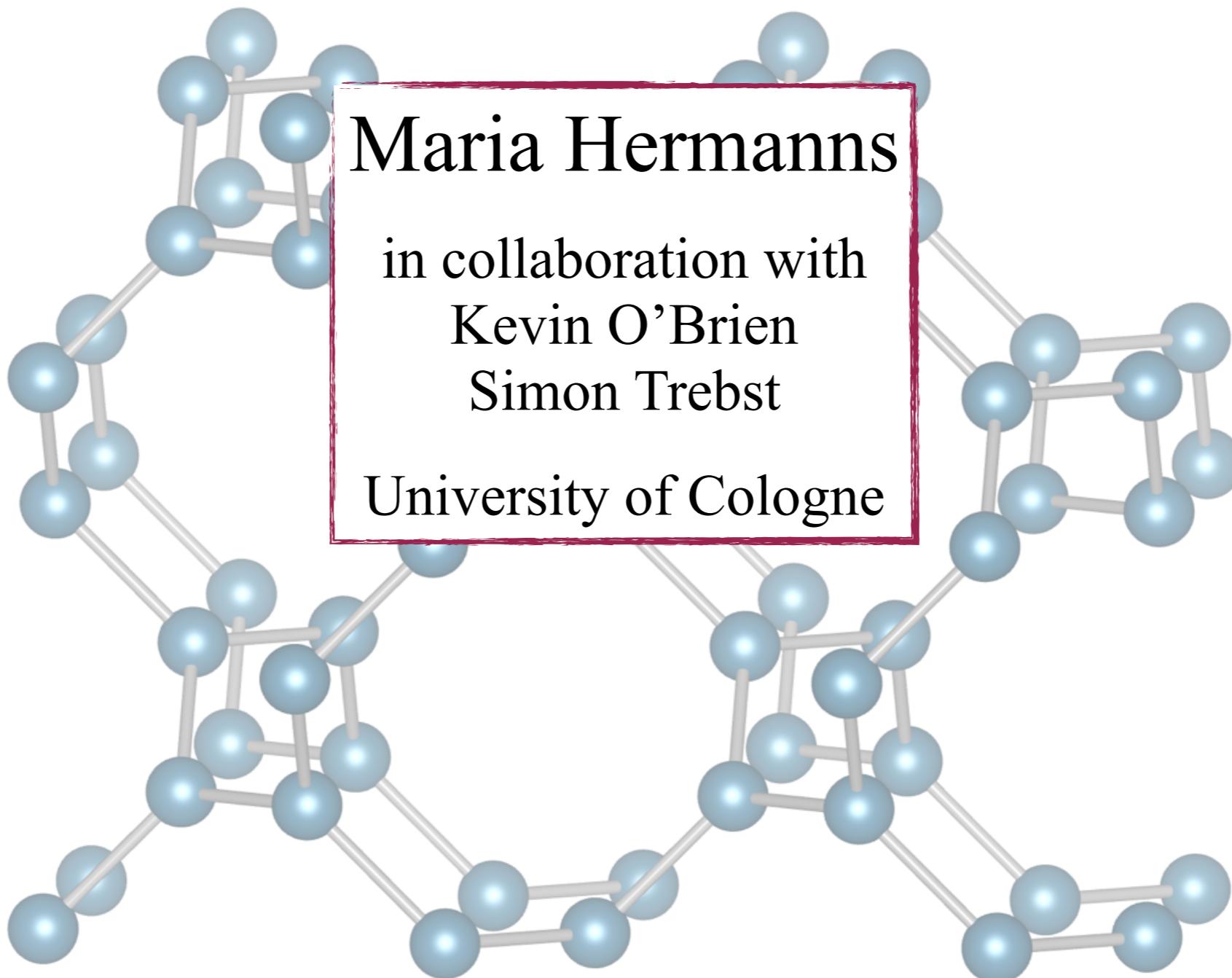


# Quantum spin liquid with a Majorana Fermi surface



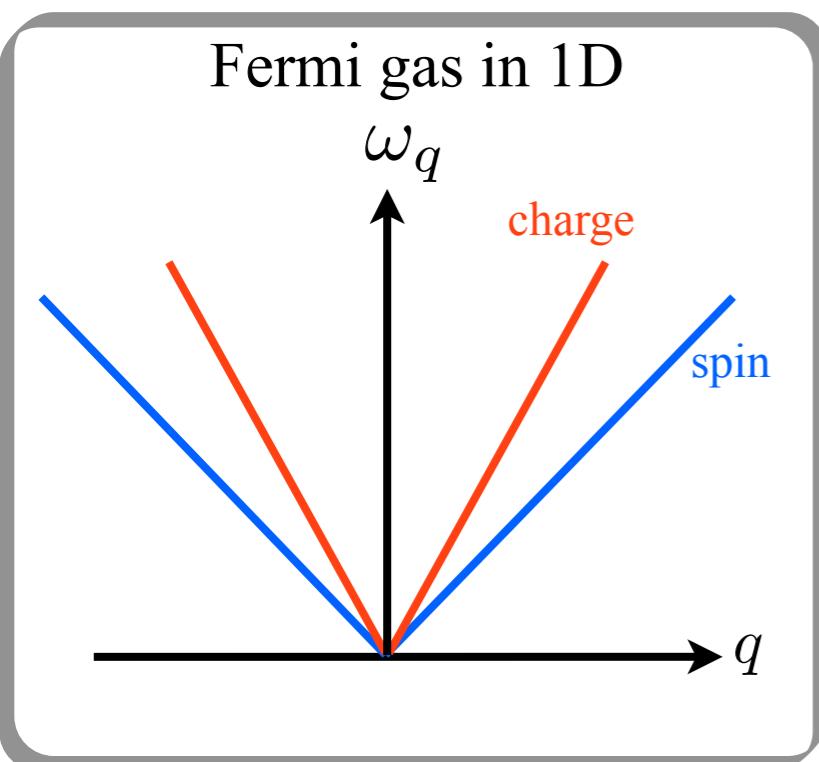
# Fractionalization in strongly correlated systems

(Strong) interactions can lead to emergent quasiparticles  
with ‘fractional’ quantum numbers

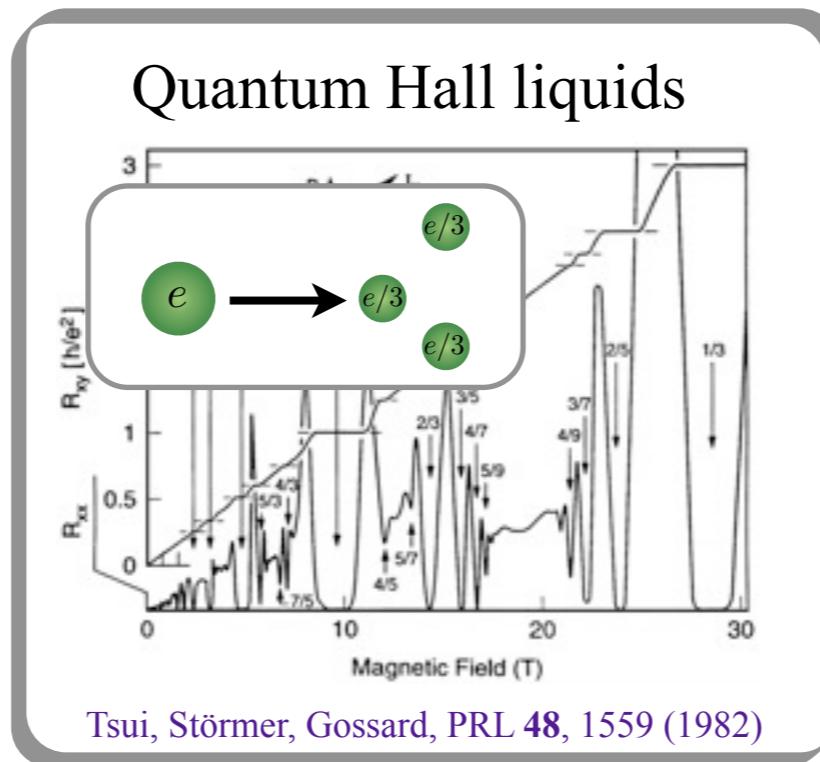
# Fractionalization in strongly correlated systems

(Strong) interactions can lead to emergent quasiparticles  
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## Spin-charge separation

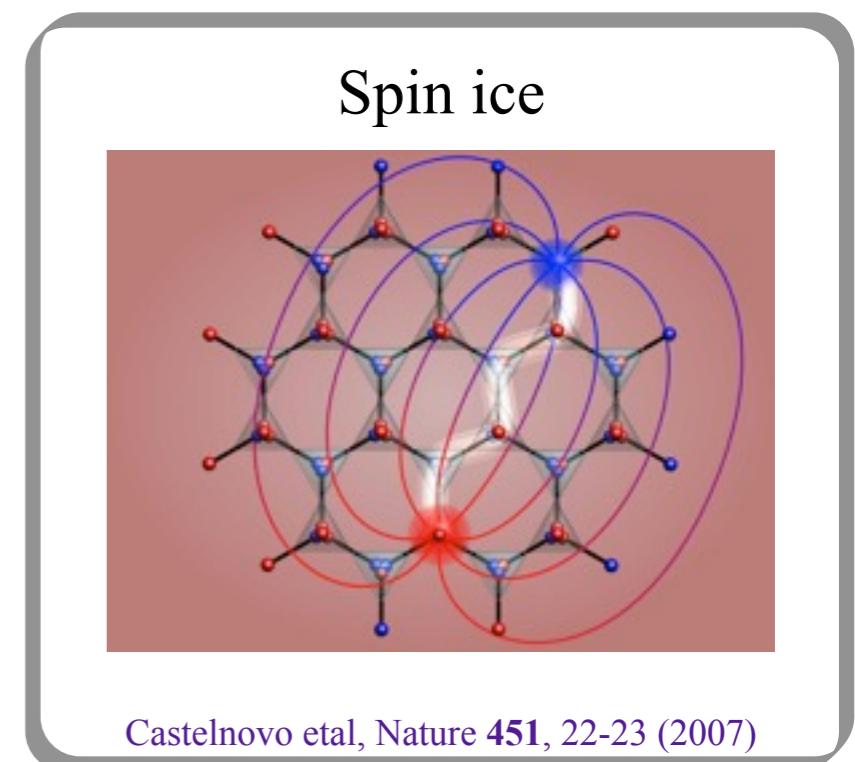


## Electron fractionalization



Tsui, Störmer, Gossard, PRL 48, 1559 (1982)

