Topological Phases of Quantum Matter
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Dissipatively Induced Quantum Phases of Atomic Fermions

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Collaborations:
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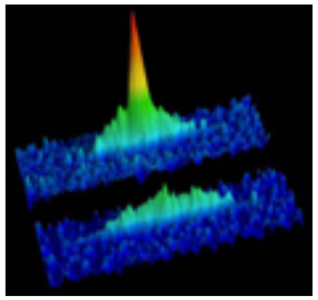
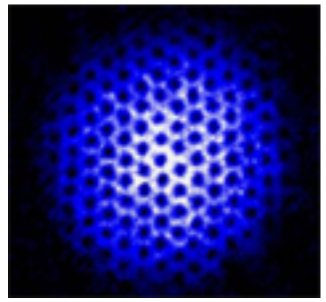
C. Bardyn (Caltech), A. Imamoglu (ETH)

FWF Der Wissenschaftsfonds.

Motivation

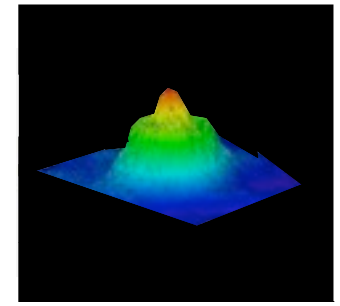


Bose-Einstein Condensate
(1995)



Vortices
(1999)

Many-body physics
with cold atoms



Mott Insulator
(2002)

Fermion superfluid
(2003)

Common theme:

many-body system

Temperature T,
particle number N

- closed system (isolated from environment)
- stationary states in thermodynamic equilibrium

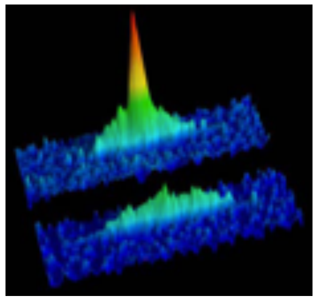
➡ thermalization/equilibration (PennState, Berkeley, Chicago, ...)

➡ sweep and quench many-body dynamics (Munich, Vienna)

➡ metastable excited many-body states (Innsbruck, MIT, ...)

➡ ...

Motivation



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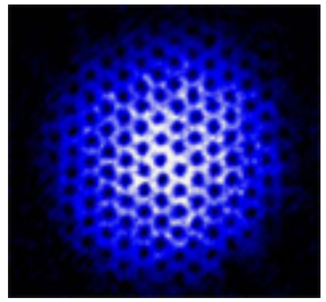

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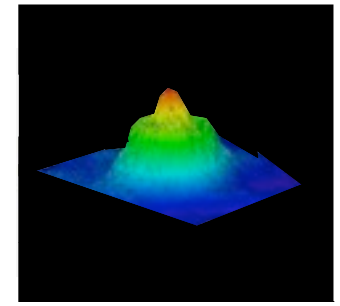

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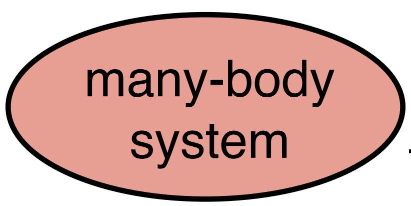
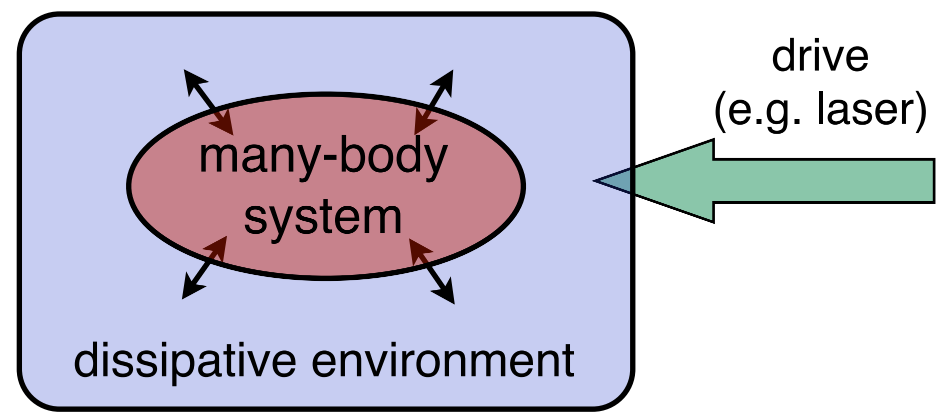


Temperature T,
particle number N

- closed system (isolated from environment)
- stationary states in thermodynamic equilibrium

Novel Situation: Cold atoms as open many-body systems



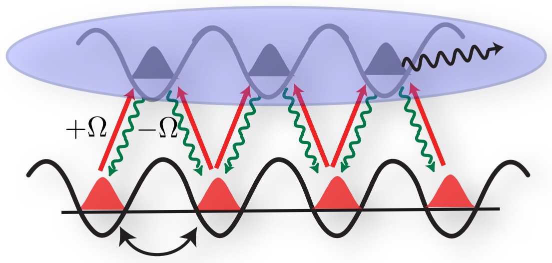
- natural occurrences of dissipation

➡ no immediate condensed matter counterpart

- use manipulation tools of quantum optics

➡ drive/dissipation as dominant resource of many-body dynamics!

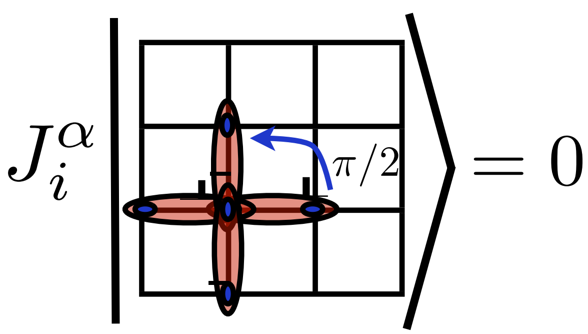
Outline



Many-body physics with tailored dissipation

- basic idea

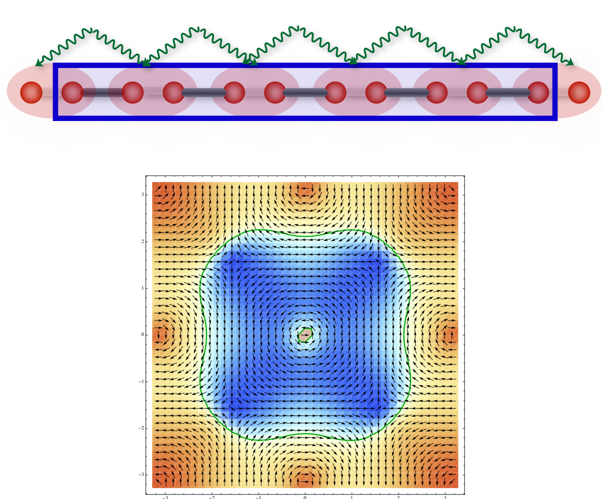
↓



Dissipatively induced fermionic pairing

- pairing mechanism
- potential application: Cooling of atomic Fermi-Hubbard model

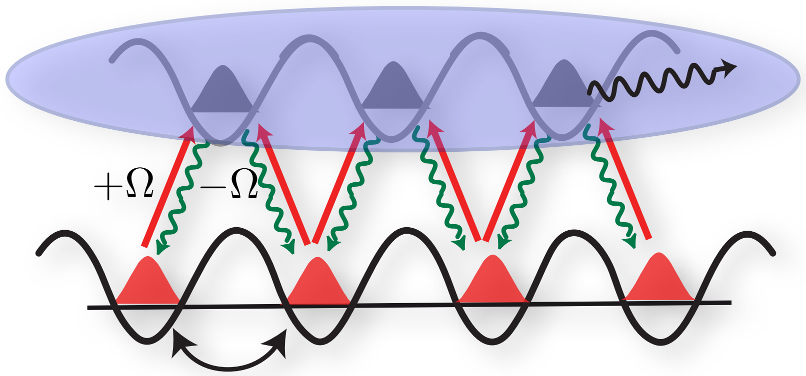
↓



Topology by dissipation

- targeted cooling into topological states
- phys. realization with cold atoms
- characteristic many-body properties in 1 and 2 dimensions

Many-body physics with tailored dissipation



SD et al., Nature Physics (2008)

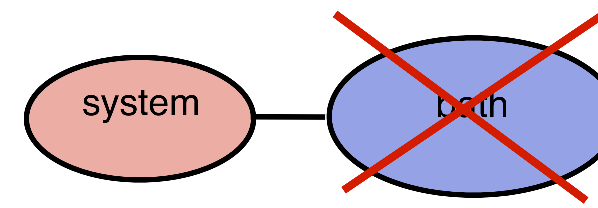
B. Kraus, SD, et al PRA (2008)

Many-Body Physics with Dissipation: Description

- Many-Body master equations

Lindblad operators

$$\partial_t \rho = \underbrace{-i[H,\rho]}_{\text{coherent evolution}} + \underbrace{\kappa \sum_i (J_i \rho J_i^\dagger - \tfrac{1}{2}\{J_i^\dagger J_i, \rho\})}_{\text{dissipative evolution}}$$



$\mathcal{L}[\rho]$ -- Liouvillian operator

➡ extend notion of Hamiltonian engineering to dissipative sector

➡ microscopically well controlled non-equilibrium many-body quantum systems

➡ here: focus on H = 0

- Important concept: Dark states

$$J_i|D\rangle = 0 \quad \forall i$$

$$\Rightarrow \mathcal{L}[|D\rangle\langle D|] = 0$$

➡ time evolution stops when $\rho = |D\rangle\langle D|$

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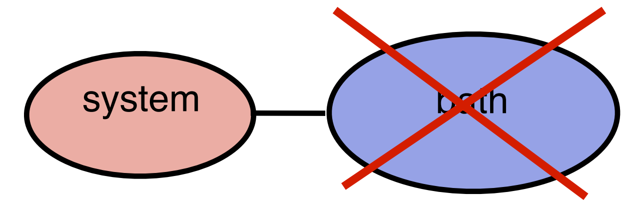
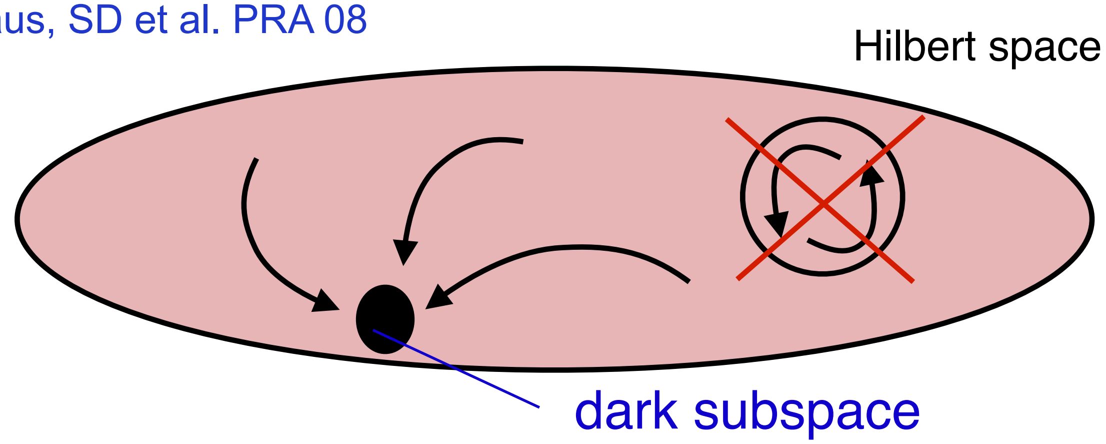


$\mathcal{L}[\rho]$ -- Liouvillian operator

- Interesting situation: unique dark state solution

B. Kraus, SD et al. PRA 08



- dark subspace one-dimensional
- no other stationary solutions

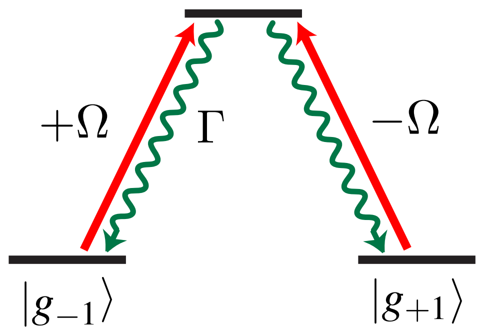
➡ directed motion in Hilbert space $\rho \overset{t\to\infty}{\longrightarrow} |D\rangle\langle D|$

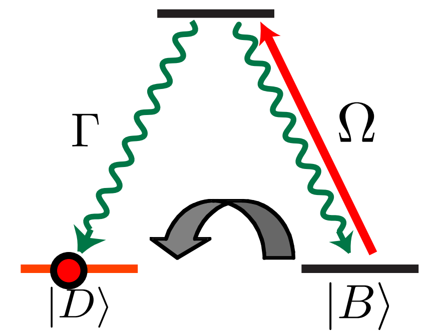
➡ dissipation increases purity

Dark states: An analogy

SD et al. Nat. Phys. (2008)
F. Verstraete et al. Nat. Phys. (2009)

- optical pumping: three internal (electronic) levels (Aspect, Cohen-Tannoudji; Kasevich, Chu)

