Topological Kondo effect in Majorana devices

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Overview

Coulomb charging effects on quantum transport in a Majorana device:

„Topological Kondo effect“ with stable non-Fermi liquid behavior
  Beri & Cooper, PRL 2012

- With interactions in the leads: new unstable fixed point
  Altland & Egger, PRL 2013

- ‘Majorana quantum impurity spin‘ dynamics near strong coupling
  Altland, Beri, Egger & Tsvelik, PRL 2014

- Non-Fermi liquid manifold: coupling to bulk superconductor
  Eriksson, Mora, Zazunov & Egger, PRL 2014
Majorana bound states

- Majorana fermions
  - Non-Abelian exchange statistics \( \gamma_j = \gamma_j^+ \) \( \{\gamma_i, \gamma_j\} = 2\delta_{ij} \)
  - Two Majoranas = nonlocal fermion \( d = \gamma_1 + i\gamma_2 \)
  - Occupation of single Majorana ill-defined: \( \gamma^+\gamma = \gamma^2 = 1 \)
  - Count state of Majorana pair \( d^+d = 0, 1 \)

- Realizable (for example) as end states of spinless 1D p-wave superconductor (Kitaev chain)
  - Recipe: Proximity coupling of 1D helical wire to s-wave superconductor
  - For long wires: Majorana bound states are zero energy modes

Experimental Majorana signatures

InSb nanowires expected to host Majoranas due to interplay of
• strong Rashba spin orbit field
• magnetic Zeeman field
• proximity-induced pairing
  Oreg, Refael & von Oppen, PRL 2010
  Lutchyn, Sau & Das Sarma, PRL 2010

Transport signature of Majoranas:
Zero-bias conductance peak due to resonant Andreev reflection
  Bolech & Demler, PRL 2007
  Law, Lee & Ng, PRL 2009
  Flensberg, PRB 2010

See also:  Rokhinson et al., Nat. Phys. 2012;
  Deng et al., Nano Lett. 2012;  Das et al., Nat.
  Phys. 2012;  Churchill et al., PRB 2013
Zero-bias conductance peak

Mourik et al., Science 2012

Possible explanations:
- Majorana state (most likely!)
- Disorder-induced peak
- Smooth confinement
- Kondo effect

Bagrets & Altland, PRL 2012
Kells, Meidan & Brouwer, PRB 2012
Lee et al., PRL 2012
Suppose that Majorana mode is realized...

- Quantum transport features beyond zero-bias anomaly peak? Coulomb interaction effects?
- Simplest case: Majorana single charge transistor
  - 'Overhanging' helical wire parts serve as normal-conducting leads
  - Nanowire part coupled to superconductor hosts pair of Majorana bound states
  - Include charging energy of this 'dot'

\[ \gamma_L \quad \gamma_R \]
Majorana single charge transistor

- Floating superconducting 'dot' contains two Majorana bound states tunnel-coupled to normal-conducting leads
- Charging energy finite

Consider universal regime:
- Long superconducting wire: Direct tunnel coupling between left and right Majorana modes is assumed negligible
- No quasi-particle excitations: Proximity-induced gap is largest energy scale of interest
Hamiltonian: charging term

- Majorana pair: nonlocal fermion \( d = \gamma_L + i\gamma_R \)
- Condensate gives another zero mode
  - Cooper pair number \( N_c \), conjugate phase \( \phi \)
- Dot Hamiltonian (gate parameter \( n_g \))

\[
H_{\text{island}} = E_C \left( 2N_c + d^+d - n_g \right)^2
\]

Majorana fermions couple to Cooper pairs through the charging energy
Tunneling

- Normal-conducting leads: effectively spinless helical wire
  - Applied bias voltage $V = \text{chemical potential difference}$

- Tunneling of electrons from lead to dot:
  - Project electron operator in superconducting wire part to Majorana sector
  - Spin structure of Majorana state encoded in tunneling matrix elements

*Flensberg, PRB 2010*
Tunneling Hamiltonian

Source (drain) couples to left (right) Majorana only:

\[ H_t = \sum_{j=L,R} t_j c_j^+ \eta_j + h.c. \]

- respeacts current conservation
- Hybridizations: \( \Gamma_j \sim \nu |t_j|^2 \)

Normal tunneling \( \sim c^+ d, d^+ c \)
- Either destroy or create nonlocal d fermion
- Condensate not involved

Anomalous tunneling \( \sim c^+ e^{-i\phi} d^+, de^{i\phi} c \)
- Create (destroy) both lead and d fermion & split (add) a Cooper pair
Absence of even-odd effect