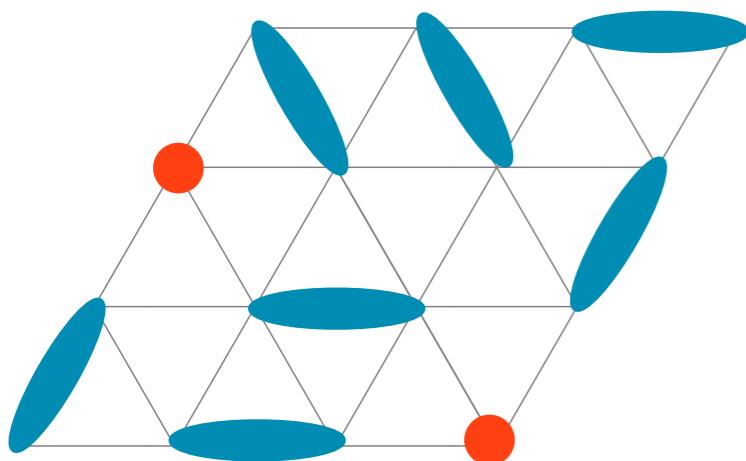
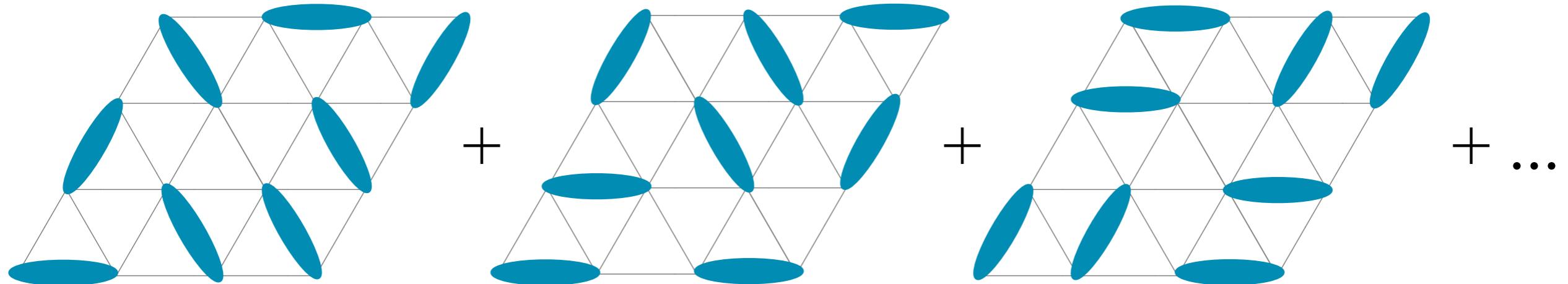
## Magnetic monopoles



# Fractionalization in magnetic systems

Moessner, Sondhi PRL 86, 1881 (2000)

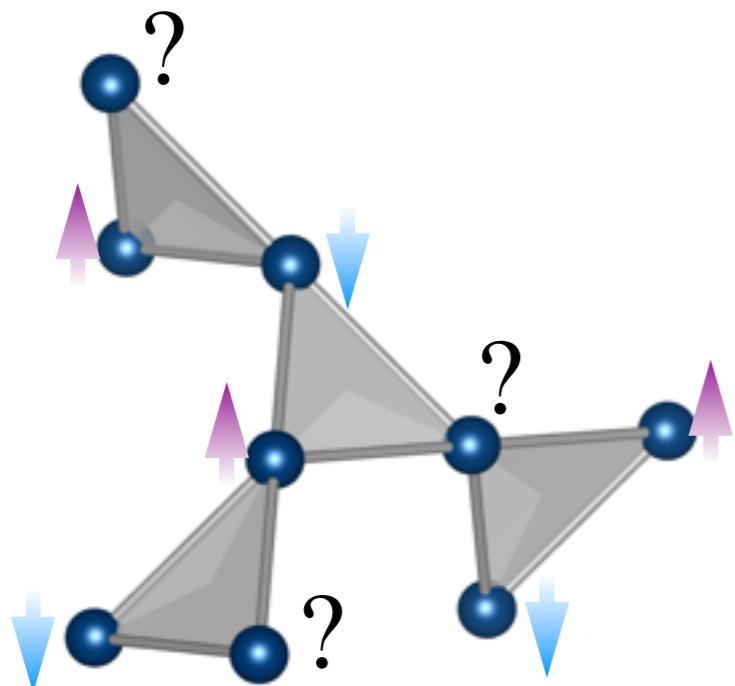
Resonating Valence Bond liquid:



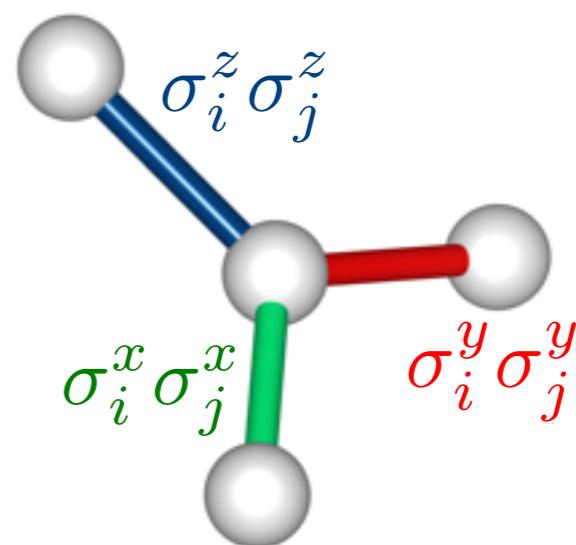
- short-range dimer correlations
- no magnetic order
- gapped, deconfined **spinon** excitations – spin 1/2