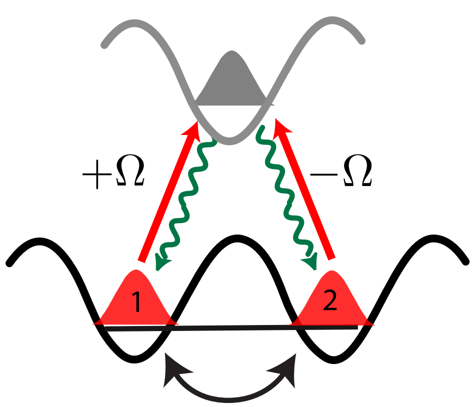


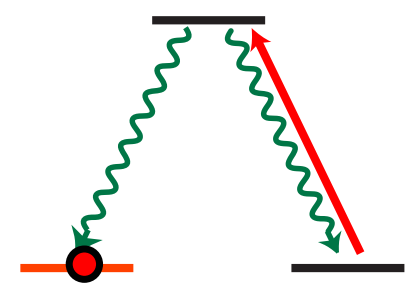


dark state — $|D\rangle \sim |g_{+1}\rangle + |g_{-1}\rangle$

bright state — $|B\rangle \sim |g_{+1}\rangle - |g_{-1}\rangle$

- 1 atom on 2 sites: external (spatial) degrees of freedom

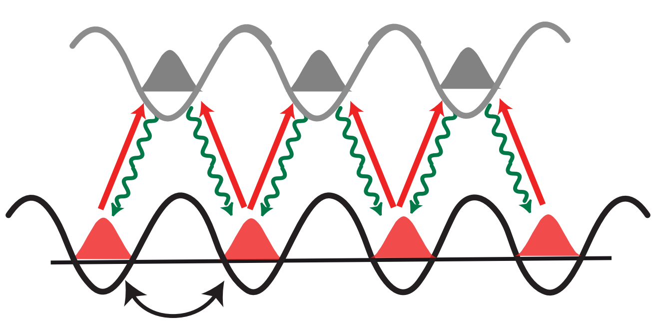




$(a_1^\dagger + a_2^\dagger)\,|\mathrm{vac}\rangle$ symmetric

$(a_1^\dagger - a_2^\dagger)\,|\mathrm{vac}\rangle$ anti-symmetric

- N atoms on M sites



$$|BEC\rangle = \frac{1}{N!}\Big(\sum_\ell a_\ell^\dagger\Big)^N |vac\rangle$$

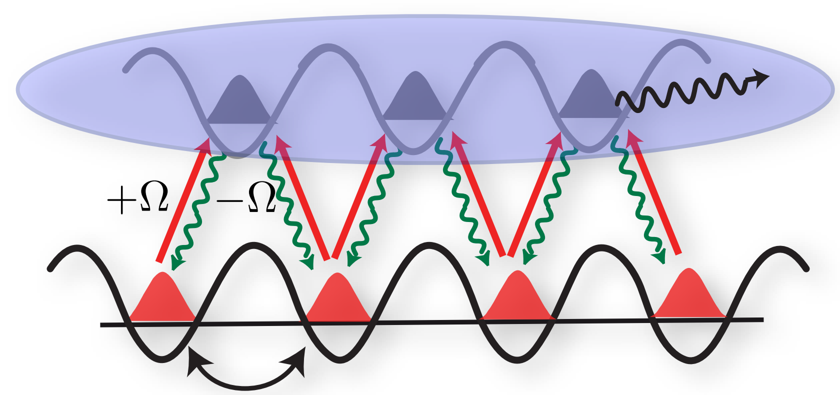
➡ combination of drive and dissipation enables purification
(no conflict with second law of thermodynamics)

Sketch of implementation with cold bosonic atoms

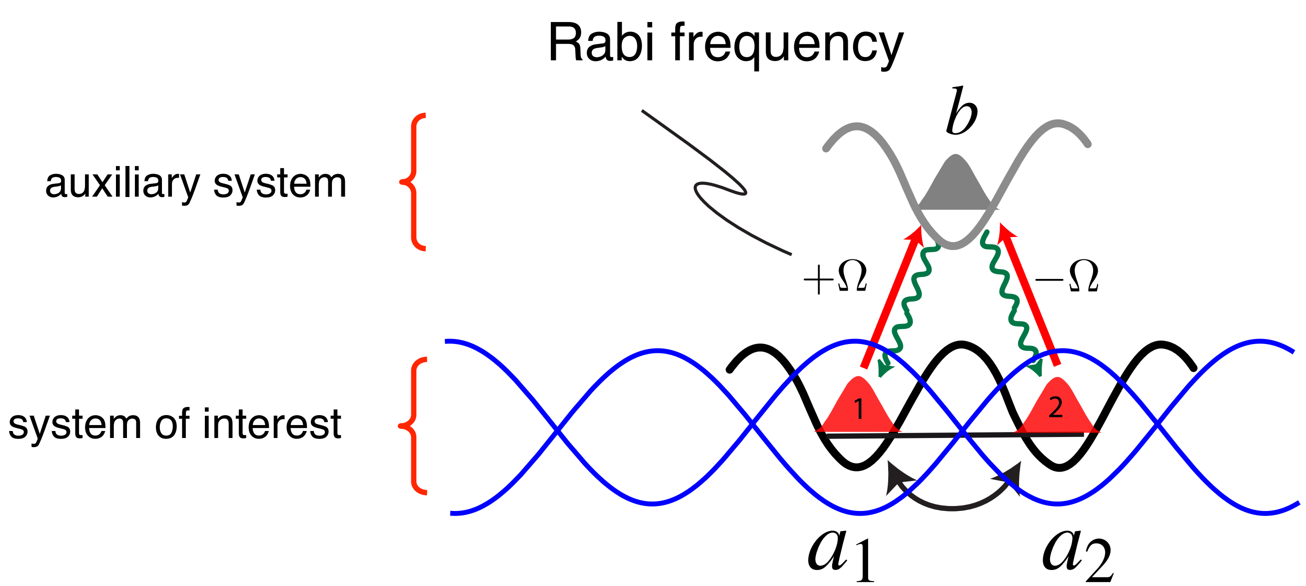
- Lindblad operators for BEC dark state: locally mapping any antisymmetric component into the symmetric one

$$\boxed{J_i = (a_i^\dagger + a_{i+1}^\dagger)(a_i - a_{i+1})}$$

by immersion of driven system into BEC reservoir

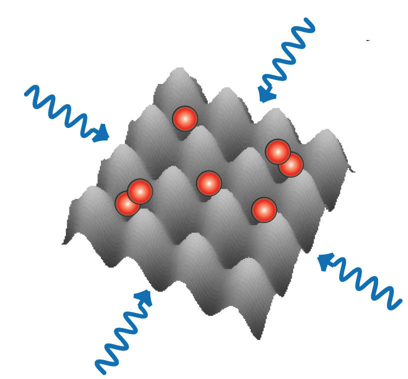


(i) Drive: coherent coupling to auxiliary system with double wavelength Raman laser



$$\lambda_{\text{laser}} = 2\lambda_{\text{lattice}}$$

driving laser



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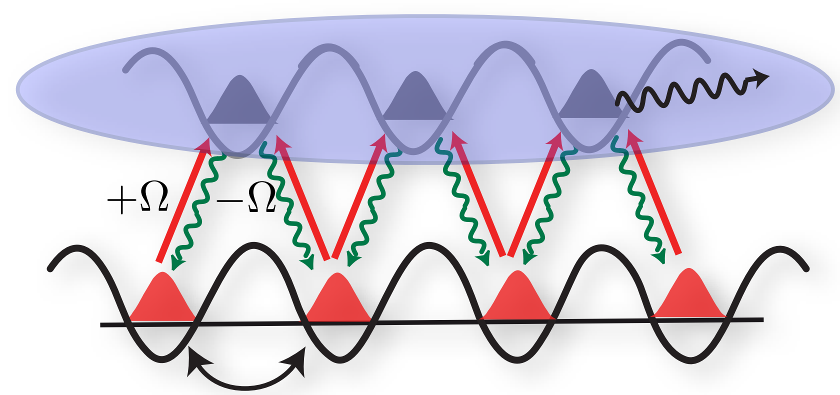
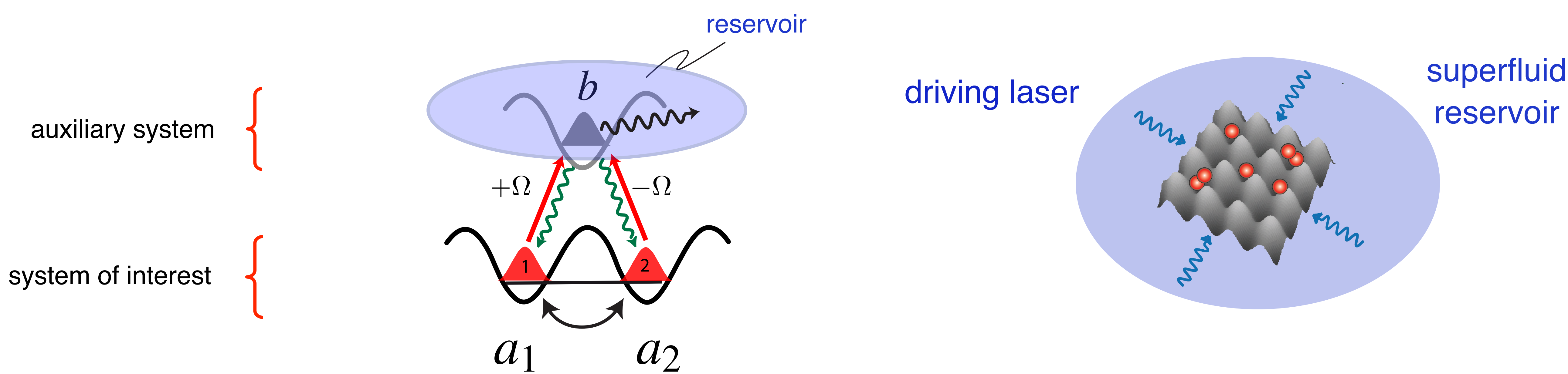


(ii) Dissipation: phonon emission into superfluid reservoir



Summary: Dissipative Many-Body State Preparation

- Lindblad operators for BEC dark state:

$$J_i = (a_i^\dagger + a_{i+1}^\dagger)(a_i - a_{i+1}) \qquad J_i|\mathrm{BEC}\rangle = 0$$

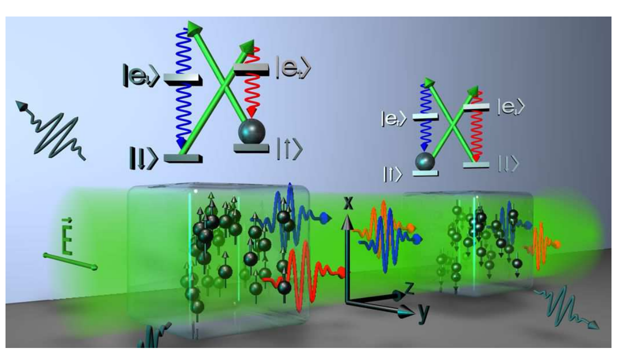
➡ Long range phase coherence/ boson condensation builds up from quasilocal dissipative operations

- Uniqueness of stationary solution can be shown (for fixed particle number)

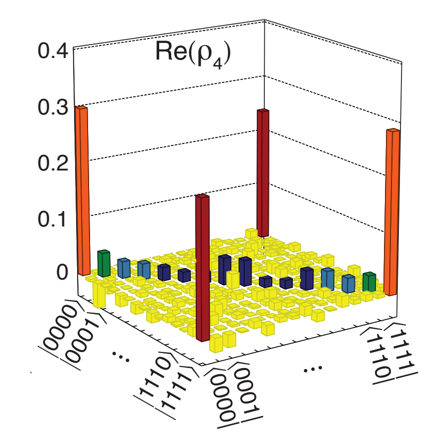
➡ Ordered phase reached from arbitrary initial state

$$\Longrightarrow \rho(t) \longrightarrow |BEC\rangle\langle BEC| \text{ for } t \to \infty$$

First experimental realizations

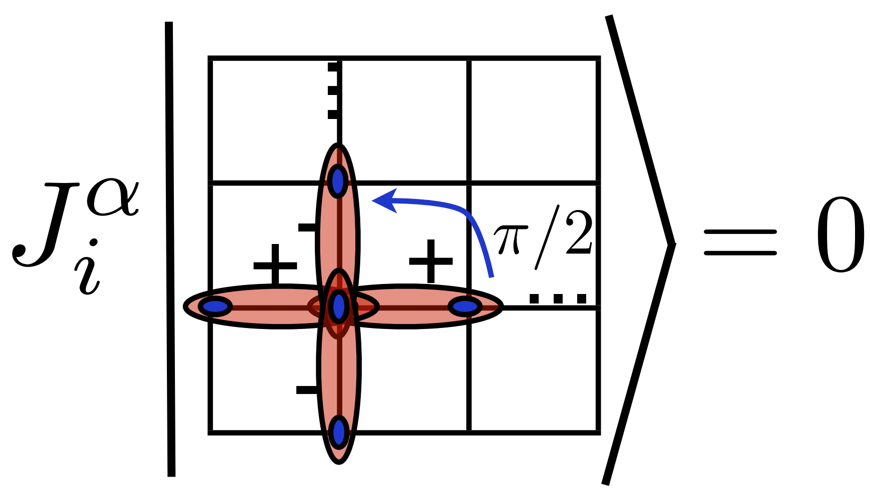


Entanglement by dissipation in atomic spin system
(Polzik group, Kopenhagen, PRL 2011)



Open-system simulator with trapped ions
(Blatt group, Innsbruck, Nature 2011)

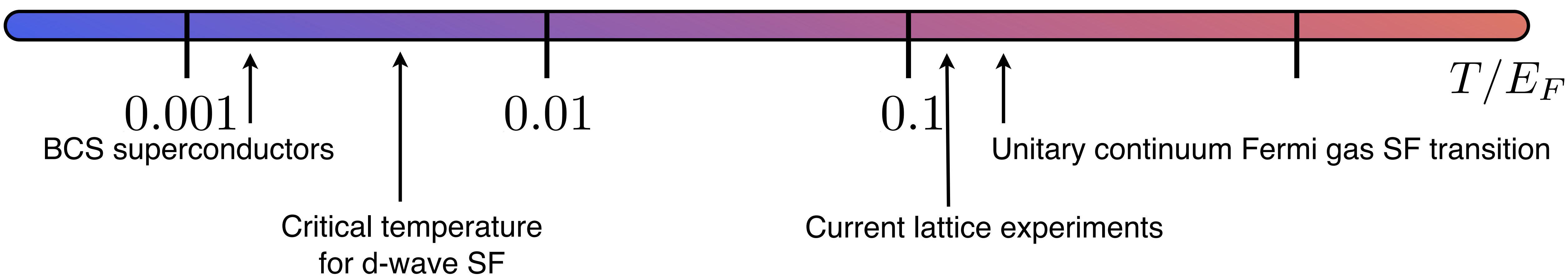
Dissipatively Induced Fermion Pairing



SD, W. Yi, A. Daley, P. Zoller, PRL (2010);
W. Yi, SD, A. Daley, P. Zoller, New J. Phys. (2012);