- Without Majorana states: Even-odd effect
- With Majoranas: no even-odd effect!
- Tuning wire parameters into the topological phase removes even-odd effect

picture from: Fu, PRL 2010
Noninteracting case: Resonant Andreev reflection

- $E_c=0$ Majorana spectral function
  \[
  - \text{Im} \, G_{\gamma_j}^{\text{ret}}(\epsilon) = \frac{\Gamma_j}{\epsilon^2 + \Gamma_j^2}
  \]

- $T=0$ differential conductance:
  \[
  G(V) = \frac{2e^2}{h} \frac{1}{1 + (eV/\Gamma)^2}
  \]

- Currents $I_L$ and $I_R$ fluctuate independently, superconductor is effectively grounded

- Perfect Andreev reflection via Majorana state
  - Zero-energy Majorana bound state leaks into lead
Strong blockade: Electron teleportation

- Peak conductance for half-integer $n_g$
- Strong charging energy then allows only two degenerate charge configurations
- Model maps to spinless resonant tunneling model
- Linear conductance ($T=0$): $G = e^2 / h$
- Interpretation: Electron teleportation due to nonlocality of $d$ fermion
Now $N>1$ helical wires: $M$ Majorana states tunnel-coupled to helical Luttinger liquid wires with $g \leq 1$

Strong charging energy, with nearly integer $n_g$: unique equilibrium charge state on the island

$2^{N-1}$-fold ground state degeneracy due to Majorana states (taking into account parity constraint)

Need $N>1$ for interesting effect!

Beri & Cooper, PRL 2012
Altland & Egger, PRL 2013
Beri, PRL 2013
Altland, Beri, Egger & Tsvelik, PRL 2014
Zazunov, Altland & Egger, NJP 2014
Abelian bosonization of lead fermions
- Klein factors are needed to ensure anticommutation relations between different leads
- Klein factors can be represented by additional Majorana fermion for each lead

Combine Klein-Majorana and 'true' Majorana fermion at each contact to build auxiliary fermions, \( f_j \)
- All occupation numbers \( f_j^+ f_j \) are conserved and can be gauged away
- purely **bosonic problem** remains…
Charging effects: dipole confinement

- High energy scales $> E_C$: charging effects irrelevant
  - Electron tunneling amplitudes from lead $j$ to dot renormalize independently upwards
    $$ t_j(E) \sim E^{-1+\frac{1}{2g}} $$
  - RG flow towards resonant Andreev reflection fixed point
- For $E < E_C$: charging induces 'confinement'
  - In- and out-tunneling events are bound to 'dipoles' with coupling $\lambda_{j\neq k}$: entanglement of different leads
  - Dipole coupling describes amplitude for 'cotunneling' from lead $j$ to lead $k$
  - 'Bare' value
    $$ \lambda_{jk}^{(1)} = \frac{t_j(E_C) t_k(E_C)}{E_C} \sim E_C^{-3+\frac{1}{g}} $$
    large for small $E_C$
RG equations in dipole phase

- Energy scales below $E_c$: effective phase action
  \[ S = \frac{g}{2\pi} \sum_j \int \frac{d\omega}{2\pi} |\omega| \Phi_j(\omega)^2 - \sum_{j \neq k} \lambda_{jk} \int d\tau \cos(\Phi_j - \Phi_k) \]

- One-loop RG equations
  \[ \frac{d\lambda_{jk}}{dl} = -(g^{-1} - 1)\lambda_{jk} + \nu \sum_{m \neq (j,k)}^{M} \lambda_{jm}\lambda_{mk} \]

- Suppression by Luttinger liquid tunneling DoS
- Enhancement by dipole fusion processes

- RG-unstable intermediate fixed point with isotropic couplings (for $M > 2$ leads)
  \[ \lambda_{j \neq k} = \lambda^* = \frac{g^{-1} - 1}{M - 2} \nu \]
RG flow

- RG flow towards strong coupling for \( \left\langle \lambda^{(1)} \right\rangle > \lambda^* \)
  - Always happens for moderate charging energy
- Flow towards isotropic couplings: anisotropies are RG irrelevant
- Perturbative RG fails below Kondo temperature

\[
T_K \approx E_C e^{-\lambda^*/\left\langle \lambda^{(1)} \right\rangle}
\]
Topological Kondo effect

- Refermionize for $g=1$:

$$H = -i \int dx \sum_{j=1}^{M} \psi_j^+ \partial_x \psi_j + i \lambda \sum_{j \neq k} \psi_j^+(0) S_{jk} \psi_k(0)$$

- Majorana bilinears

$$S_{jk} = i \gamma_j \gamma_k$$

- 'Reality' condition: SO(M) symmetry [instead of SU(2)]
- Nonlocal realization of 'quantum impurity spin'
- Nonlocality ensures stability of Kondo fixed point