# Towards spin liquids – frustration

geometric frustration



exchange frustration

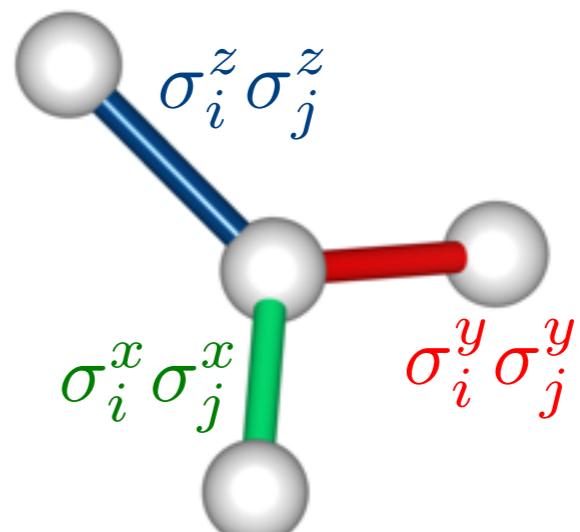


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geometric frustration

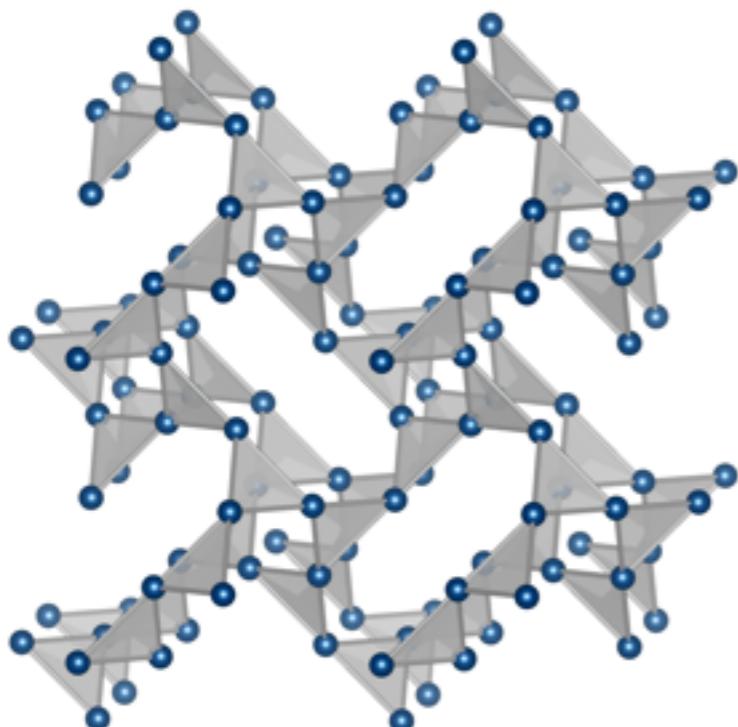


exchange frustration



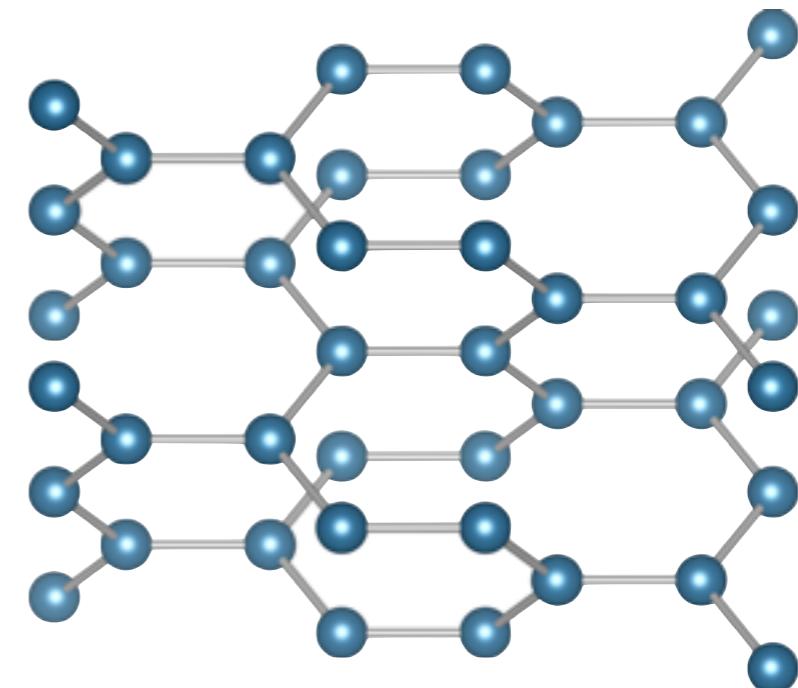
# Towards spin liquids – frustration

geometric frustration



hyperkagome lattice  
 $\text{Na}_4\text{Ir}_3\text{O}_8$

exchange frustration

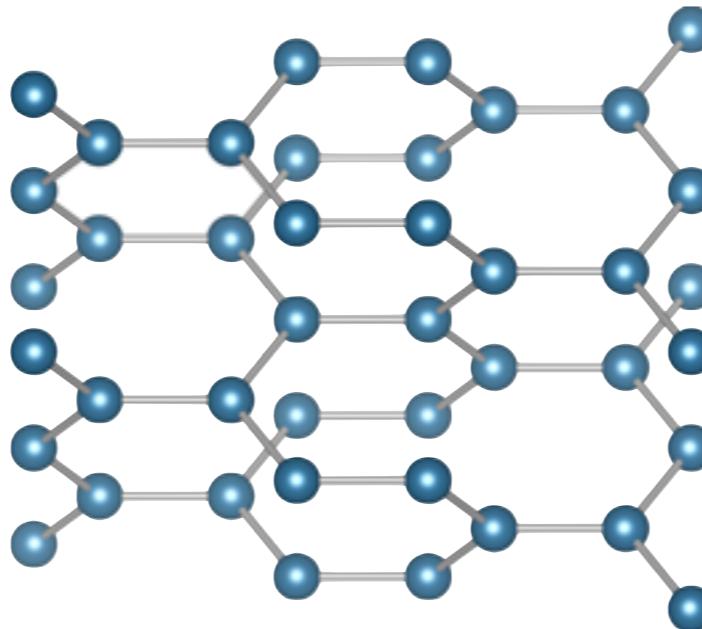


hyperhoneycomb lattice  
 $\beta\text{-Li}_2\text{IrO}_3$

Y. Okamoto, M. Nohara, H. Aruga-Katori, and H. Takagi,  
PRL 99, 137207 (2007)

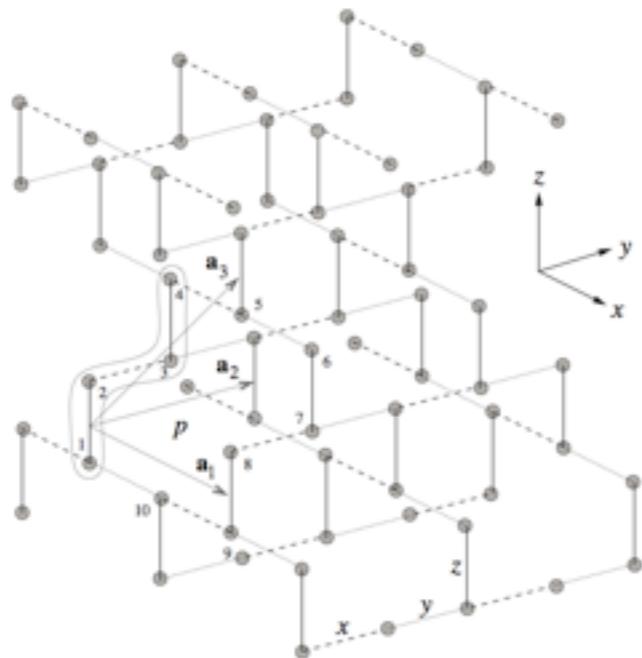
T. Takayama et al., arXiv:1403.3296 (2014)  
K.A. Modic et al. arXiv:1402.3254 (2014)

# Zoo of 3D tri-coordinated lattices

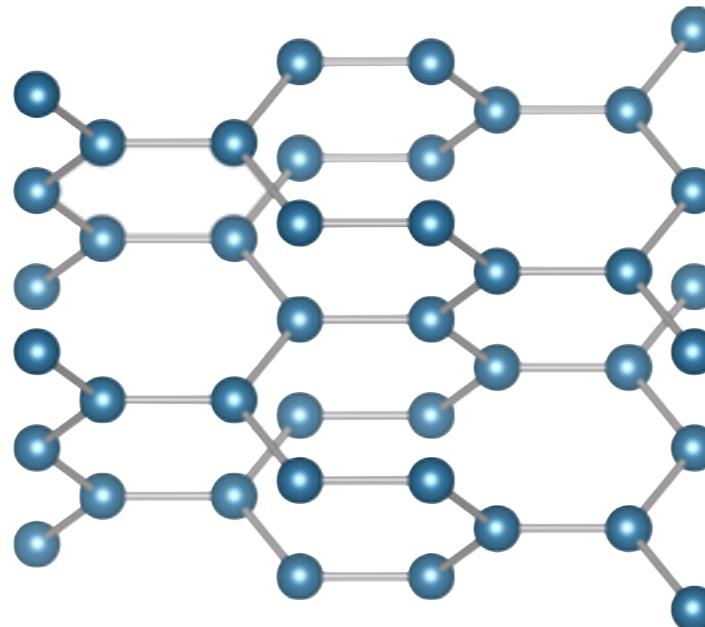


Takayama et al., arXiv:1403.3296 (2014)

# Zoo of 3D tri-coordinated lattices



Mandal, Surendran, PRB 79, 024426 (2009)

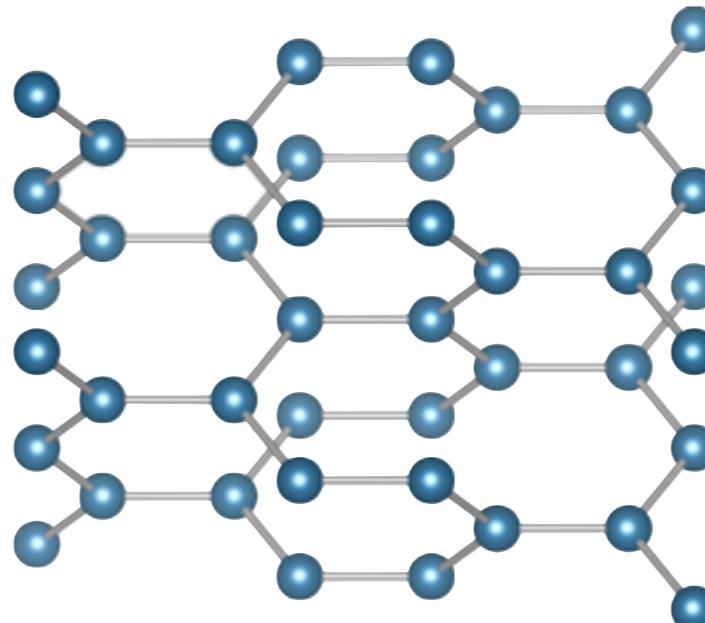


Takayama et al., arXiv:1403.3296 (2014)

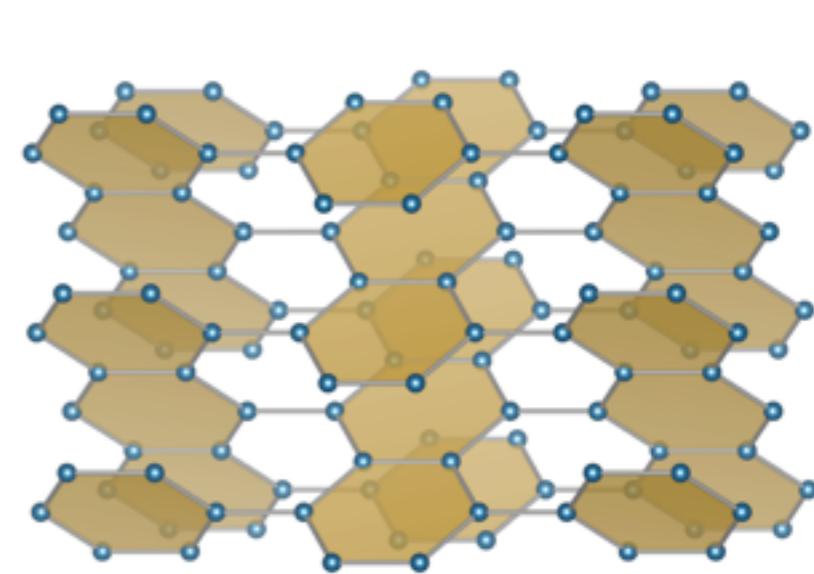
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Takayama et al., arXiv:1403.3296 (2014)