Motivation: Fermi-Hubbard Model Quantum Simulation

- Goal: finding ground state of Fermi-Hubbard model
- Clean realization of fermion Hubbard model possible
  - Detection of Fermi surface in 40K (M. Köhl et al. PRL 05)
  - Fermionic Mott Insulators (R. Jördens et al. Nature 08; U. Schneider et al., Science 08)
- Cooling problematic: small d-wave gap sets tough requirements



➡ Still need to be 10-100x cooler

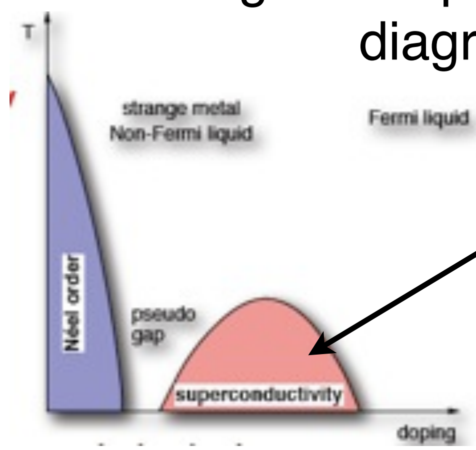
- Roadmap via dissipative quantum state preparation approach:

(1) Dissipatively prepare pure (zero entropy) state close to the expected ground state

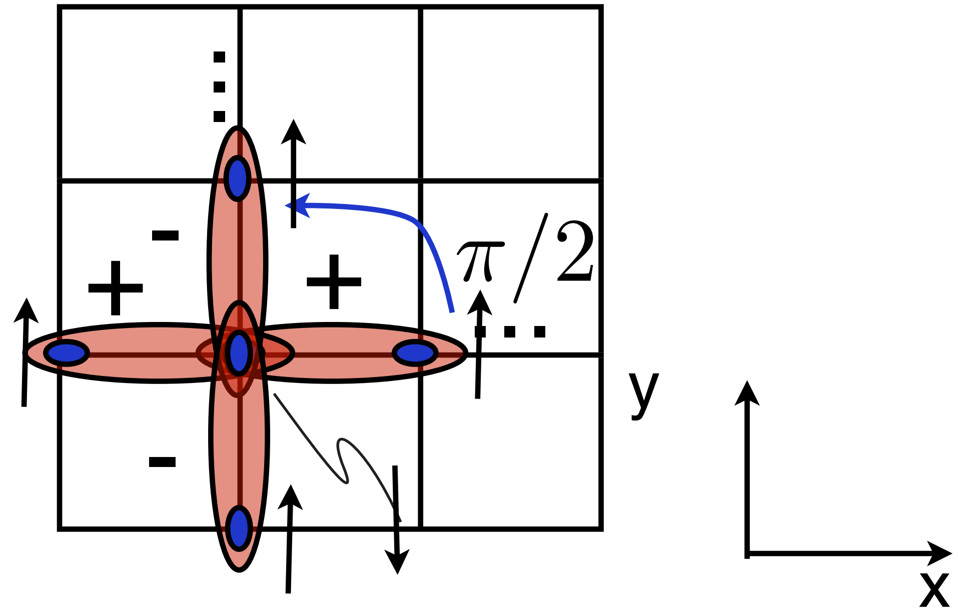
(2) Adiabatic passage to the Hubbard ground state

The State to Be Prepared

High-Tc cuprate phase diagram



d-wave SC



product state

$$|\mathrm{BCS}_N\rangle \sim (d^\dagger)^{N/2}|\mathrm{vac}\rangle \qquad d^\dagger = \sum_i [\underbrace{c^\dagger_{i+\mathbf{e}_x,\uparrow} + c^\dagger_{i-\mathbf{e}_x,\uparrow}} - (\underbrace{c^\dagger_{i+\mathbf{e}_y,\uparrow} + c^\dagger_{i-\mathbf{e}_y,\uparrow}})]c^\dagger_{i,\downarrow}$$

- Features shared with expected Hubbard ground state:

  (1) Quantum numbers

  ➡ no phase transition crossed in preparation process: gap protection

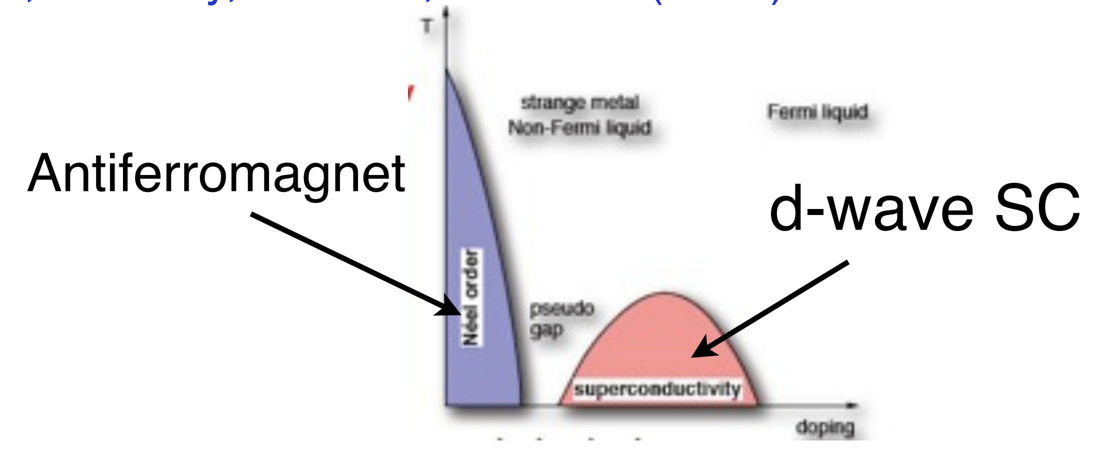
  (2) Energetically close?

  ➡ off-site pairing avoids excessive double occupancy

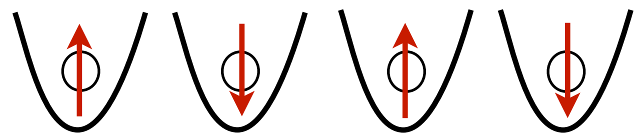
➡ Task: find “parent Liouvillian” for this state
➡ “cooling” into the d-wave

SD, W. Yi, A. Daley, P. Zoller, PRL 105 (2010)

Pairing mechanism



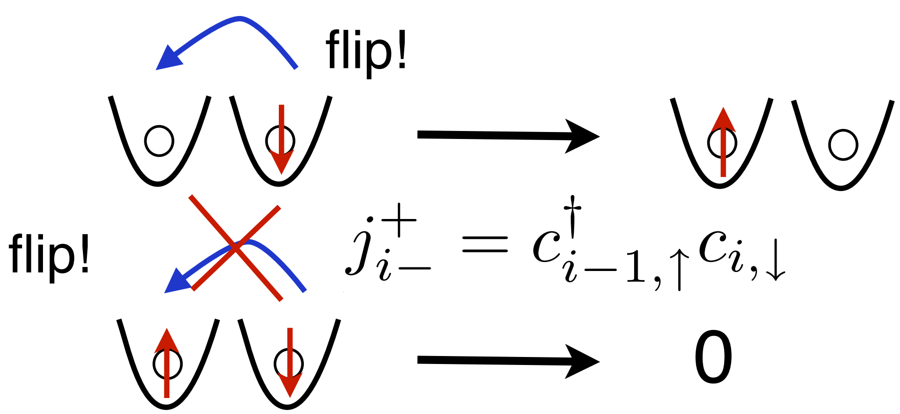
- Consider 1D cut only
- Half filling: Neel state for antiferromagnetism



- Lindblad operators (1D): e.g. $j_{i-}^{+} = c_{i-1,\uparrow}^{\dagger} c_{i,\downarrow}$

full set:

$$j_\ell = \{ j_{i\pm}^{\pm},\ j_{i\pm}^{z} \}$$

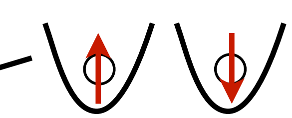


$$j_{i-}^{+} = c_{i-1,\uparrow}^{\dagger} c_{i,\downarrow}$$

➡ dark state based on Fermi statistics

- D-wave (analog) state: interpret the state as a symmetrically delocalized Neel order

$$|\mathrm{BCS}_1\rangle = (d^\dagger)^N |\mathrm{vac}\rangle, \ d^\dagger = \sum_i (c_{i+1,\uparrow}^{\dagger} + c_{i-1,\uparrow}^{\dagger}) c_{i,\downarrow}^{\dagger}$$



- Lindblad operators (1D): e.g. $J_i^+ = j_{i,+}^{+} + j_{i,-}^{+} = (c_{i+1,\uparrow}^{\dagger} + c_{i-1,\uparrow}^{\dagger}) c_{i,\downarrow}$

phase locking

➡ Combine fermionic Pauli blocking with delocalization as for bosons

Dissipative Pairing: The d-wave jump operators

- The full set of Lindblad operators is found from

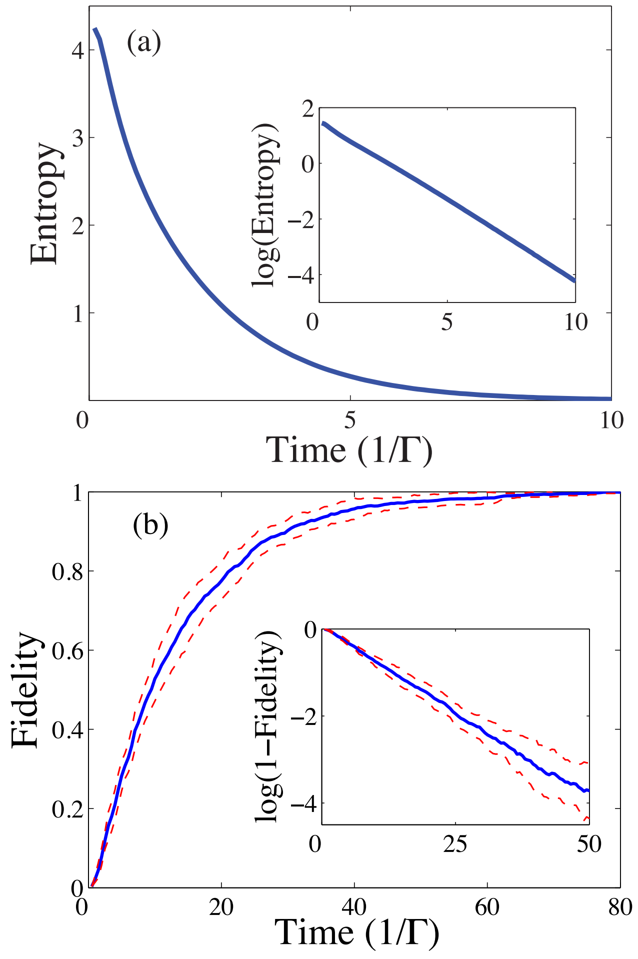
$$[J_i^\alpha, G^\dagger] = 0 \quad \forall i, \alpha \qquad\qquad |D(N)\rangle \sim G^{\dagger\, N}|\text{vac}\rangle$$

- given by

Pauli matrices

$$\boxed{J_i^\alpha = (c_{i+1}^\dagger + c_{i-1}^\dagger)\sigma^\alpha c_i} \qquad c_i = \begin{pmatrix} c_{\uparrow,i} \\ c_{\downarrow,i} \end{pmatrix}$$

- Discussion: These operators
  - form exhaustive set: d-wave steady state unique, reached for arbitrary initial state
  - bilinear: describe the redistribution of the superposition of a single particle
  - generalization to arbitrary symmetries possible



➡ Projective pair condensation mechanism, does not rely on attractive conservative forces

Fixed Number vs. Fixed Phase Lindblad Operators

- spinless fermions for simplicity

- fixed number Lindblad operators

$$J_i = C_i^\dagger A_i$$

- fixed phase Lindblad operators

$$j_i = C_i^\dagger + r e^{\mathrm{i}\theta} A_i$$

$$\Leftrightarrow$$

- resulting dark state

$$|BCS, N\rangle = G^{\dagger\, N} |\mathrm{vac}\rangle$$

- resulting dark state (with $\Delta N \sim 1/\sqrt{N}$

$$|BCS, \theta\rangle = \exp(r e^{\mathrm{i}\theta} G^\dagger)|\mathrm{vac}\rangle$$

- requirements

translation invariant creation and annihilation part

$$C_i^\dagger = \sum_j v_{i-j} a_j^\dagger \qquad C_k^\dagger = v_k a_k^\dagger$$

$$A_i = \sum_j u_{i-j} a_j \qquad A_k = u_k a_k$$

antisymmetry

$$\varphi_k = \frac{v_k}{u_k} = -\varphi_{-k}$$

$$G^\dagger = \sum_k \varphi_k c_{-k}^\dagger c_k^\dagger$$

- comment: allows us to construct exactly solvable interacting Hubbard models with parent Hamiltonian

$$H = \sum_i J_i^\dagger J_i \qquad J_i |D\rangle = 0 \,\forall i$$

Spontaneous Symmetry Breaking and Dissipative Gap

- use equivalence of fixed number and fixed phase states in thdyn limit
- use exact knowledge of stationary state: linearized long time evolution

$$\mathcal{L}[\rho] = \kappa \sum_i [j_i \rho j_i^\dagger - \tfrac{1}{2}\{j_i^\dagger j_i, \rho\}] = \sum_{\mathbf{q}} \kappa_{\mathbf{q}} [j_{\mathbf{q}} \rho j_{\mathbf{q}}^\dagger - \tfrac{1}{2}\{j_{\mathbf{q}}^\dagger j_{\mathbf{q}}, \rho\}]$$