Majorana basis

$$\psi(x) = \mu(x) + i \xi(x)$$

for leads:

**SO$_2$(M) Kondo model**

$$H = -i \int dx \mu^T \partial_x \mu + i \lambda \mu^T(0) \hat{S} \mu(0) + [\mu \leftrightarrow \xi]$$

Beri & Cooper, PRL 2012
Minimal case: M=3 Majorana states

- SU(2) representation of "quantum impurity spin"
  \[ S_j = \frac{i}{4} \epsilon_{jkl} \gamma_k \gamma_l \]
  \[ [S_1, S_2] = iS_3 \]

- Spin S=1/2 operator, nonlocally realized in terms of Majorana states
  - can be represented by Pauli matrices

- Exchange coupling of this spin-1/2 to two SO(3) lead currents → multichannel Kondo effect
Transport properties near unitary limit

- Temperature & voltages < $T_K$
  - Dual instanton version of action applies near strong coupling limit
  - Nonequilibrium Keldysh formulation

Linear conductance tensor

$$G_{jk} = e \frac{\partial I_j}{\partial \mu_k} = \frac{2e^2}{h} \left( 1 - \left( \frac{T}{T_K} \right)^{2y-2} \right) \left[ \delta_{jk} - \frac{1}{M} \right]$$

- Non-integer scaling dimension $y = 2g \left( 1 - \frac{1}{M} \right) > 1$
implies non-Fermi liquid behavior even for $g=1$

- completely isotropic multi-terminal junction
Correlated Andreev reflection

- Diagonal conductance at T=0 exceeds resonant tunneling ("teleportation") value but stays below resonant Andreev reflection limit

\[ G_{jj} = \frac{2e^2}{h} \left( 1 - \frac{1}{M} \right) \Rightarrow \frac{e^2}{h} < G_{jj} < \frac{2e^2}{h} \]

- Interpretation: Correlated Andreev reflection

- Remove one lead: change of scaling dimensions and conductance

- Non-Fermi liquid power-law corrections at finite T
Fano factor

- Backscattering correction to current near unitary limit for \( \sum_j \mu_j = 0 \)

\[
\delta I_j = -\frac{e}{\hbar} \sum_k \left| \frac{\mu_k}{T_K} \right|^{2y-2} \left( \delta_{jk} - \frac{1}{M} \right) \mu_k
\]

- Shot noise:

\[
\tilde{S}_{jk}(\omega \to 0) = \int dt \ e^{i\omega t} \left( \langle I_j(t)I_k(0) \rangle - \langle I_j \rangle \langle I_k \rangle \right)
\]

\[
\tilde{S}_{jk} = -\frac{2ge^2}{\hbar} \sum_l \left( \delta_{jl} - \frac{1}{M} \right) \left( \delta_{kl} - \frac{1}{M} \right) \left| \frac{\mu_l}{T_K} \right|^{2y-2} |\mu_l|
\]

- universal Fano factor, but different value than for SU(N) Kondo effect

Sela et al. PRL 2006; Mora et al., PRB 2009
Majorana spin dynamics

Altland, Beri, Egger & Tsvelik, PRL 2014

- Overscreened multi-channel Kondo fixed point: massively entangled effective impurity degree remains at strong coupling: "Majorana spin"
- Probe and manipulate by coupling of Majoranas

\[ H_Z = \sum_{jk} h_{jk} S_{jk} \]

- 'Zeeman fields' \( h_{jk} = -h_{kj} \): overlap of Majorana wavefunctions within same nanowire
- Couple to \( S_{jk} = i\gamma_j\gamma_k \)
Majorana spin near strong coupling

Bosonized form of Majorana spin at Kondo fixed point:

\[ S_{jk} = i \gamma_j \gamma_k \cos[\Theta_j(0) - \Theta_k(0)] \]

- Dual boson fields \( \Theta_j(x) \) describe 'charge' (not 'phase') in respective lead
- Scaling dimension \( \gamma_z = 1 - \frac{2}{M} \) \( \rightarrow \) RG relevant
- Zeeman field ultimately destroys Kondo fixed point & breaks emergent time reversal symmetry
- Perturbative treatment possible for \( T_h < T < T_K \)

\[ T_h = \left( \frac{h_{12}}{T_K} \right)^{M/2} T_K \]

dominant 1-2 Zeeman coupling:
Crossover $\text{SO}(M) \rightarrow \text{SO}(M-2)$

- Lowering $T$ below $T_h \rightarrow$ crossover to another Kondo model with $\text{SO}(M-2)$ (Fermi liquid for $M<5$)
  - Zeeman coupling $h_{12}$ flows to strong coupling $\rightarrow \gamma_1, \gamma_2$ disappear from low-energy sector
  - Same scenario follows from Bethe ansatz solution