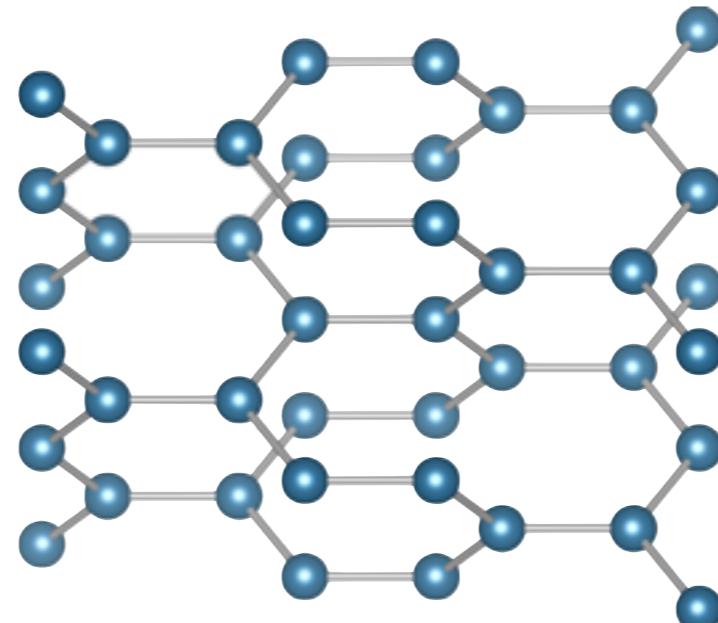


Modic et al. arXiv:1402.3254 (2014)

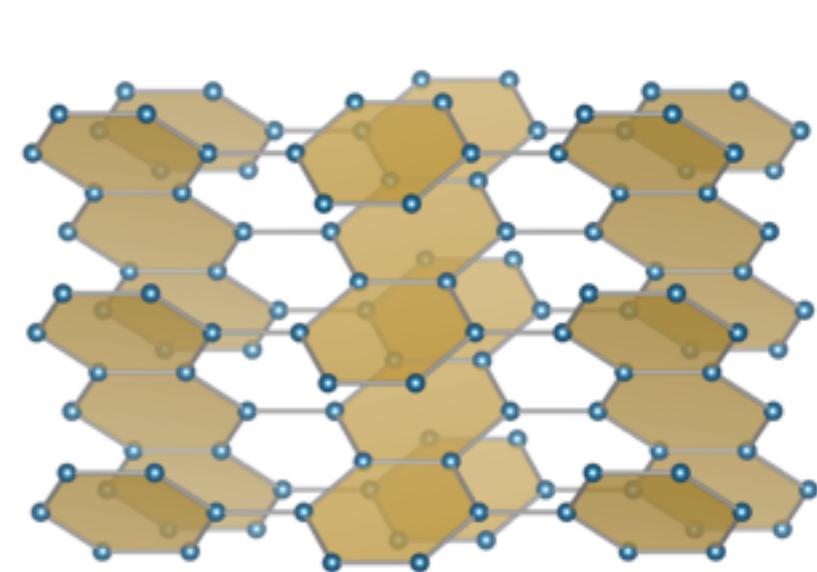
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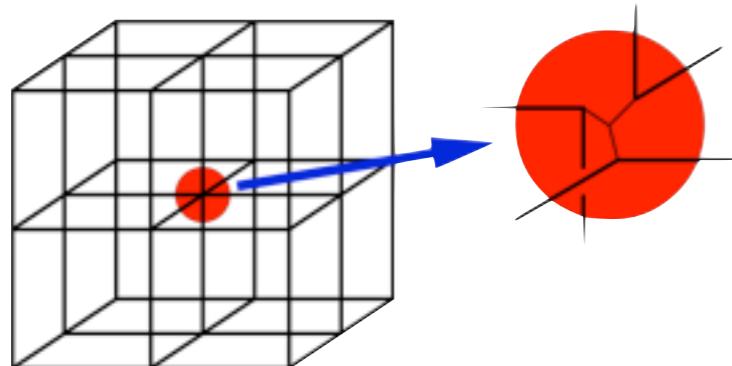
Mandal, Surendran, PRB 79, 024426 (2009)



Takayama et al., arXiv:1403.3296 (2014)

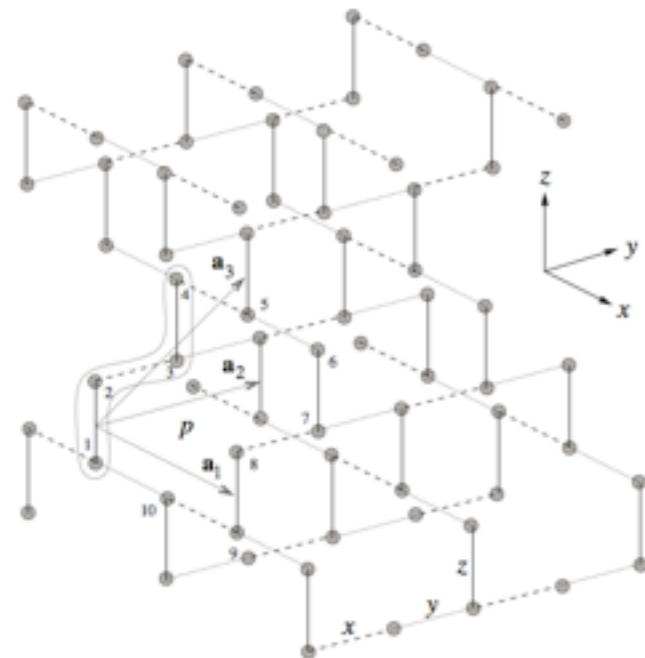


Modic et al. arXiv:1402.3254 (2014)

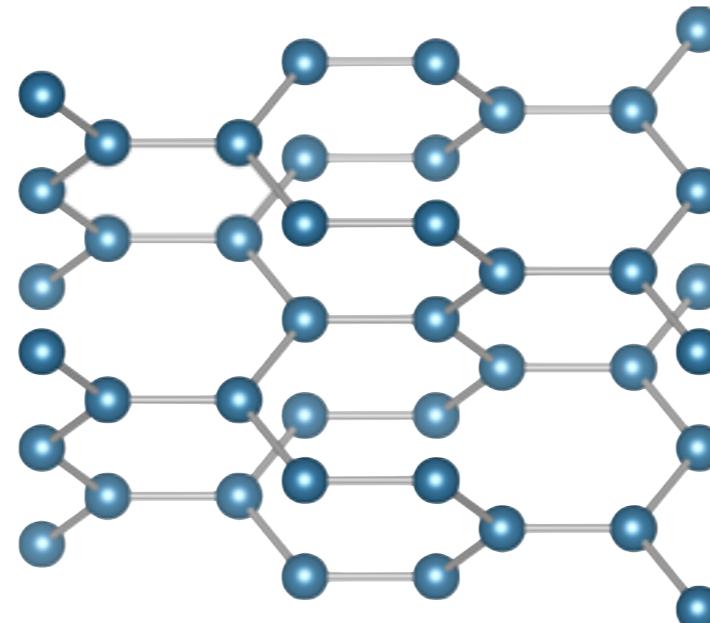


Levin, Wen, PRB 71, 045110 (2005)

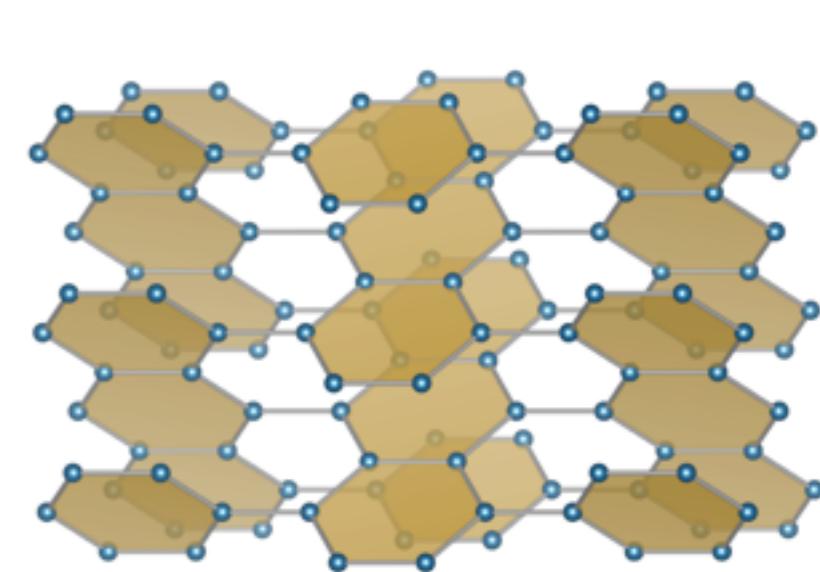
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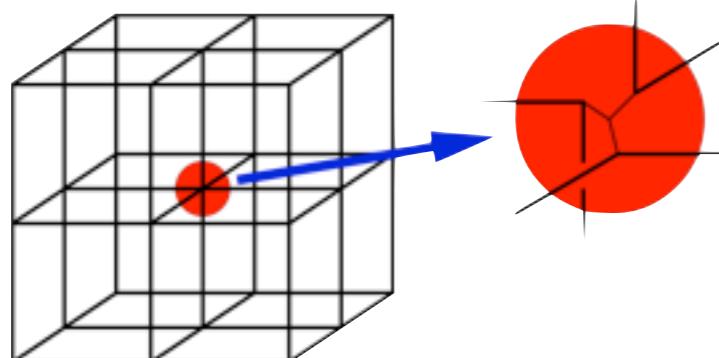
Mandal, Surendran, PRB 79, 024426 (2009)