- properties
  - relation to microscopic operators

fixed by average particle number

fixed spontaneously

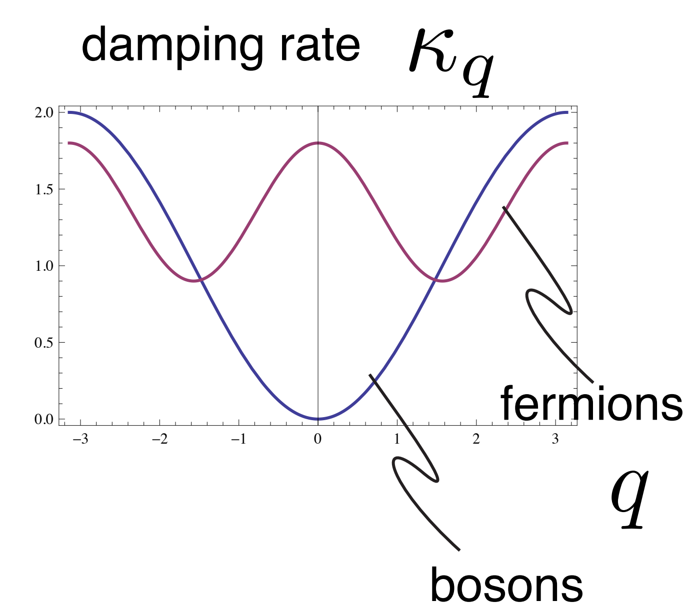
$$\underbrace{J_i = C_i^\dagger A_i}_{\text{fixed number}} \quad \xrightarrow[\text{“low energy limit”}]{t \to \infty} \quad \underbrace{j_i = C_i^\dagger + r e^{\mathrm{i}\theta} A_i}_{\text{fixed phase}}$$

  - effective fermionic quasiparticle operators

$$j_{\mathbf{q}} |BCS, \theta\rangle = 0$$; fulfill Dirac algebra -> uniqueness

  - effective damping rate with a “dissipative gap”

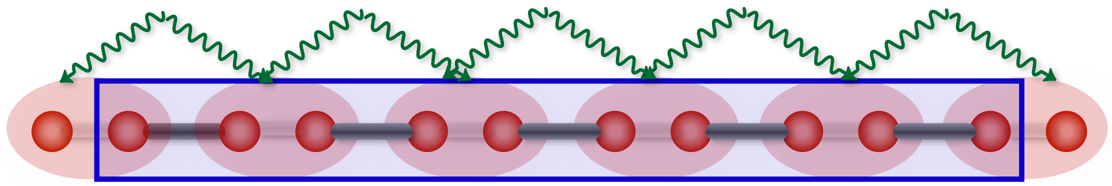
$$\kappa_{\mathbf{q}} = \kappa_0 \int_{\mathrm{BZ}} \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{|u_{\mathbf{k}} v_{\mathbf{k}}|^2}{|u_{\mathbf{k}}|^2 + |\alpha v_{\mathbf{k}}|^2} (|u_{\mathbf{q}}^2| + |v_{\mathbf{q}}^2|) \geq \kappa_0 n$$



➡ Scale generated in long time evolution ; exponentially fast approach of steady state

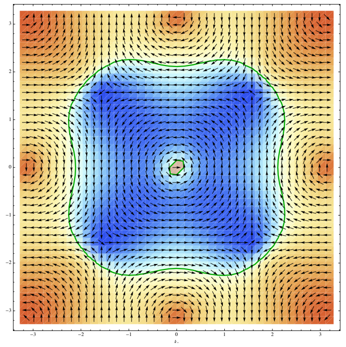
➡ Robustness of prepared state against perturbations

Topology by Dissipation



One Dimension

SD, E. Rico, M. A. Baranov, P. Zoller,
Nat. Phys. (2011)



Two Dimensions

C. Bardyn, E. Rico, M. Baranov, A. Imamoglu, P. Zoller, SD, PRL (2012);
New J. Phys. (2013);
J. C. Budich, P. Zoller, SD, in preparation

Key Questions:

- Is topological order an exclusive feature of Hamiltonian ground states, or pure states?
- Which topological states be reached by a targeted, dissipative cooling process?
- What are proper microscopic, experimentally realizable models?
- What are the parallels and differences to the equilibrium (ground state) scenario?

Topological States of Matter [Hamiltonian setting]

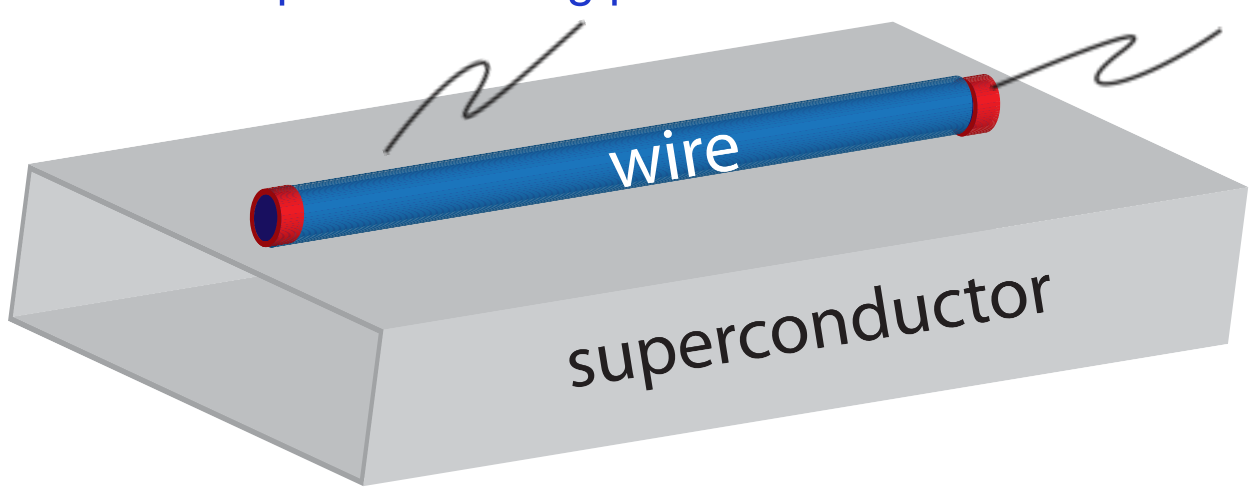
- **topological states of matter (noninteracting fermions)**
  - beyond the Landau paradigm — Hasan and Kane, RMP (2010)
  - robust edge states and non-Abelian excitations — Qi and Zhang, RMP (2011)
  - topological protected quantum memory and quantum computing — Nayak et al., RMP (2008)

- **Quantum Hall systems and topological insulators/ BdG superconductors**

  classification: Schnyder et al. PRB (2008); Kitaev (2009)
  based on Altland and Zirnbauer, PRB (1997)

  - minimal model: Kitaev's quantum wire

  Kitaev, Phys. Usp. (2001)



- Wire Hamiltonian (spinless [spin-polarized] fermions)

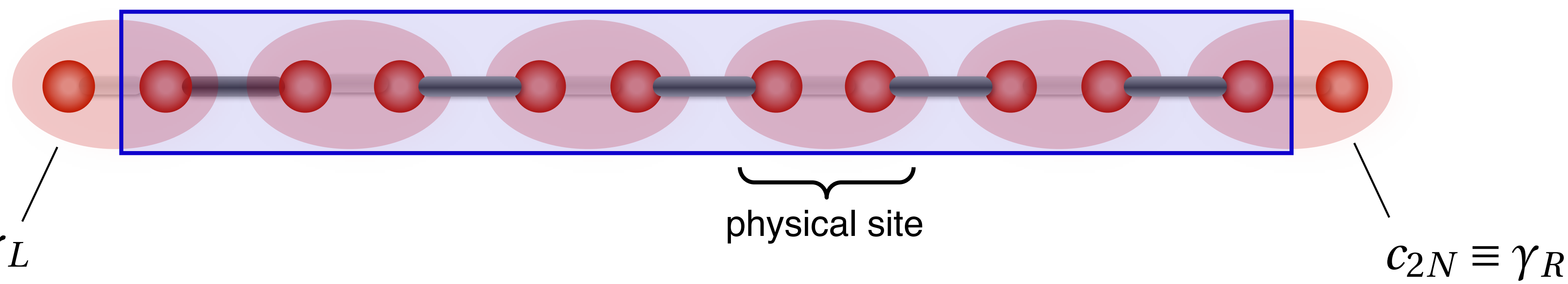
$$H = \sum_i \Big[\underbrace{-J}_{\text{hopping}} a_i^\dagger a_{i+1} + \underbrace{\Delta}_{\text{superconducting order parameter}} a_i a_{i+1} + \text{h.c.} - \underbrace{\mu}_{\text{chemical potential}} \Big(a_i^\dagger a_i - \frac{1}{2} \Big) \Big]$$

Reminder: Kitaev's quantum wire (Hamiltonian scenario)

Kitaev, Phys. Usp. (2001)

- Hamiltonian $$H = 2J \sum_{i=1}^{N-1} (\tilde{a}_i^\dagger \tilde{a}_i - \tfrac{1}{2}) + 0 \cdot \tilde{a}_N^\dagger a_N$$ with $\tilde{a}_i = \frac{1}{2}\mathrm{i}(a_{i+1} + a_{i+1}^\dagger - a_i + a_i^\dagger)$ for $(\Delta = J,\, \mu = 0)$ quasilocal

- "Majorana fermions" $a_j \equiv \frac{1}{2}(c_{2j-1} + \mathrm{i}c_{2j})$ $\quad c_j^\dagger = c_j,\ \{c_j, c_l\} = 2\delta_{jl}$



unpaired Majorana edge modes

bulk

- p-wave superfluid in ground state

$$\tilde{a}_i|\mathrm{p-wave}\rangle = 0 \ (i = 1, \ldots, N-1)$$

- *off-site* paired Majoranas
- gap in spectrum: 2J

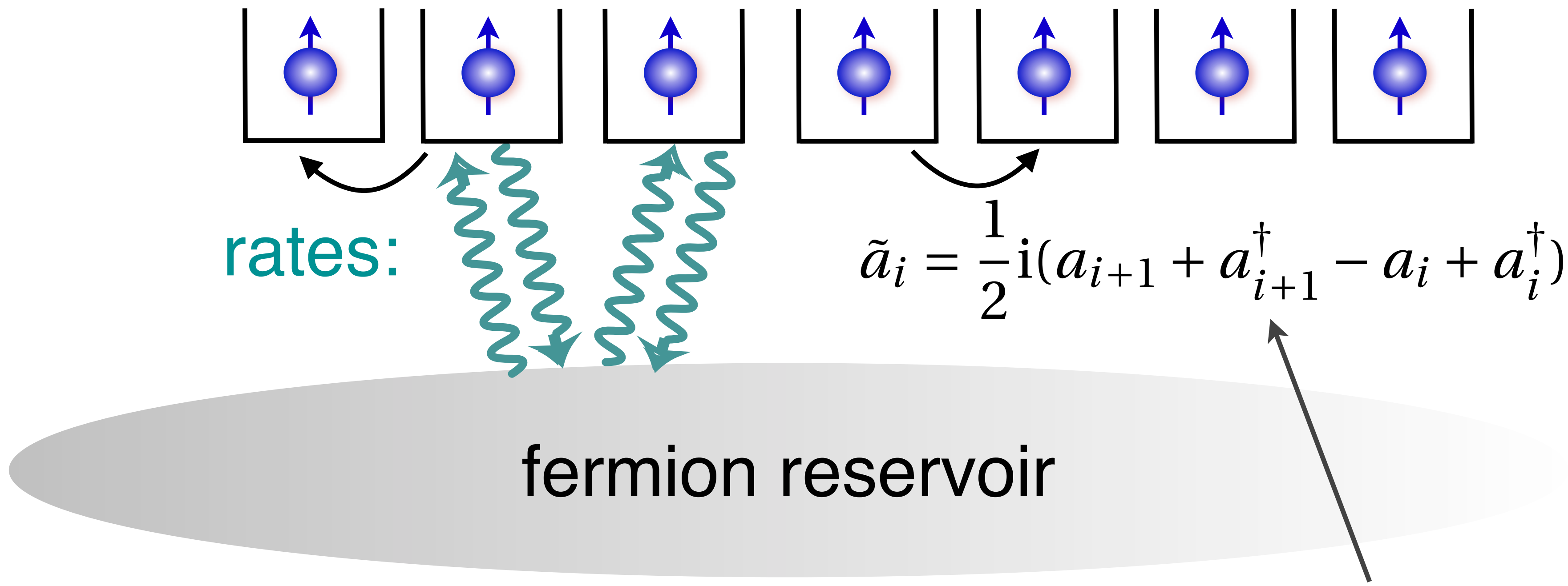
edge

- unpaired Majoranas γ_L, γ_R
- non-local Bogoliubov fermion

$$|0\rangle,\ |1\rangle = \tilde{a}_N^\dagger |0\rangle$$

- zero energy

Dissipative Topological Quantum Wire



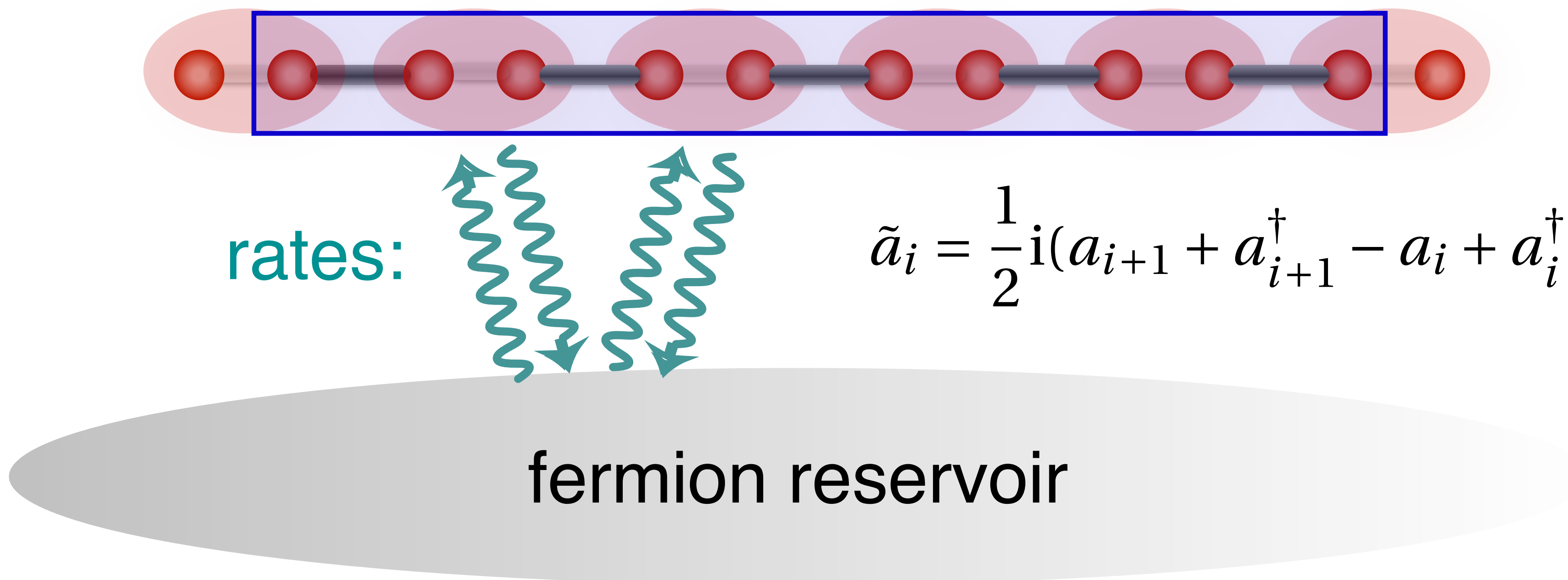
$$\tilde{a}_i = \frac{1}{2}\mathrm{i}(a_{i+1} + a_{i+1}^\dagger - a_i + a_i^\dagger)$$