Altland, Beri, Egger & Tsvelik, JPA 2014

- Observable in conductance & in thermodynamic properties
SO(M) → SO(M-2): conductance scaling

for single Zeeman component $h_{12} \neq 0$ consider $G_{jj}$ ($j \neq 1,2$)
(diagonal element of conductance tensor)
Multi-point correlations

- Majorana spin has nontrivial multi-point correlations at Kondo fixed point, e.g. for $M=3$ (absent for SU(N) case!)

$$\langle T_\tau S_j(\tau_1)S_k(\tau_2)S_l(\tau_3) \rangle \sim \frac{\epsilon_{jkl}}{T_K(\tau_{12}\tau_{13}\tau_{23})^{1/3}}$$

- Observable consequences for time-dependent 'Zeeman‘ field $B_j = \epsilon_{jkl} h_{kl}$ with $\tilde{B}(t) = (B_1 \cos(\omega_1 t), B_2 \cos(\omega_2 t), 0)$
  - Time-dependent gate voltage modulation of tunnel couplings
  - Measurement of 'magnetization‘ by known read-out methods

- Nonlinear frequency mixing $\langle S_3(t) \rangle \sim B_1 B_2 \cos[(\omega_1 \pm \omega_2) t]$

- Oscillatory transverse spin correlations (for $B_2=0$)

$$\langle S_2(t)S_3(0) \rangle \sim B_1 \frac{\cos(\omega_1 t)}{(\omega_1 t)^{2/3}}$$
Adding Josephson coupling: Non Fermi liquid manifold

Eriksson, Mora, Zazunov & Egger, PRL 2014

\[ H_{\text{island}} = E_C \left( 2N_c + \hat{n} - n_g \right)^2 - E_J \cos \varphi \]

Yet another bulk superconductor: Topological Cooper pair box

Effectively harmonic oscillator for \( E_J >> E_C \)

with Josephson plasma oscillation frequency: \( \Omega = \sqrt{8E_J E_C} \)
Low energy theory

- Tracing over phase fluctuations gives two coupling mechanisms:
  - Resonant Andreev reflection processes:
    \[ H_A = \sum_j t_j \gamma_j \left( \psi_j^+(0) - \psi_j(0) \right) \]
  - Kondo exchange coupling, but of SO\(_1(M)\) type:
    \[ H_K = \sum_{j \neq k} \lambda_{jk} \left( \psi_j^+(0) + \psi_j(0) \right) \left( \psi_k^+(0) + \psi_k(0) \right) \gamma_j \gamma_k \]
  - Interplay of resonant Andreev reflection and Kondo screening for \( \Gamma < T_K \)
Quantum Brownian Motion picture

Abelian bosonization now yields (M=3)

\[ H_A + H_K \propto -\sum_j \sqrt{\Gamma_j} \sin \Phi_j - \sqrt{T_K} \sum_{j \neq k} \cos \Phi_j \cos \Phi_k \]

Simple cubic lattice

bcc lattice
Quantum Brownian motion

- Leading irrelevant operator (LIO): tunneling transitions connecting nearest neighbors
- Scaling dimension of LIO from n.n. distance $d$
  \[ y_{LIO} = \frac{d^2}{2\pi^2} \]
  \[ Yi & Kane, PRB 1998 \]
- Pinned phase field configurations correspond to Kondo fixed point, but unitarily rotated by resonant Andreev reflection corrections
- Stable non-Fermi liquid manifold as long as LIO stays irrelevant, i.e. for $y_{LIO} > 1$
Scaling dimension of LIO

- M-dimensional manifold of non-Fermi liquid states spanned by parameters $\delta_j = \sqrt{\frac{\Gamma_j}{T_k}}$
- Scaling dimension of LIO

$$y = \min \left\{ 2, \frac{1}{2} \sum_{j=1}^{M} \left[ 1 - \frac{2}{\pi} \arcsin \left( \frac{\delta_j}{2(M-1)} \right) \right] \right\}$$

- Stable manifold corresponds to $y > 1$
- For $y < 1$: standard resonant Andreev reflection scenario applies
- For $y > 1$: non-Fermi liquid power laws appear in temperature dependence of conductance tensor
Conclusions

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Beri & Cooper, PRL 2014

➢ With interactions in the leads: new unstable fixed point  
Altland & Egger, PRL 2013  

➢ 'Majorana quantum impurity spin‘ dynamics near strong coupling  
Altland, Beri, Egger & Tsvelik, PRL 2014

➢ Non-Fermi liquid manifold: coupling to bulk superconductor  
Eriksson, Mora, Zazunov & Egger, PRL 2014

THANK YOU FOR YOUR ATTENTION