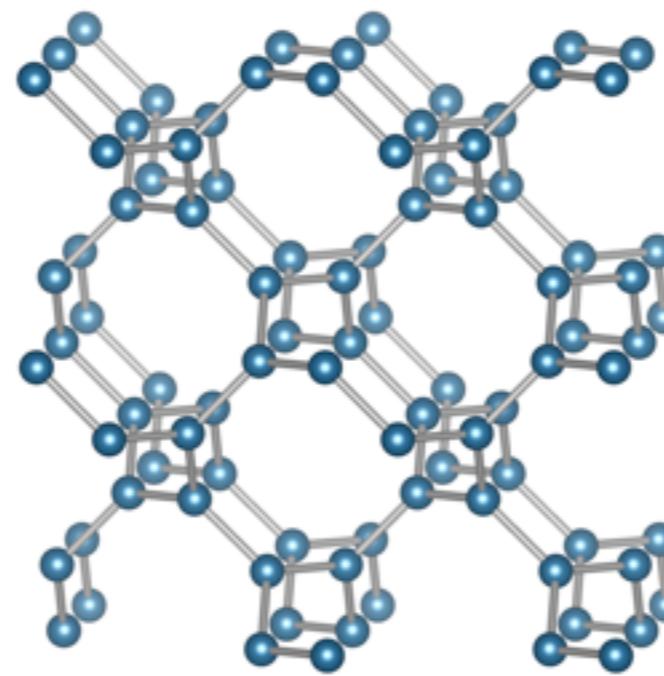
Takayama et al., arXiv:1403.3296 (2014)



Modic et al. arXiv:1402.3254 (2014)

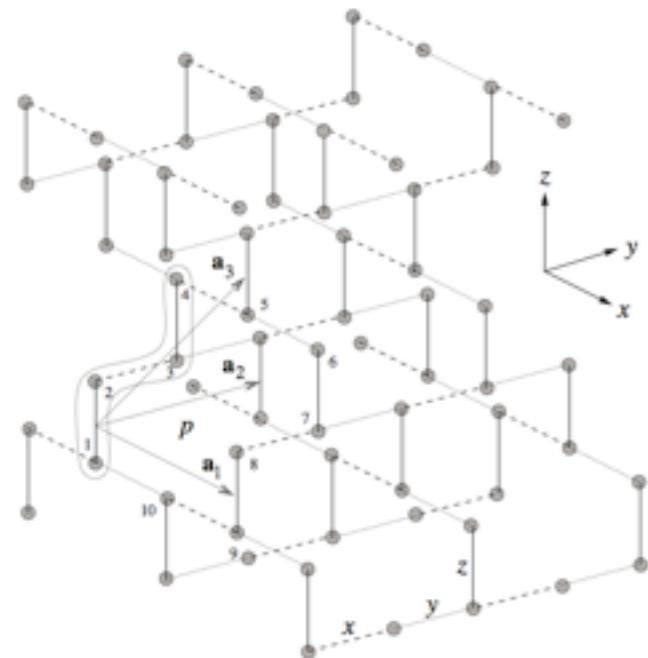


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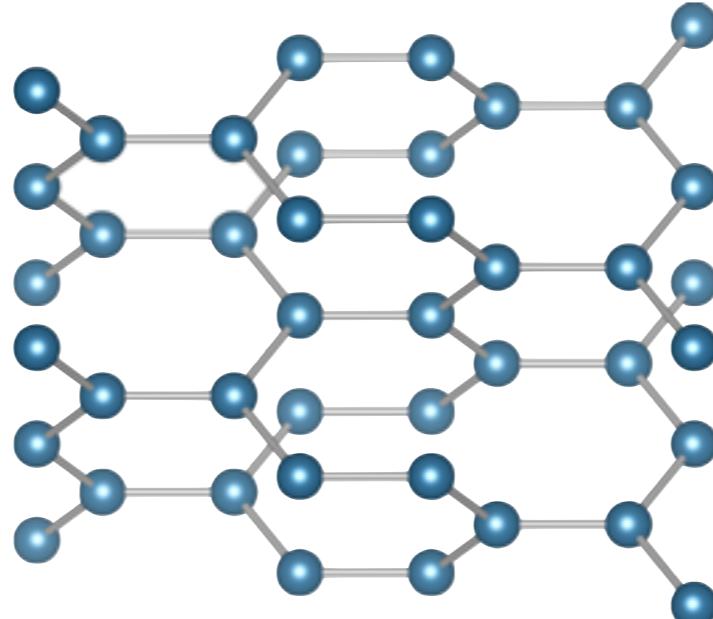


Hermanns, Trebst, PRB 89, 235102 (2014)

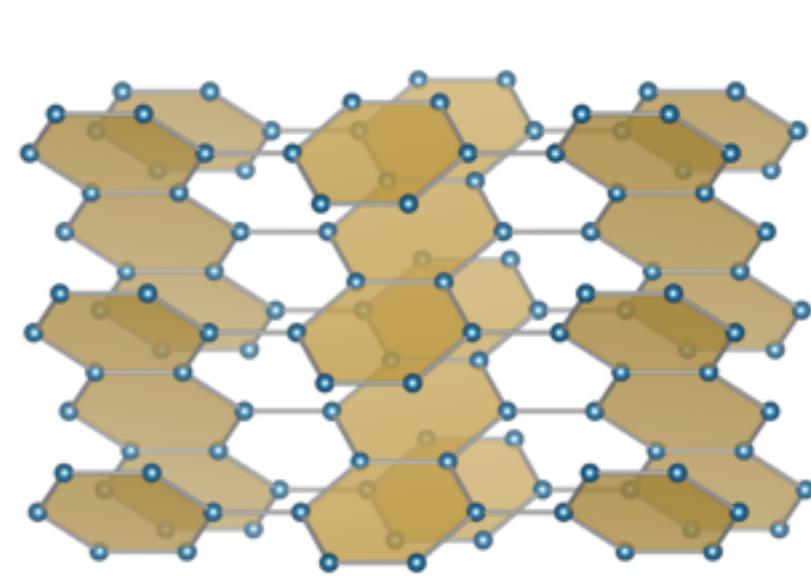
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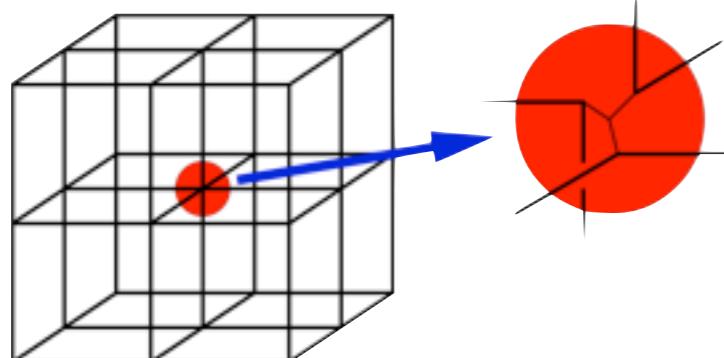
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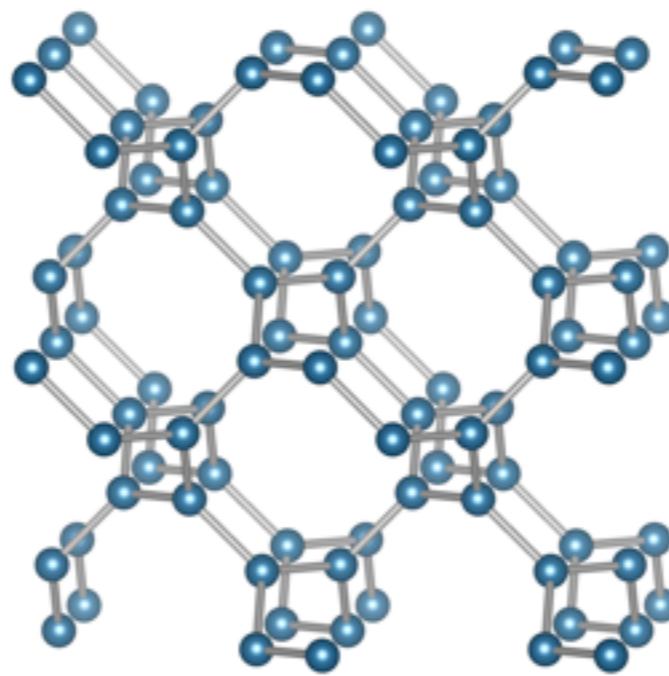
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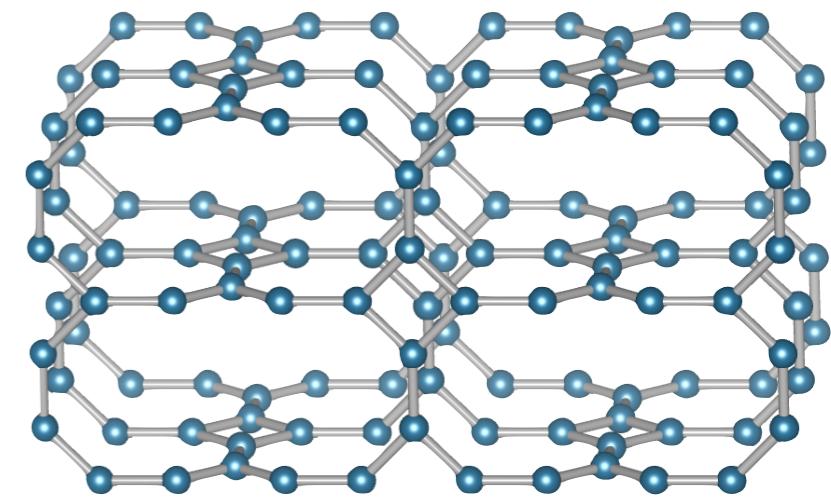
Modic et al. arXiv:1402.3254 (2014)