Lindblad operators
~ Bogoliubov operators
(quasilocal)

- master equation

$$\dot{\rho} = \underbrace{\kappa \sum_{i=1}^{N-1} \left(\tilde{a}_i \rho \tilde{a}_i^\dagger - \frac{1}{2} \tilde{a}_i^\dagger \tilde{a}_i \rho - \rho \frac{1}{2} \tilde{a}_i^\dagger \tilde{a}_i \right)}_{\text{Liouville operator}}$$

Dissipative Topological Quantum Wire



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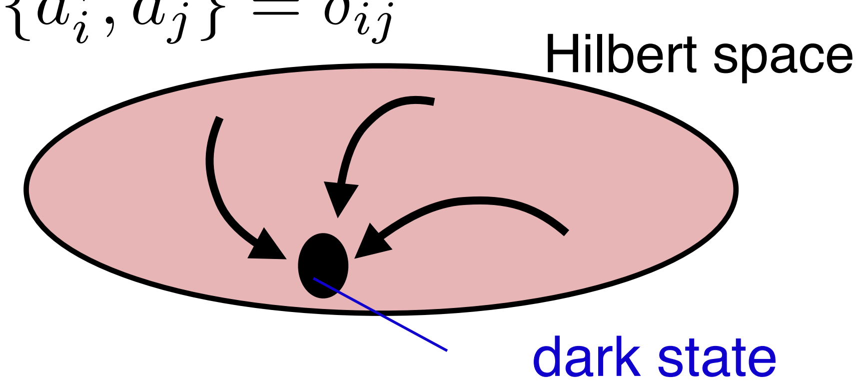
bulk driven to pure steady state:
Kitaev's ground state

$$\tilde{a}_i |\mathrm{p-wave}\rangle = 0 \ (i = 1, \ldots, N-1)$$

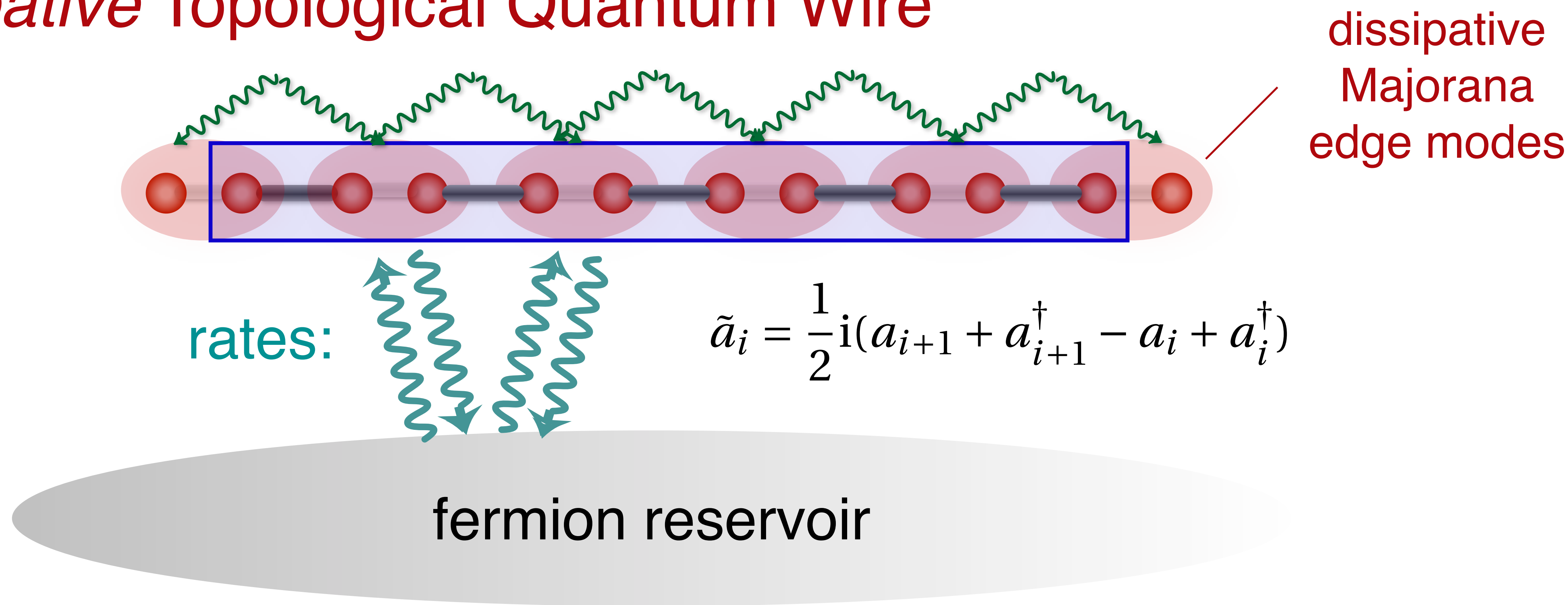
dark state = topological p-wave

$$\{\tilde{a}_i, \tilde{a}_j\} = 0 \quad \{\tilde{a}_i^\dagger, \tilde{a}_j\} = \delta_{ij}$$

=> dark state unique



Dissipative Topological Quantum Wire



$$\tilde{a}_i = \frac{1}{2}\mathrm{i}(a_{i+1} + a_{i+1}^\dagger - a_i + a_i^\dagger)$$

- master equation

$$\dot{\rho} = \kappa \sum_{i=1}^{N-1} \left(\tilde{a}_i \rho \tilde{a}_i^\dagger - \frac{1}{2} \tilde{a}_i^\dagger \tilde{a}_i \rho - \rho \frac{1}{2} \tilde{a}_i^\dagger \tilde{a}_i \right)$$

bulk driven to pure steady state: Kitaev's ground state

$$\tilde{a}_i |\mathrm{p-wave}\rangle = 0 \;\; (i = 1, \ldots, N-1)$$

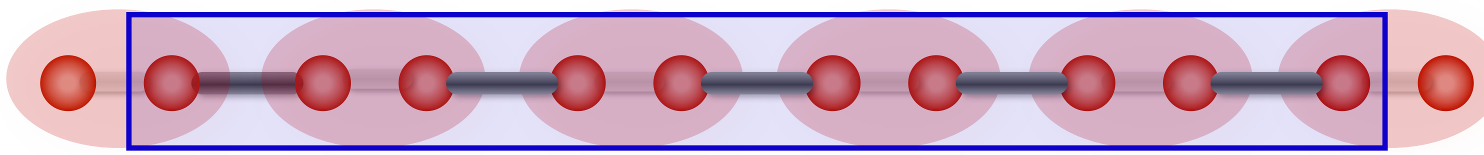
dark state = topological p-wave

Majorana edge modes decoupled from dissipation

$$|0\rangle, \; |1\rangle = \tilde{a}_N^\dagger |0\rangle$$

non-local decoherence free subspace

Dissipative Topological Quantum Wire



Edge - Bulk:

- dynamically isolated from each other

$$\rho_{\text{bulk-edge}} \lesssim e^{-\lambda_{\text{gap}} t} \rho_{\text{bulk-edge}}(0) \to 0$$

$$\Rightarrow t \to \infty : \rho \to \rho_{\text{edge}} \otimes \rho_{\text{bulk}}$$

- edge mode subspace protected by dissipative gap

$$\rho_{\text{bulk}}(\infty) = |\mathrm{p-wave}\rangle \langle \mathrm{p-wave}|$$

bulk cooled to pure steady state:
Kitaev's ground state

$$\tilde{a}_i |\mathrm{p-wave}\rangle = 0 \ (i = 1, \ldots, N-1)$$

dark state = topological p-wave

$$\dot{\rho}_{\text{edge}} = 0 \quad (\rho_{\text{edge}})_{\alpha\beta} \equiv \langle \alpha | \rho_{\text{edge}} | \beta \rangle \qquad |\alpha\rangle \in \{|0\rangle, |1\rangle\}$$

Majorana edge modes decoupled from dissipation

$$|0\rangle, \ |1\rangle = \tilde{a}_N^\dagger |0\rangle$$

non-local decoherence free subspace

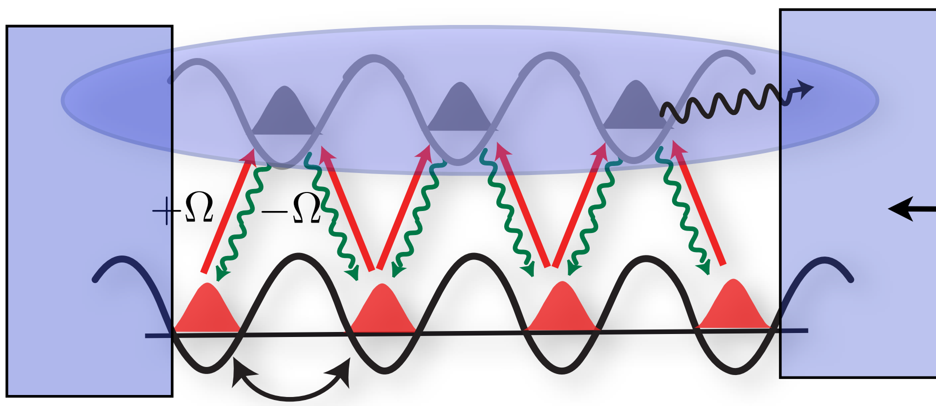
Implementation with Fermionic Atoms

- Microscopic implementation with spinless fermions (cold atoms)

$$J_i = (a_i^\dagger + a_{i+1}^\dagger)(a_i - a_{i+1})$$

immersion of driven system into BEC reservoir

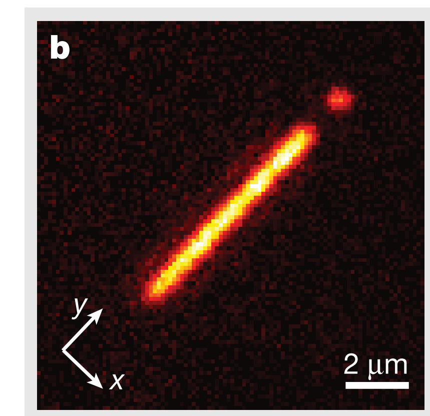
(similar to bosonic case above)



+

- physical edge via single site addressability

(Bakr et al. Nature 2009; Weitenberg et al, Nature 2011 also: Chicago, Pennstate, ...)



