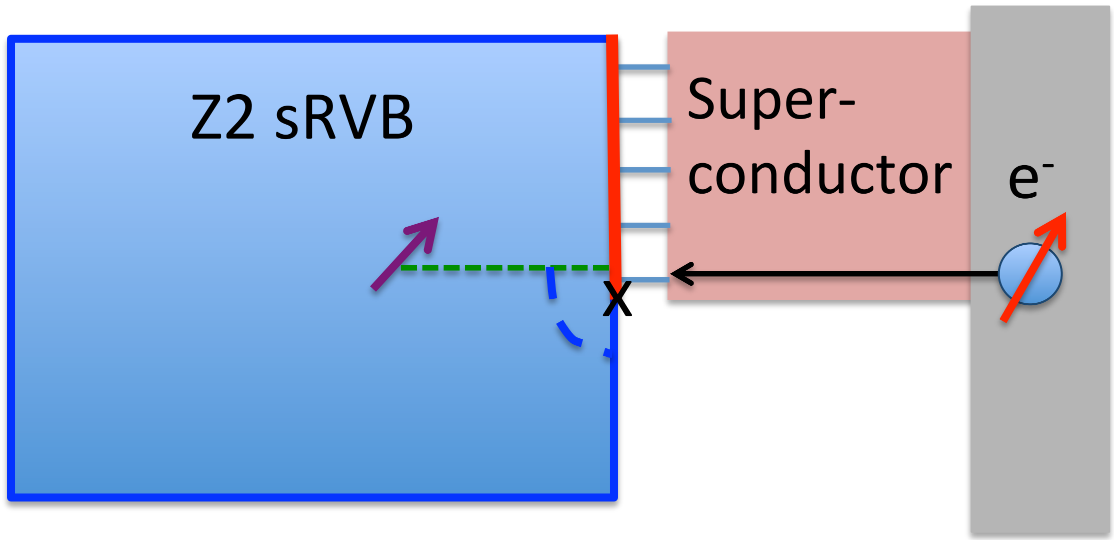
- XXZ spin system: magnetic field applied to boundary can induce topological transition to e- edge





Coherent fermion-boson transmutation

Signature of Majorana fermion zero mode:



- Near domain wall, fermionic spinon can emit/absorb a vortex from m edge

- Electron coherently enters as **bosonic** spinon

Conclusion

- Theory of gapped boundaries and domain walls
  - Topologically distinct gapped edges → Lagrangian subgroups
  - Domain walls → Topological degeneracies, exotic zero modes,

    Non-abelian statistics in an abelian phase.

- Possible new experimental probes of topological order
  - Direct coupling to fractionalized quasiparticles
  - Experimental proposal to detect topology-dependent ground state degeneracies in FQH states.

Barkeshli, Qi 2013
Barkeshli, Oreg, Qi 2014