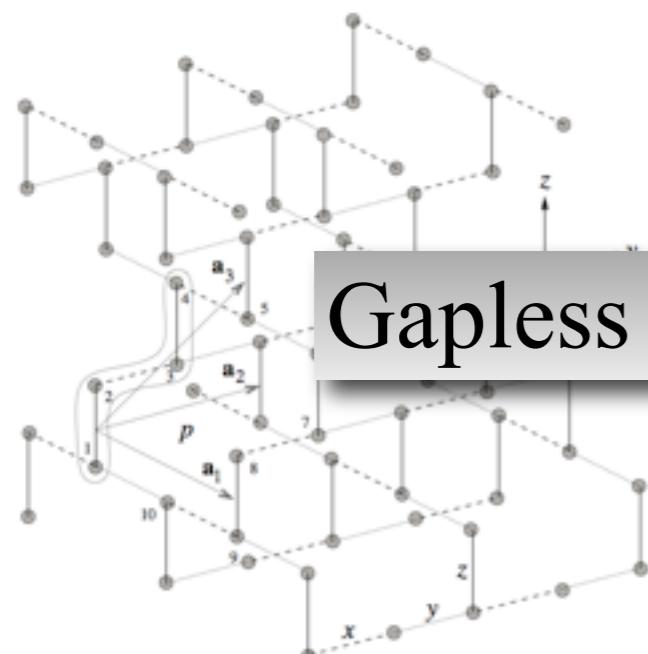
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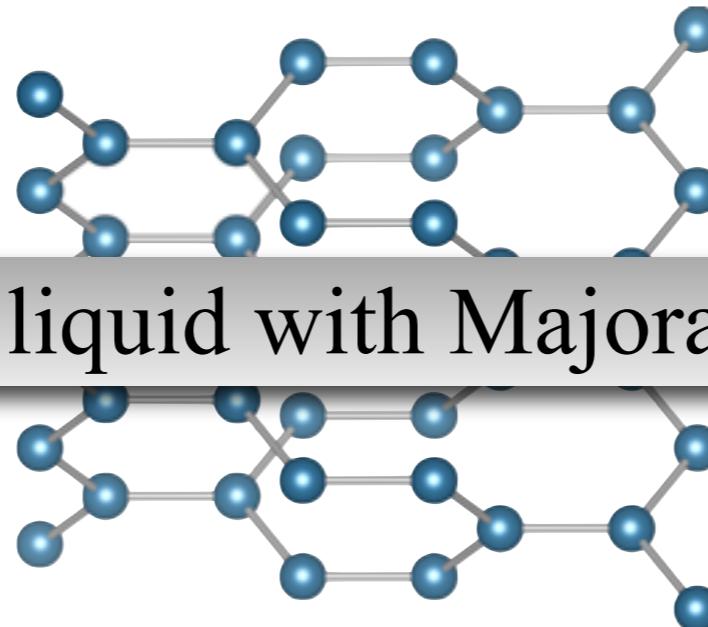


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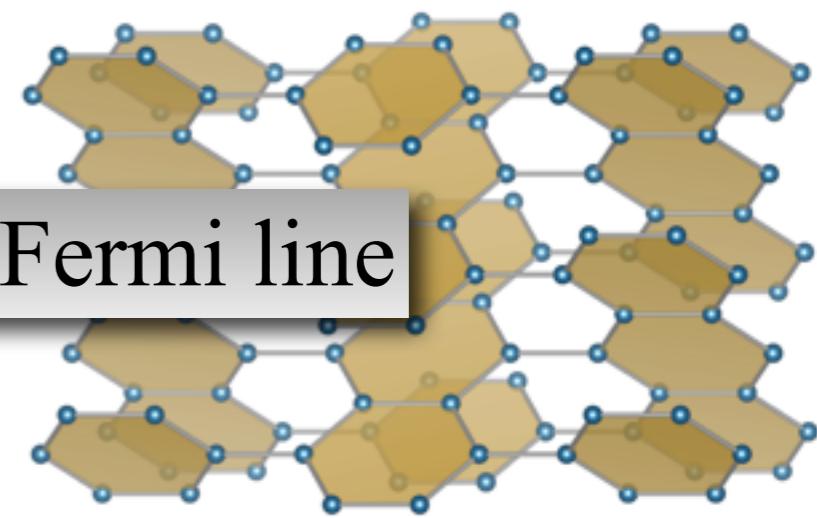


Gapless spin liquid with Majorana Fermi line

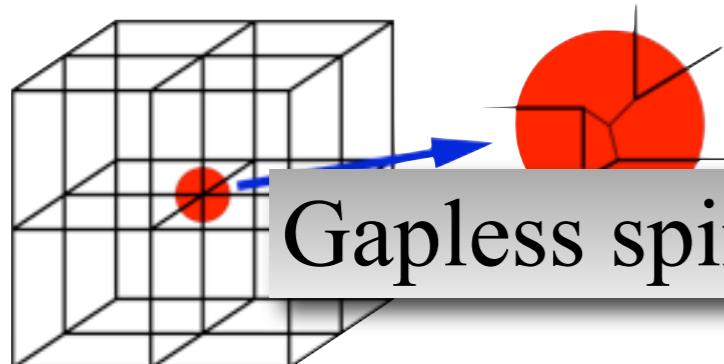
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Takayama et al., arXiv:1403.3296 (2014)