- Connection to quadratic theory: we obtain

interacting

linearized

$$J_i = \underbrace{(a_i^\dagger + a_{i+1}^\dagger)}\underbrace{(a_i - a_{i+1})}_{\hat{=}\ \text{“low energies”}} \xrightarrow{\text{long times}} j_i = \underbrace{(a_i^\dagger + a_{i+1}^\dagger}_{\propto\ \tilde{a}_i} + \underbrace{a_i - a_{i+1})}$$

dissipative gap emerges naturally

Kitaev’s Majorana operators

Properties: “Topology by Dissipation”

✓ generic features of topological states

➡ Insensitivity of edge modes against microscopic details in the bulk:

➡disorder

➡non-pure bulk states $\{j_i, j_j\} \neq 0$

✓ Adiabatic moving of dissipative Majoranas by changing $\mathcal{L}$

cf. work by Avron, Fraas, Graf, J. Stat. Phys. (2012); Avron, Fraas, Graf, Kennth, New J. Phys. (2010)

✓ topological origin

➡ Topological invariant of the bulk (for mixed, dissipative systems)

- Reason:

$$\frac{d}{dt}\rho = -i[A,\rho] + \sum_{a,b} |a\rangle \dot{\rho}_{ab} \langle b|,$$

adiabatic connection $A = \mathrm{i}\dot{U}^\dagger U$

$\dot{\rho}_{ab} \equiv \langle a(t)|\partial_t \rho | b(t)\rangle$
phys. evolution

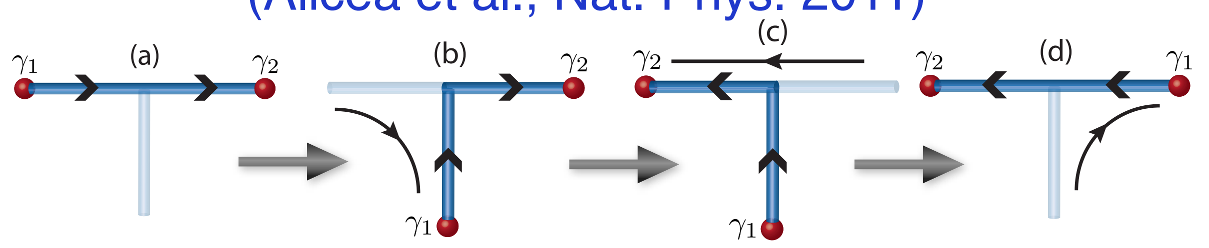
universal

- Implication:

➡ dissipative braiding in networks

➡ non-abelian statistics

(Alicea et al., Nat. Phys. 2011)



parallels Hamiltonian case

Topological invariant for mixed density matrices

- A Gaussian translationally invariant state is completely characterized by (spinless fermions):

$$\begin{pmatrix} \langle [a_k^\dagger, a_k] \rangle & \langle [a_k^\dagger, a_{-k}^\dagger] \rangle \\ \langle [a_{-k}, a_k] \rangle & \langle [a_{-k}, a_{-k}^\dagger] \rangle \end{pmatrix} = \vec{n}_k \vec{\sigma} = Q_k \qquad |\vec{n}_k| \leq 1 \quad \forall k \in (-\pi, \pi]$$

- Chiral symmetry for the state: There is $\Sigma \text{ s.t. } \{\Sigma, Q_k\} = 0 \quad \forall k$ $\quad \Sigma$ unitary, $\Sigma^2 = 1$

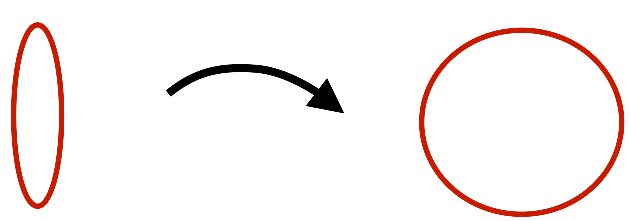
- Winding number: $$W = \frac{1}{4\pi \mathrm{i}} \int_{-\pi}^{\pi} dk \, \mathrm{tr}(\Sigma Q_k^{-1} \partial_k Q_k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk \vec{a} \cdot \left(\hat{\vec{n}}_k \times \partial_k \hat{\vec{n}}_k \right)$$

- pure states: $\forall k : \; |\vec{n}_k| = 1$ $\qquad \hat{\vec{n}}_k = \frac{\vec{n}_k}{|\vec{n}_k|}$

- defined if topology of circle is preserved

$$\forall k : \; |\vec{n}_k| > 0$$

i.e. mixed states



- circle collapses to line:

$$\exists k_0 : \; |\vec{n}_{k_0}| = 0$$

modes k_0 completely mixed

"purity gap" closes

Topological invariant for mixed density matrices

- A Gaussian translationally invariant state is completely characterized by (spinless fermions):

$$\begin{pmatrix} \langle [a_k^\dagger, a_k] \rangle & \langle [a_k^\dagger, a_{-k}^\dagger] \rangle \\ \langle [a_{-k}, a_k] \rangle & \langle [a_{-k}, a_{-k}^\dagger] \rangle \end{pmatrix} = \vec{n}_k \vec{\sigma} = Q_k \qquad |\vec{n}_k| \leq 1 \quad \forall k \in (-\pi, \pi]$$

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- pure states: $\forall k : \; |\vec{n}_k| = 1$ $\qquad \hat{\vec{n}}_k = \frac{\vec{n}_k}{|\vec{n}_k|}$

- non-pure states motivate the definition of spectral projector by smooth deformation

$$\mathcal{P}_k = \tfrac{1}{2}(\mathbb{I} - \hat{\vec{n}}_k \vec{\sigma}) \qquad \text{for} \qquad |\vec{n}_k| > 0 \qquad \text{finite “purity gap”}$$

➡ two gaps required for topological stability: damping and purity gap

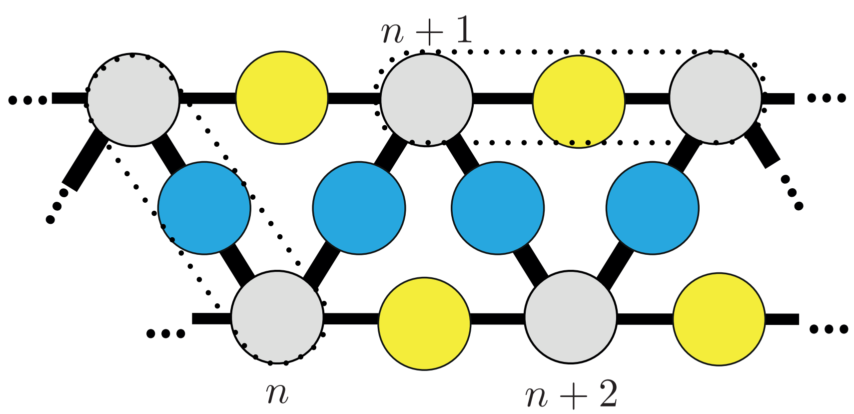
Two Gaps: Physical Implications

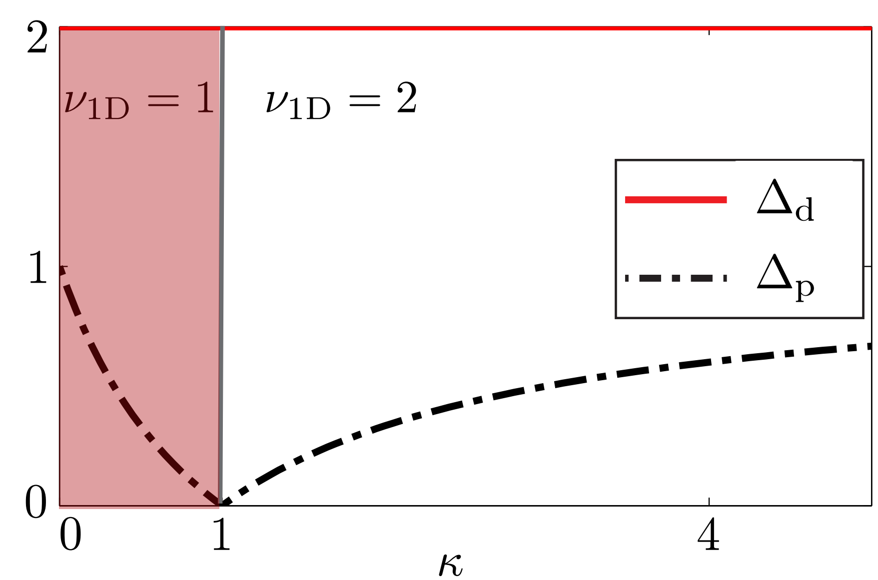
- topological phase transitions via different patterns of gap closing
  - $\Delta_d = 0,\ \Delta_p > 0$
  - $\Delta_d = 0,\ \Delta_p = 0$
  - $\Delta_d > 0,\ \Delta_p = 0$

critical behavior

- chiral Zigzag ladder: incoherent sum of two Liouvillians

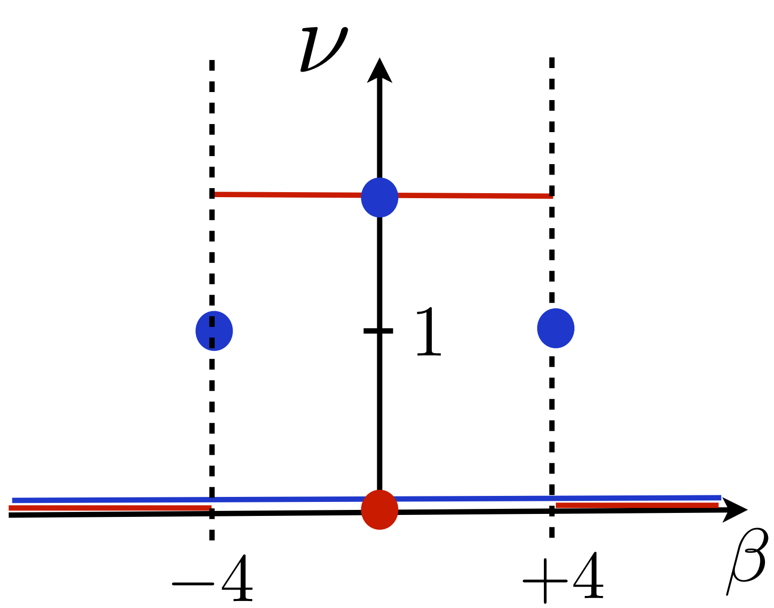
$$\mathcal{L} \propto \mathcal{L}^{(1)} + \kappa \mathcal{L}^{(2)}$$

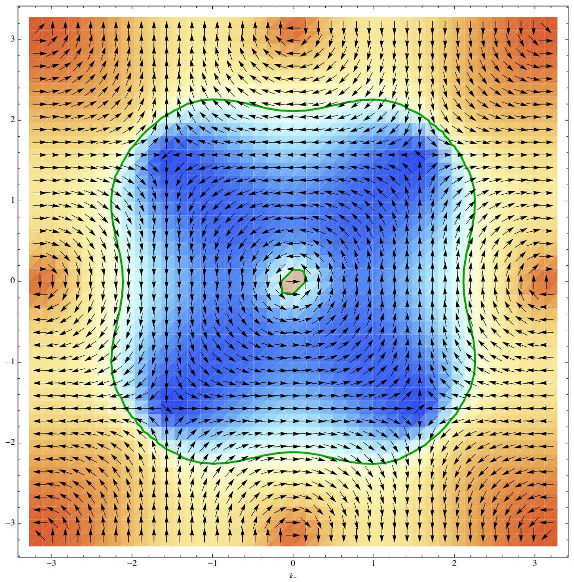




➡ topological phase transition with and without criticality (via purity gap closing)

Dissipative Topological Superfluid in 2 Dimensions





J. C. Budich, P. Zoller, SD, in preparation (2014)

Dissipative Chern Insulators (BdG Superfluids/-conductors)

- Goal: Extend scope of dissipatively preparable topologically non-trivial states
  - D > 1
  - in particular, states with nonzero Chern number

- recipe for pure dissipative topological states (so far)
  - Bogoliubov eigenoperators as Lindblad operators $$H_{\text{parent}} = \sum_i L_i^\dagger L_i \qquad L_i|G\rangle = 0 \forall i$$
  - quasi-locality of Wannier functions key requirement for physical realization

$$L_i = \sum_j u_{j-i} a_j + v_{j-i} a_j^\dagger$$

- Hurdle: Exponentially (let alone compactly supported) Wannier functions do not exist when Chern number nonzero

Competition of Topology and Locality in Chern insulator/ BdG superconductor

- first Chern number

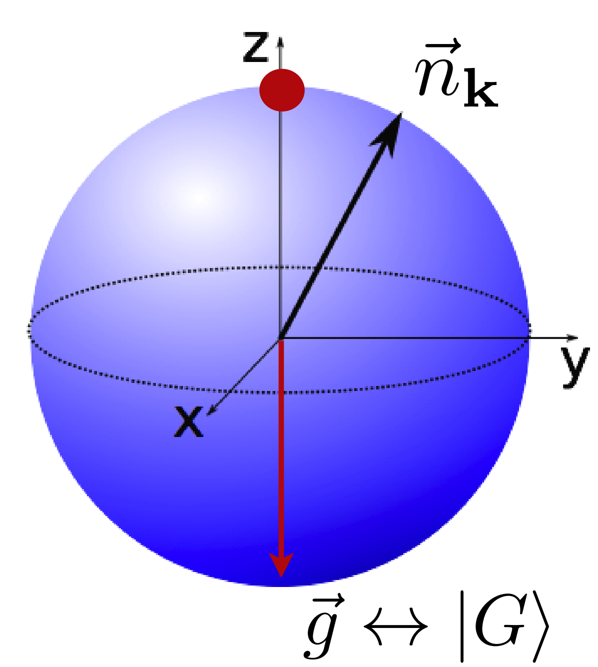
$$\mathcal{C} = \frac{i}{2\pi} \int_{\mathrm{BZ}} \mathrm{d}^2 k \, \mathrm{Tr} \left(\mathcal{P}_{\mathbf{k}} \left[(\partial_{k_x} \mathcal{P}_{\mathbf{k}}), (\partial_{k_y} \mathcal{P}_{\mathbf{k}}) \right] \right)$$

projector onto occupied bands; e.g. spinless fermions

$$\mathcal{P}_{\mathbf{k}} = \tfrac{1}{2}(\mathbf{1} - \vec{n}_{\mathbf{k}} \vec{\sigma}) = |u_{\mathbf{k}}\rangle\langle u_{\mathbf{k}}| \qquad |\vec{n}_{\mathbf{k}}| = 1$$

- nonzero Chern number <=> whole Bloch sphere covered by $\vec{n}_{\mathbf{k}}$
- then, no global gauge of Bloch functions exists:

$$|u_{\mathbf{k}}\rangle = \frac{\mathcal{P}_{\mathbf{k}}|G\rangle}{\sqrt{\langle G|\mathcal{P}_{\mathbf{k}}|G\rangle}}$$



- implication: exponentially localized Wannier functions exist if and only if Chern number is zero

Landau levels: D. J. Thouless, J. Phys. C (1984);
general band structures: C. Brouder et al. PRL (2007)

➡ previous preparation strategy requires to physically realize algebraically decaying Lindblad operators

➡ circumvent by using intrinsic open system properties

Model

- Strategy: combine
  - critical (topological) quasi-local Lindblad operators
  - non-topological Lindblad stabilizing critical point

- Lindblad operators generating dissipative dynamics:
  - starting point: interacting Liouvillian with $L_i = C_i^\dagger A_i$ & long time linearization
  - e.g. half filling $L_i = C_i^\dagger + A_i$
  - creation part