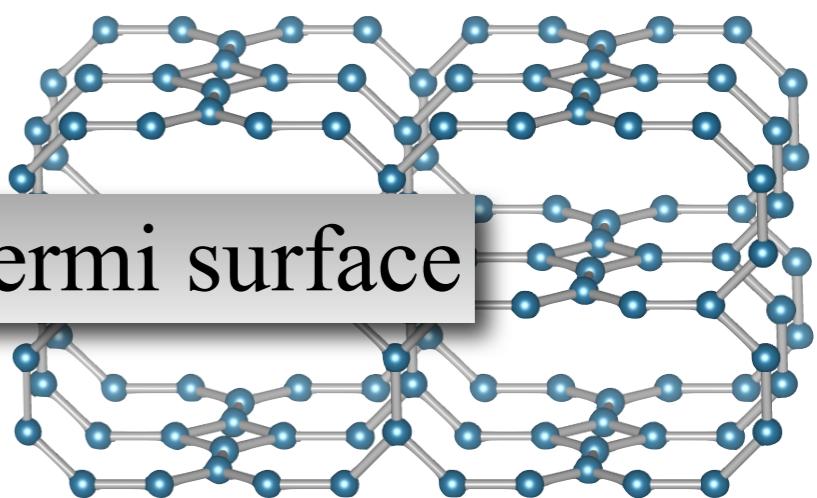
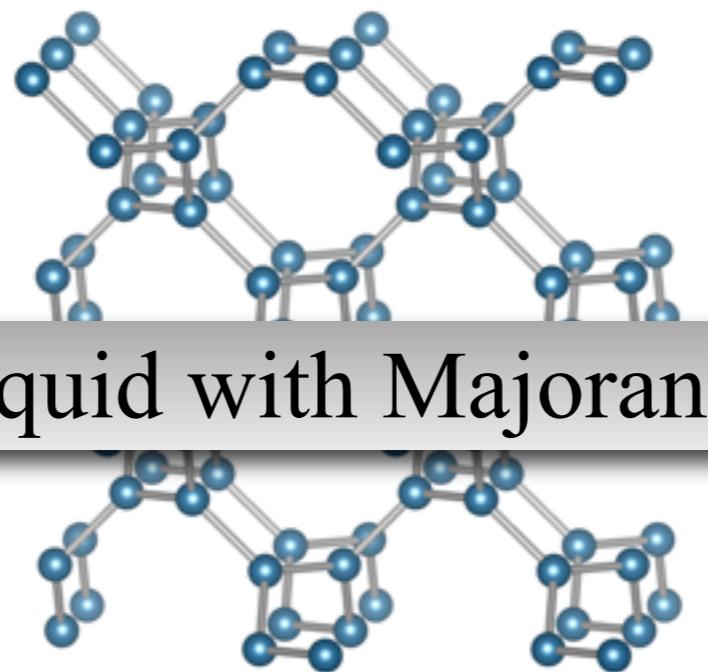
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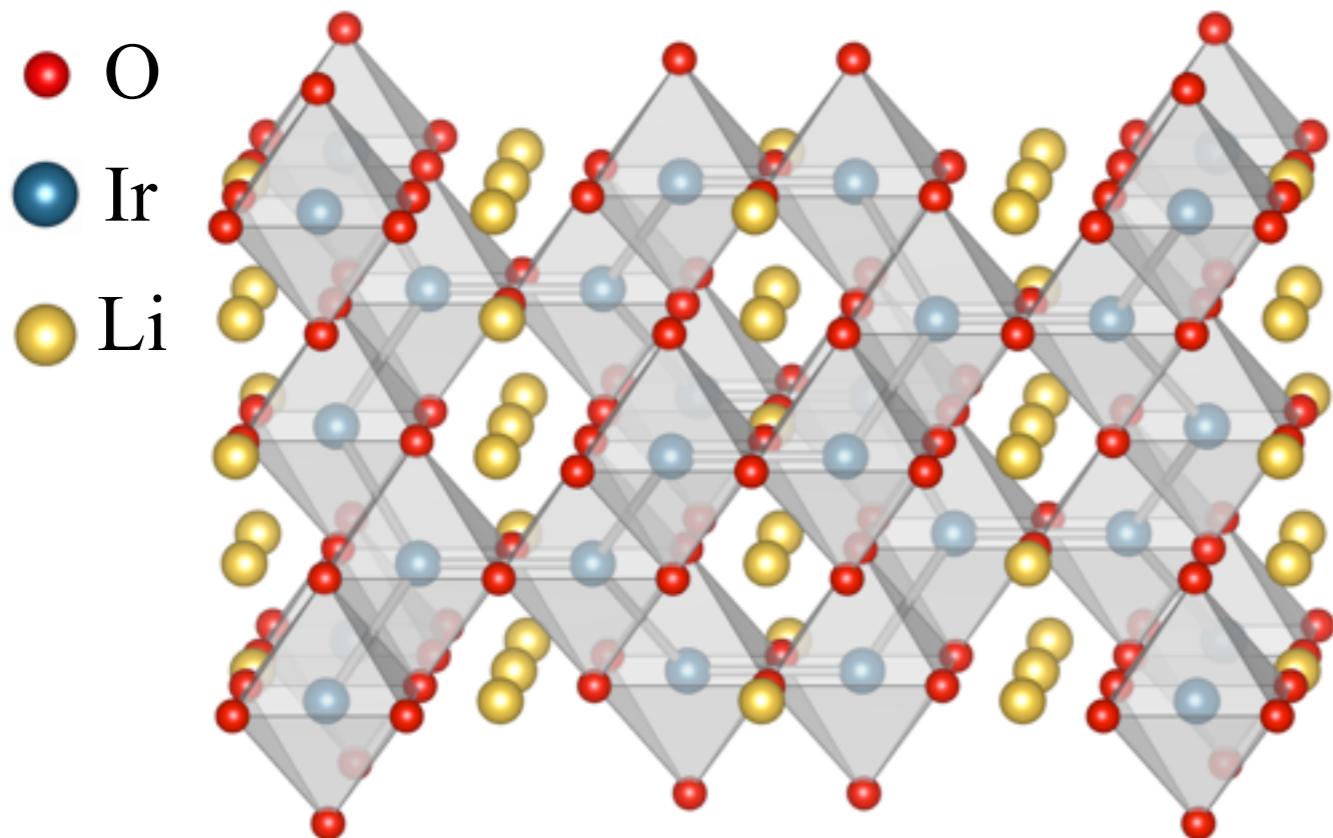
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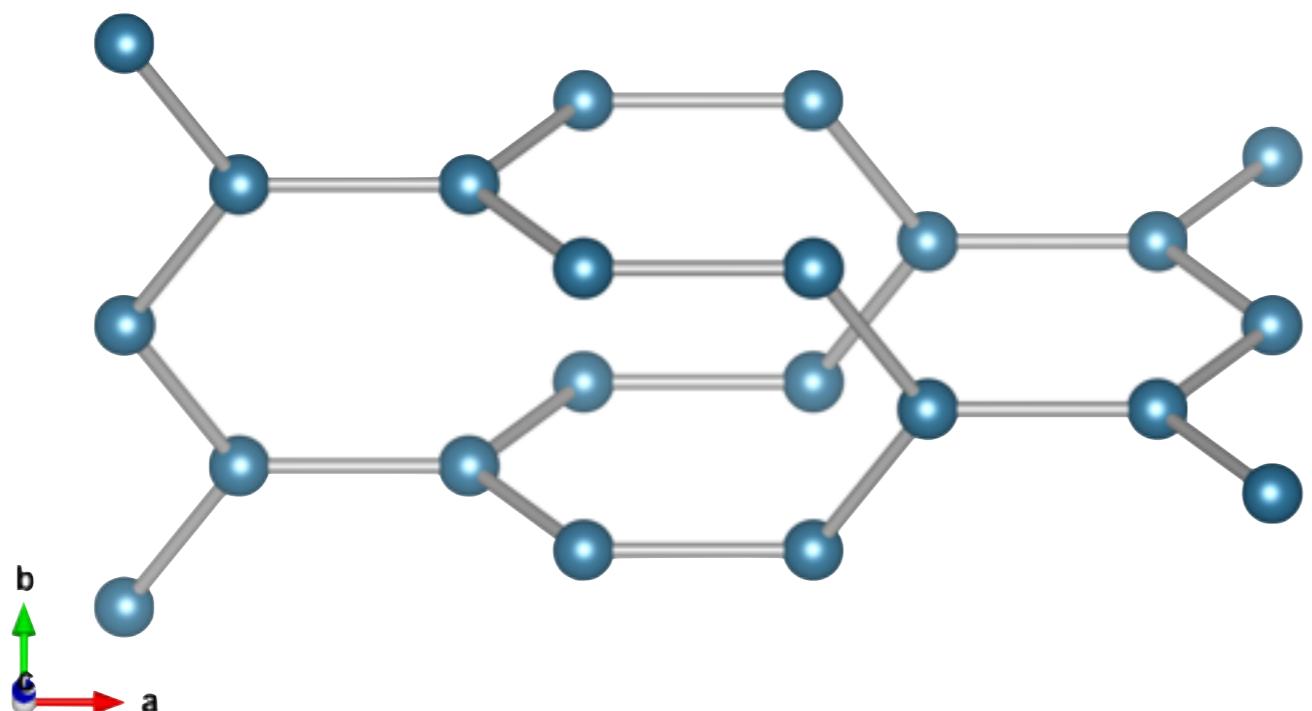
Hermanns, Trebst, PRB 89, 235102 (2014)



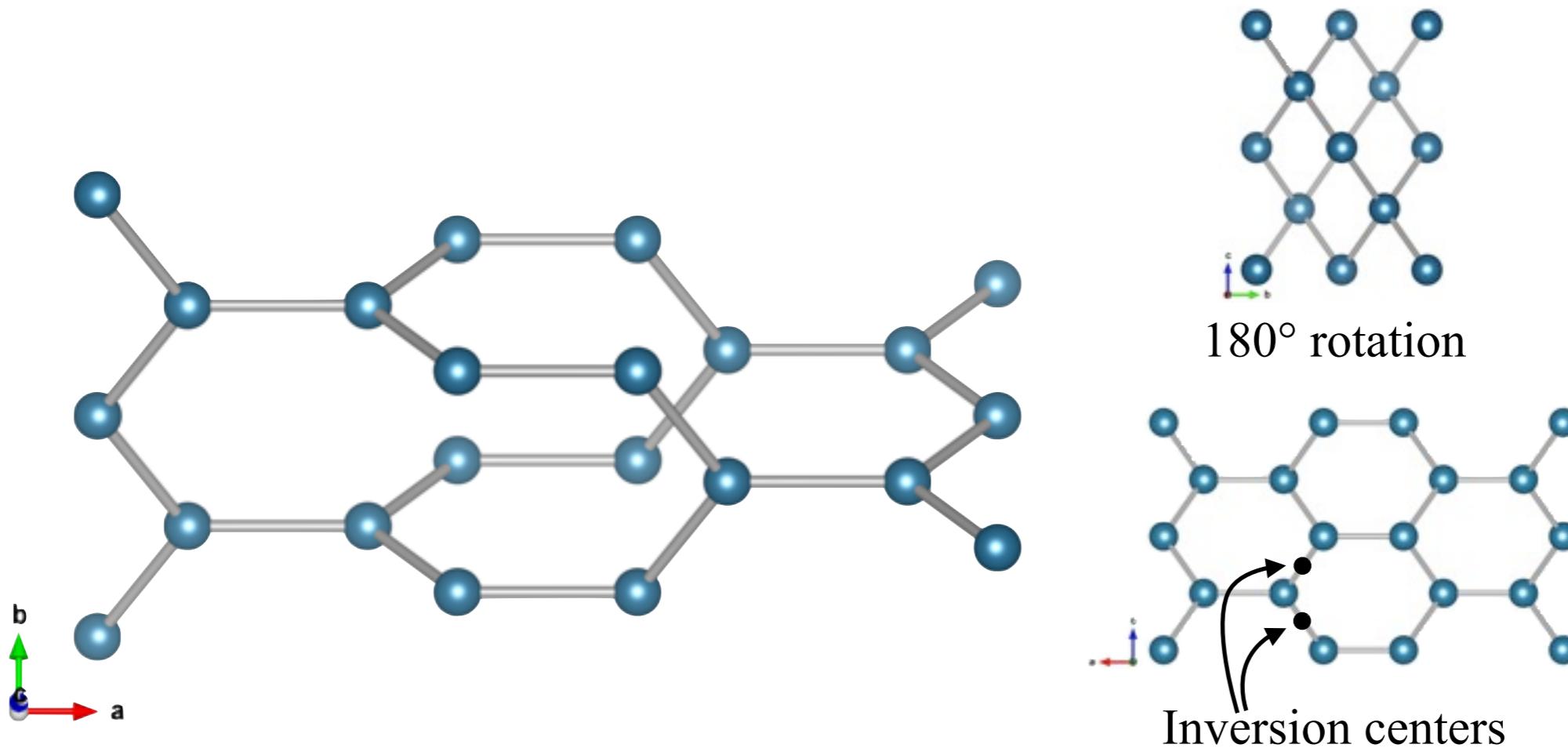
# Hyperhoneycomb material $\beta\text{-Li}_2\text{IrO}_3$