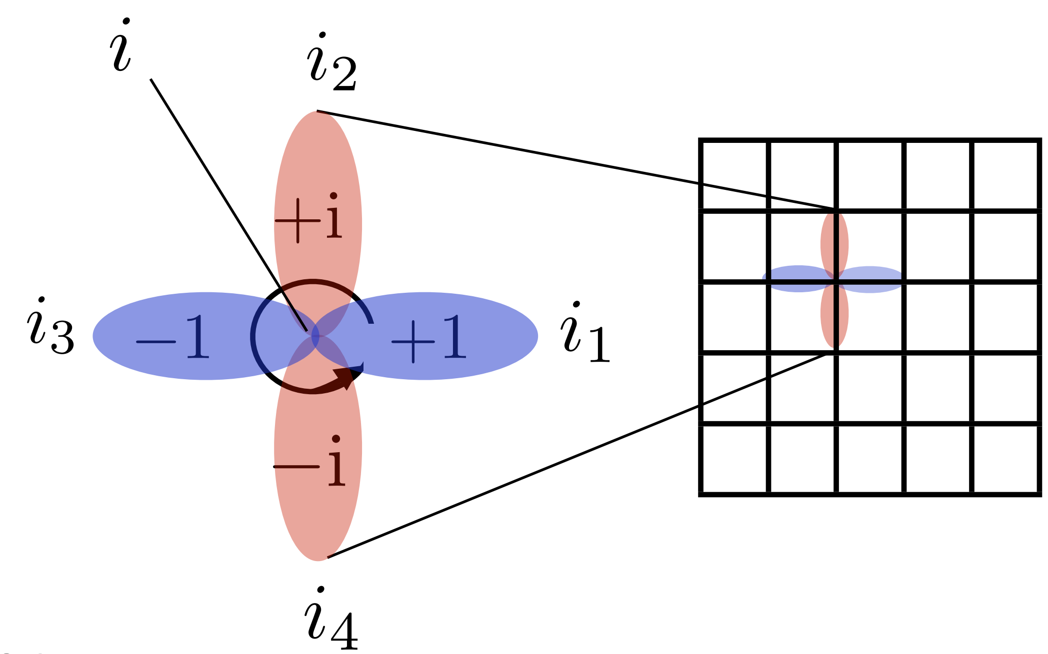
$$C_i^\dagger = \beta\, a_i^\dagger + (a_{i_1}^\dagger + a_{i_2}^\dagger + a_{i_3}^\dagger + a_{i_4}^\dagger)$$

s-wave symmetric creation part

  - annihilation part

$$A_i = (a_{i_1} + \mathrm{i} a_{i_2} - a_{i_3} - \mathrm{i} a_{i_4})$$ local circulation

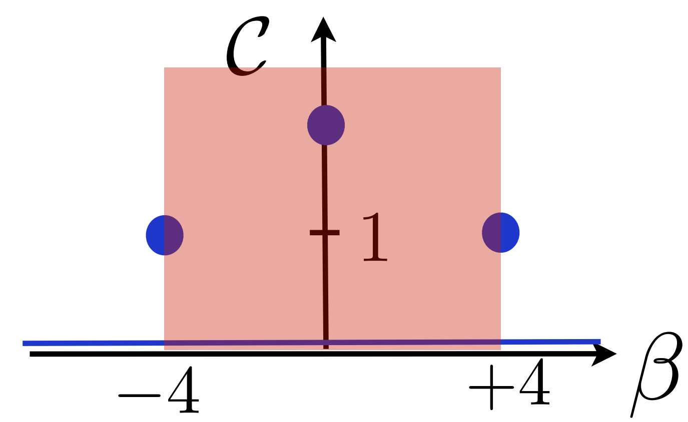
$$= \nabla_{i,x} a_i + \mathrm{i} \nabla_{i,y} a_i$$ p-wave symmetric annihilation part



Observations

- pure stationary state: $\{L_i, L_j\} = 0,\ \{L_i, L_j^\dagger\} \neq 0\ \forall i,j$
- standard 2D diagnostics via first Chern number

$$\mathcal{C} = \frac{1}{4\pi}\int d^2k\, \vec{n}_{\mathbf{k}}(\partial_{k_1}\vec{n}_{\mathbf{k}} \times \partial_{k_2}\vec{n}_{\mathbf{k}})$$



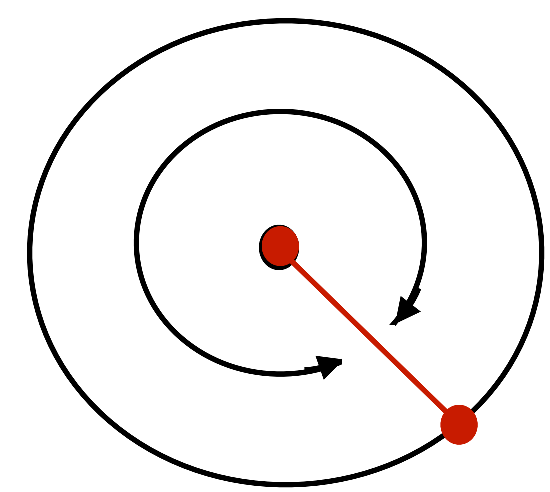
- vanishes except for special points
- special points are critical: closing of damping gap
  - not a Landau-Ginzburg transition (same symmetries)

$$\vec{n}_{\mathbf{k}}(\delta\beta) = \vec{n}_{-\mathbf{k}}(-\delta\beta)$$

distance from transition

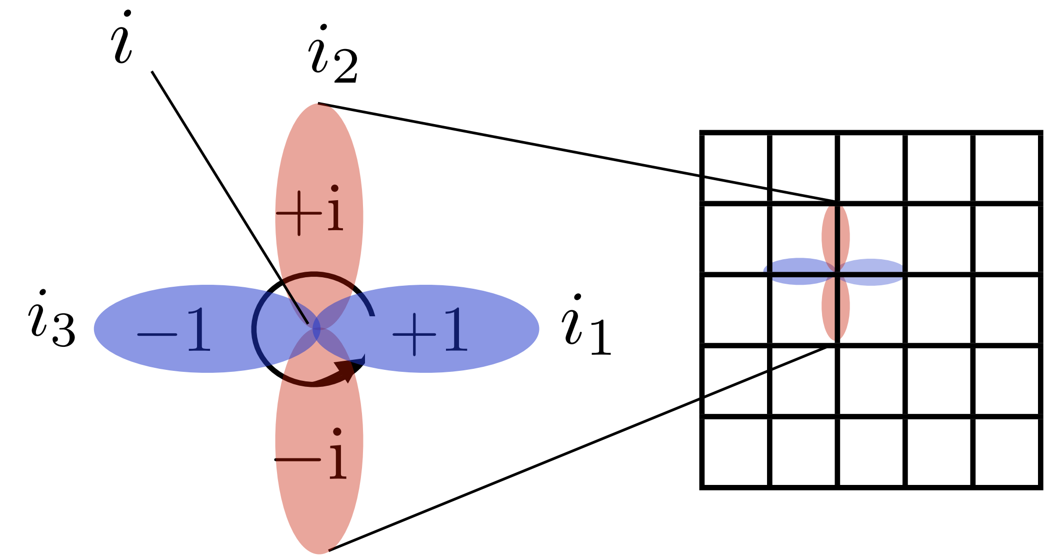
  - not obviously a topological transition

$$\mathcal{C}(\delta\beta) = \mathcal{C}(-\delta\beta)$$



- side remark C. Bardyn, E. Rico, M. Baranov, A. Imamoglu, P. Zoller, SD, PRL (2012); New J. Phys. (2013)
  - dissipative topological transition after dimensional reduction in presence of optically imprinted odd vortex
  - generic presence of unpaired Majorana mode despite topologically trivial 2D bulk

Physics at the dissipative critical point

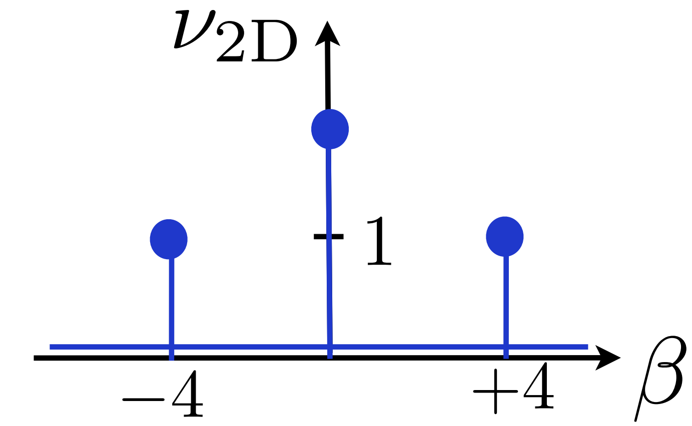


- examine critical (damping gap closing) points for quasilocal p+ip Lindblad operators

$$L_{\mathbf{k}} = \tilde{u}_{\mathbf{k}} a_{\mathbf{k}} + \tilde{v}_{\mathbf{k}} a^{\dagger}_{-\mathbf{k}}$$

$$B_{\mathbf{k}} = \begin{pmatrix} \tilde{u}_{\mathbf{k}} \\ \tilde{v}_{\mathbf{k}} \end{pmatrix} = \begin{pmatrix} 2i\left(\sin(k_x) + i\sin(k_y)\right) \\ \beta + 2(\cos(k_x) + \cos(k_y)) \end{pmatrix}$$

  - pseudo Bloch functions:
    - orthogonal, but not normalized
    - non-vanishing for all k (except at critical point)



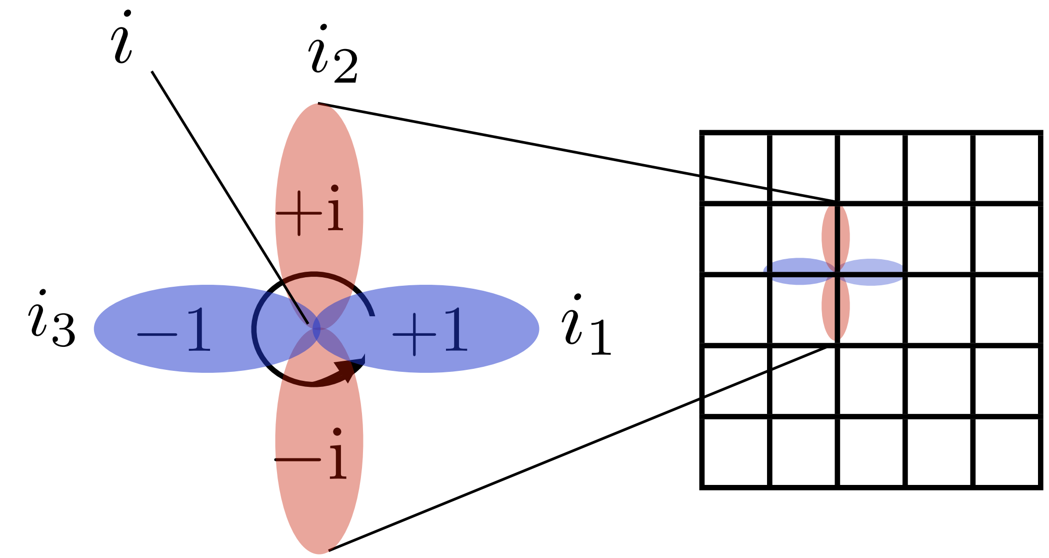
- critical point $\beta = -4$

  - there is one point k=0 where $L_{\mathbf{k}=\mathbf{0}} = 0, \; B_{\mathbf{k}=\mathbf{0}} = 0$

  - but projection smoothly defined all over BZ $\mathcal{P}_{\mathbf{k}} = \dfrac{B_{\mathbf{k}} B^{\dagger}_{\mathbf{k}}}{\mathrm{Tr}\left\{B_{\mathbf{k}} B^{\dagger}_{\mathbf{k}}\right\}} \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ for $\mathbf{k} \to 0$

  - Chern number $\mathcal{C} = -1$

Physics at the dissipative critical point



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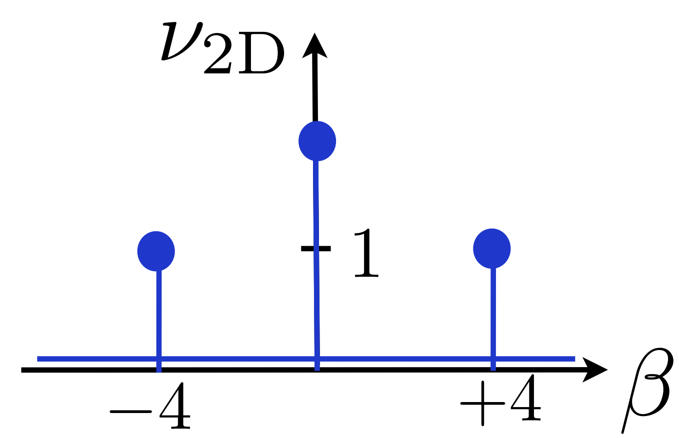


- critical point $\beta = -4$

  - there is one point k=0 where $L_{\mathbf{k=0}} = 0,\ B_{\mathbf{k=0}} = 0$

  - interpretation: over-completeness of quasi-local pseudo Wannier (and Bloch) functions necessary to obtain non-zero Chern number

E. Rashba, L. Zhukov, A. Efros, PRB (1997)

  - damping criticality of this point: $\kappa_{\mathbf{k}=0} = \{L^{\dagger}_{\mathbf{k}=0}, L_{\mathbf{k}=0}\} = \mathrm{Tr}\left\{B_{\mathbf{k=0}} B^{\dagger}_{\mathbf{k=0}}\right\} = 0$

➡ amounts to fine-tuning of damping function $\kappa_{\mathbf{k}} \geq 0$

Stabilization of the critical point

- useful decomposition of Chern number: sum of winding numbers around TRI points λ within "electron region" $\mathcal{E}$, where $\hat{n}_{3,\mathbf{k}} > 0$

$$\hat{\vec{n}}_k = \frac{\vec{n}_k}{|\vec{n}_k|}$$

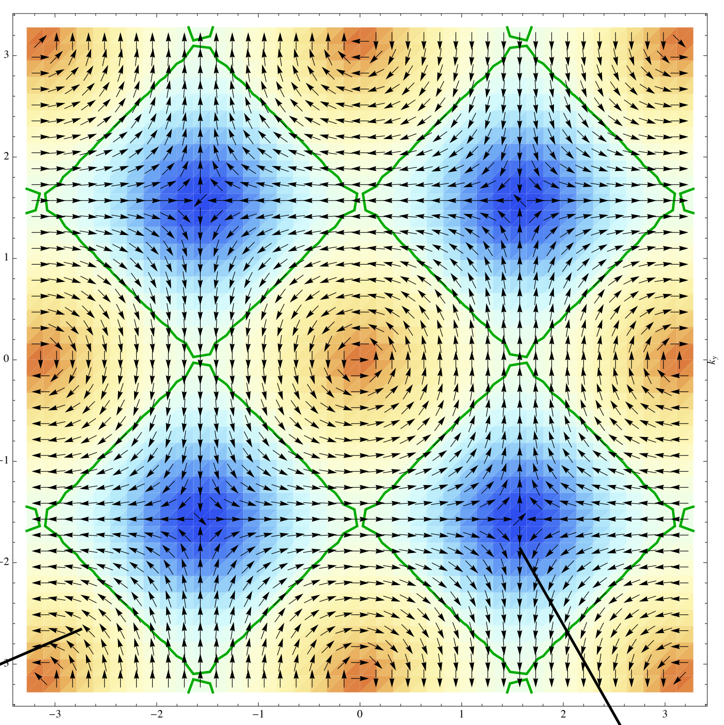
$$\mathcal{C} = \frac{1}{4\pi} \int d^2\mathbf{k}\, \hat{\vec{n}}_{\mathbf{k}} (\partial_{k_1} \hat{\vec{n}}_{\mathbf{k}} \times \partial_{k_2} \hat{\vec{n}}_{\mathbf{k}}) = \sum_{\lambda \in \mathcal{E}} \nu_\lambda$$

$$\nu_\lambda = \frac{1}{2\pi} \oint_{\mathcal{F}_\lambda} \nabla_{\mathbf{k}} \theta_{\mathbf{k}} \cdot d\mathbf{k}$$

height function: $\hat{n}_{3,\mathbf{k}} = 1 - 2n(\mathbf{k})$

fermion occ.

vector field: $\begin{pmatrix} n_{1,\mathbf{k}} \\ n_{2,\mathbf{k}} \end{pmatrix} = r_{\mathbf{k}} \begin{pmatrix} \sin\theta_{\mathbf{k}} \\ \cos\theta_{\mathbf{k}} \end{pmatrix}$



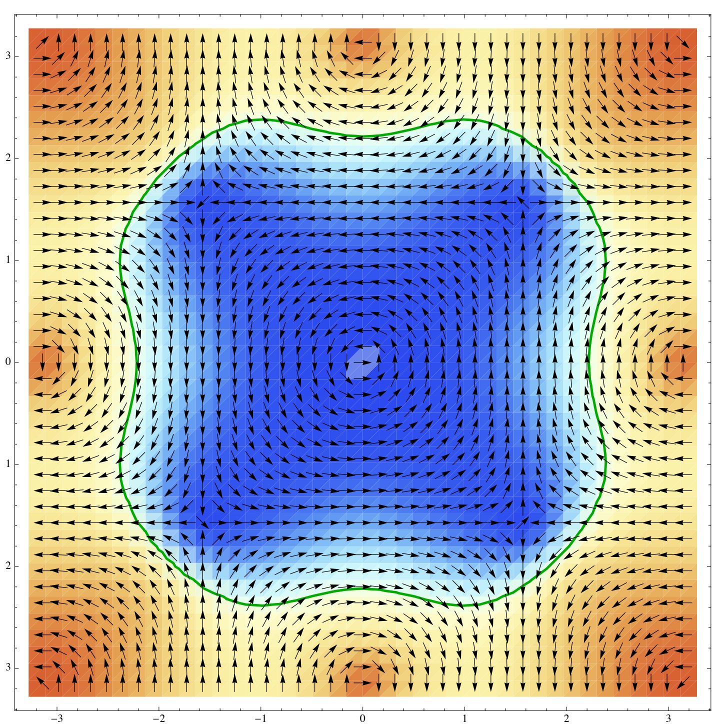
$\mathcal{E}$

$\hat{n}_{3,\mathbf{k}} > 0$

non-critical

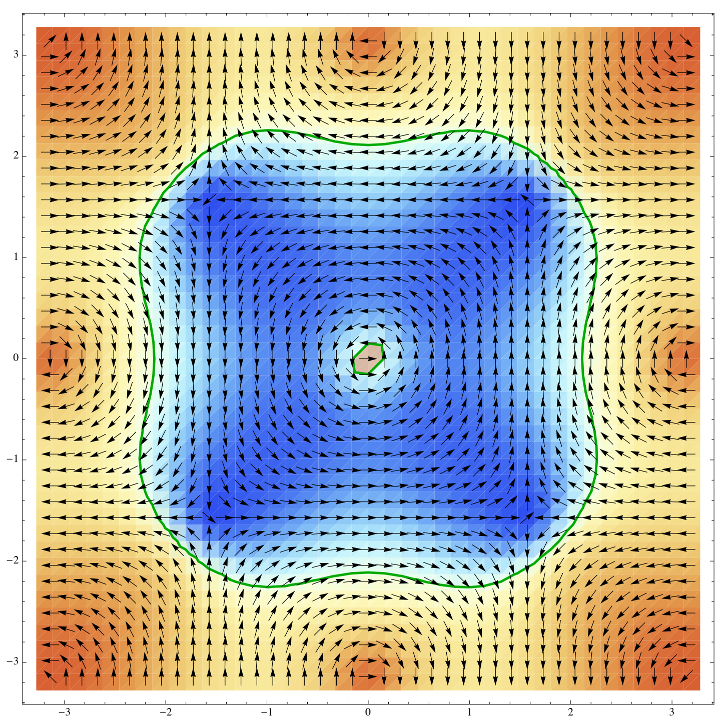
$\mathcal{C} = 0$

$n_{3,\mathbf{k}} < 0$



critical

$\mathcal{C} = -1$



near critical

$\mathcal{C} = 0$

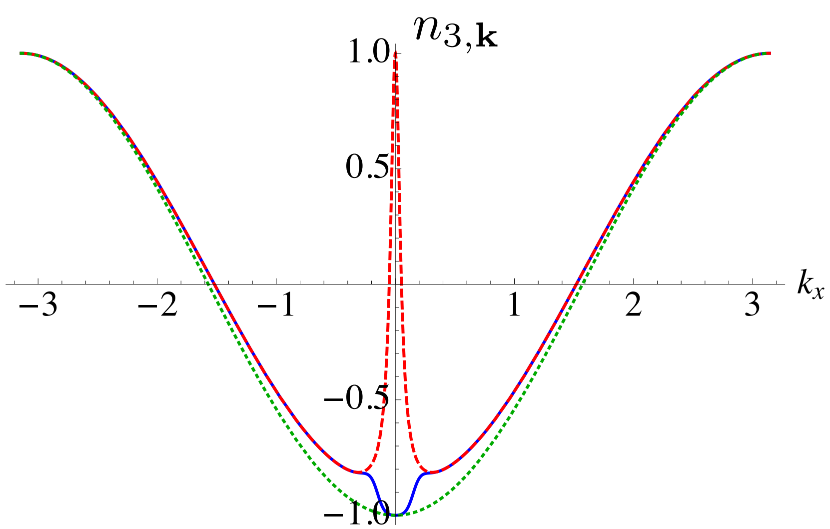
➡ need to "plug the hole" (here, near k=0)

Stabilization of the critical point

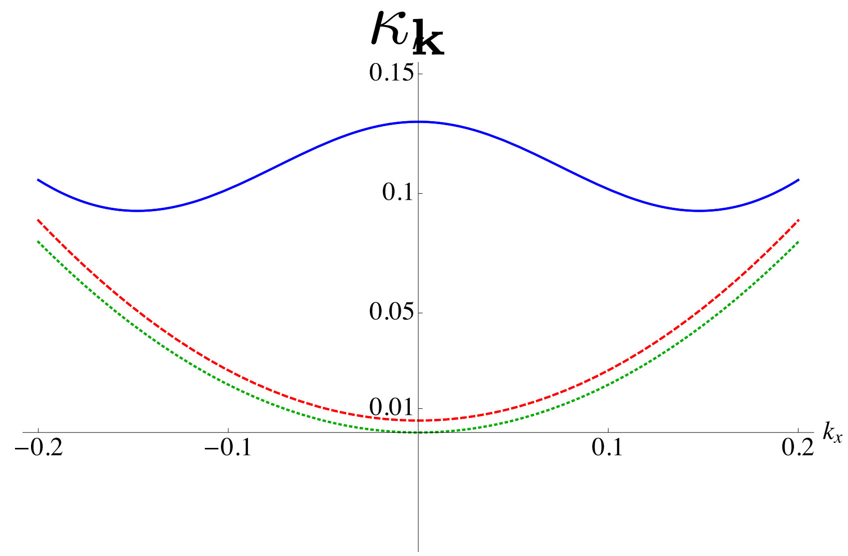
- minimal solution: add momentum selectively non-topological Lindblad operators (Raman pulse with Gaussian envelope)

$$L^A_{\mathbf{k}} = \sqrt{g} e^{-\mathbf{k}^2 d^2} a_{\mathbf{k}}$$

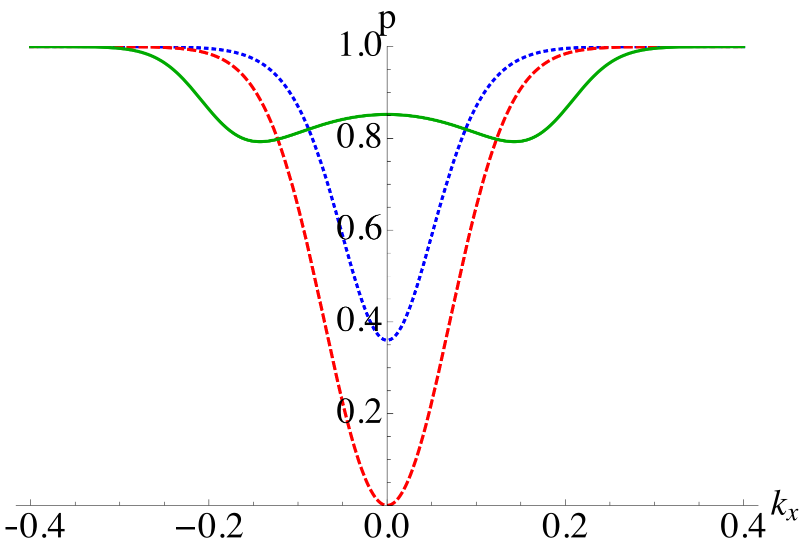
- result:



hole plugging



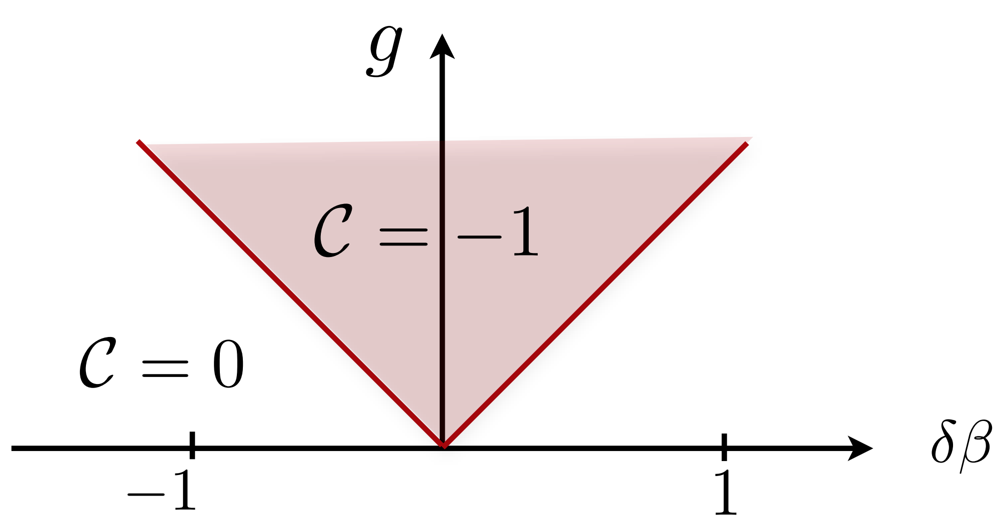
finite damping gap



purity spectrum

phase transition by purity gap closing (non-critical)

- phase diagram
  $d = 1$



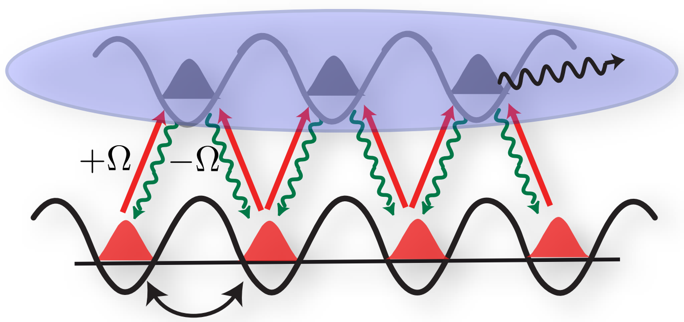
deviation from critical point

➡ dissipative stabilization of a critical topological point into a phase (extended parameter region)

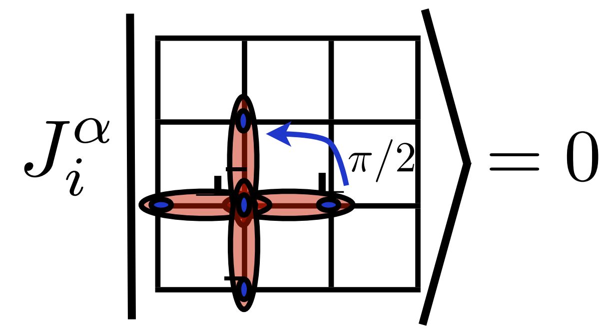
Summary

Tailored dissipation opens new perspectives for many-body physics with cold atom systems

- Pure states with long range correlations from quasilocal dissipation



- Pair condensation mechanism for fermions with potential applications for fermion cooling



- Targeted preparation of topologically nontrivial states in one and two dimensions