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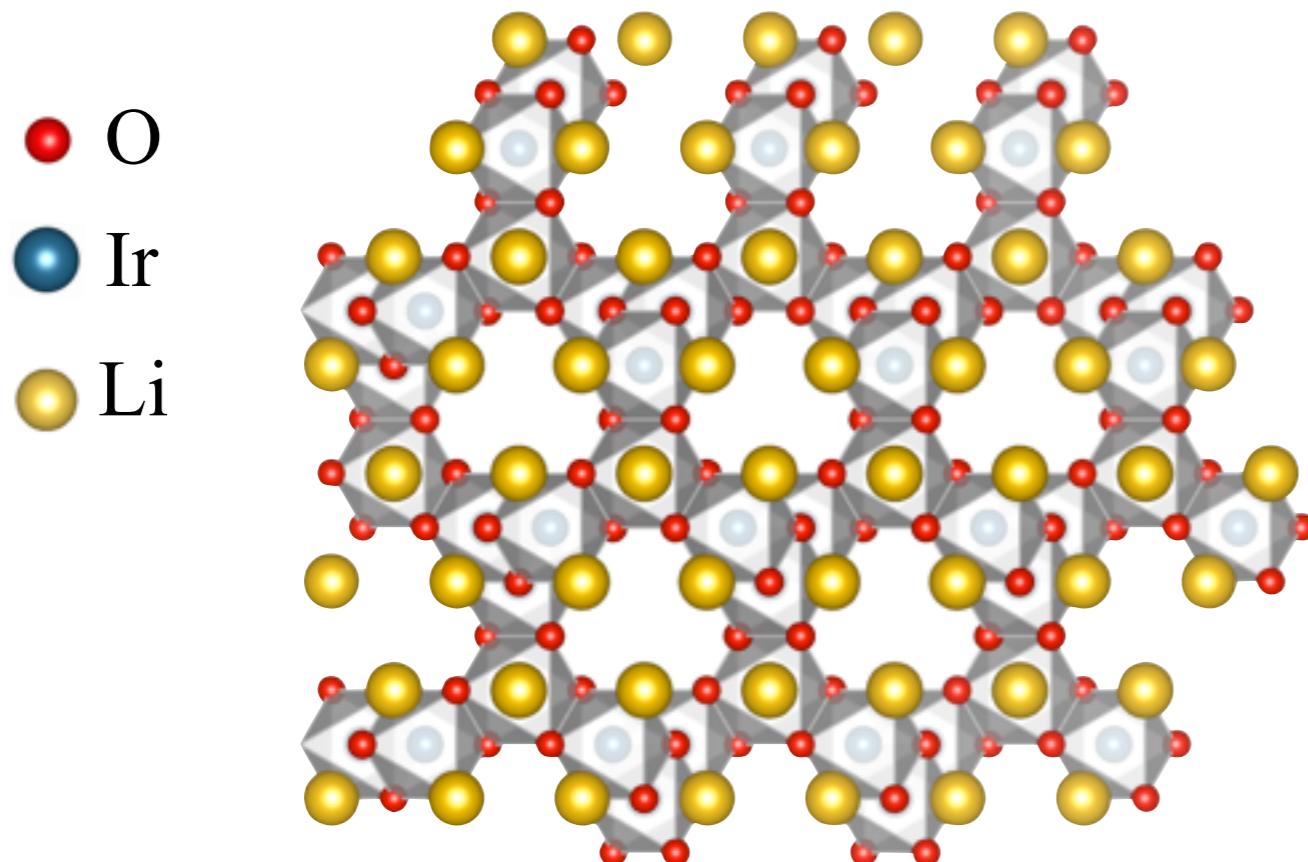


# Hyperhoneycomb material $\beta$ -Li<sub>2</sub>IrO<sub>3</sub>

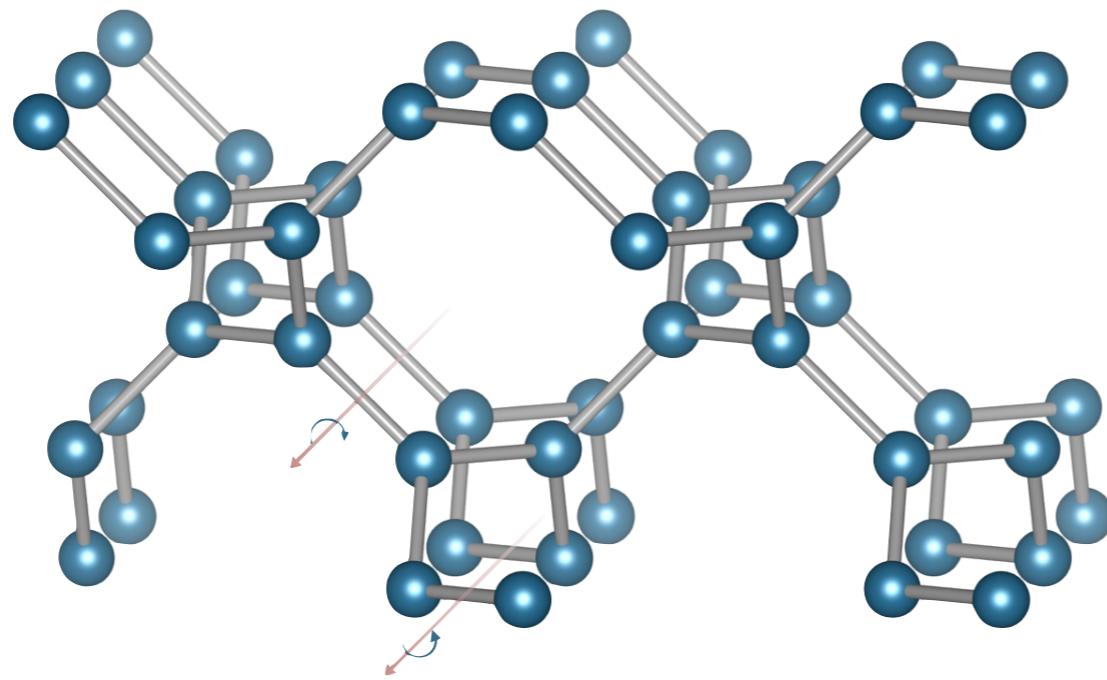


- tri-coordinated
- inversion symmetric
- preferred direction
- synthesized by - H. Takagi arxiv:1403.3296  
- J. Analytis arxiv:1402.3254

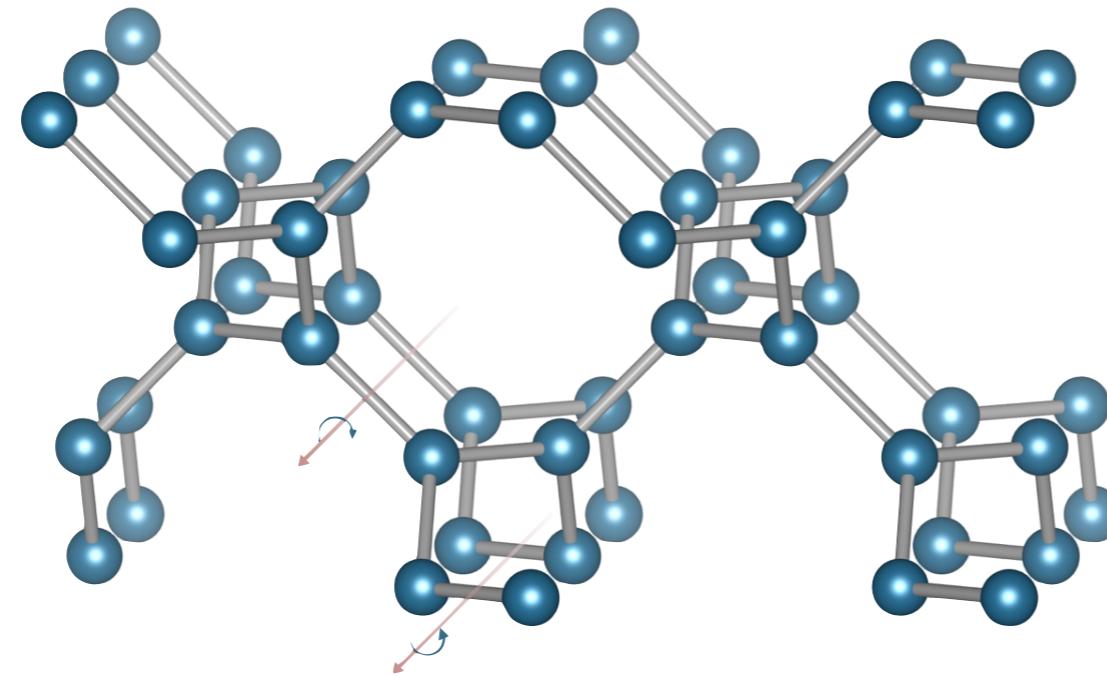
# Hyperoctagon material $\delta\text{-Li}_2\text{IrO}_3$ (?)



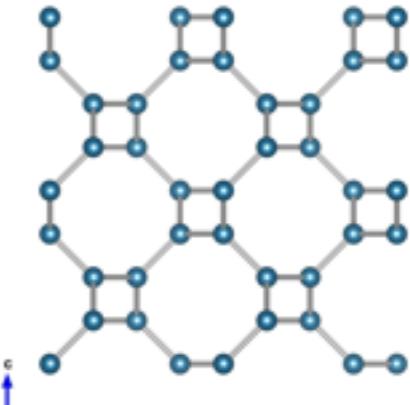
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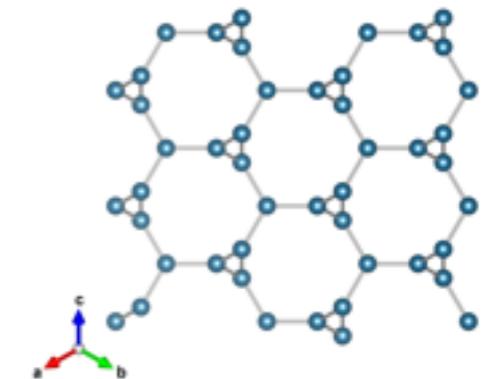
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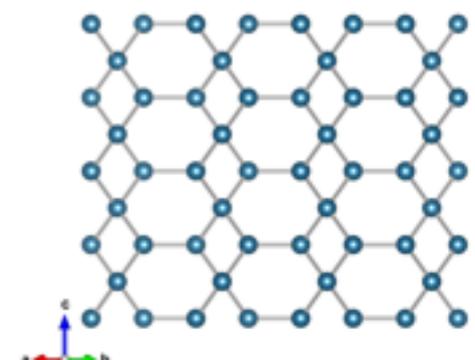
- tri-coordinated
- chiral
- cubic symmetry
- premedial lattice of the hyperkagome
- possible fourth crystalline form of  $\text{Li}_2\text{IrO}_3$



90° screw-rotation

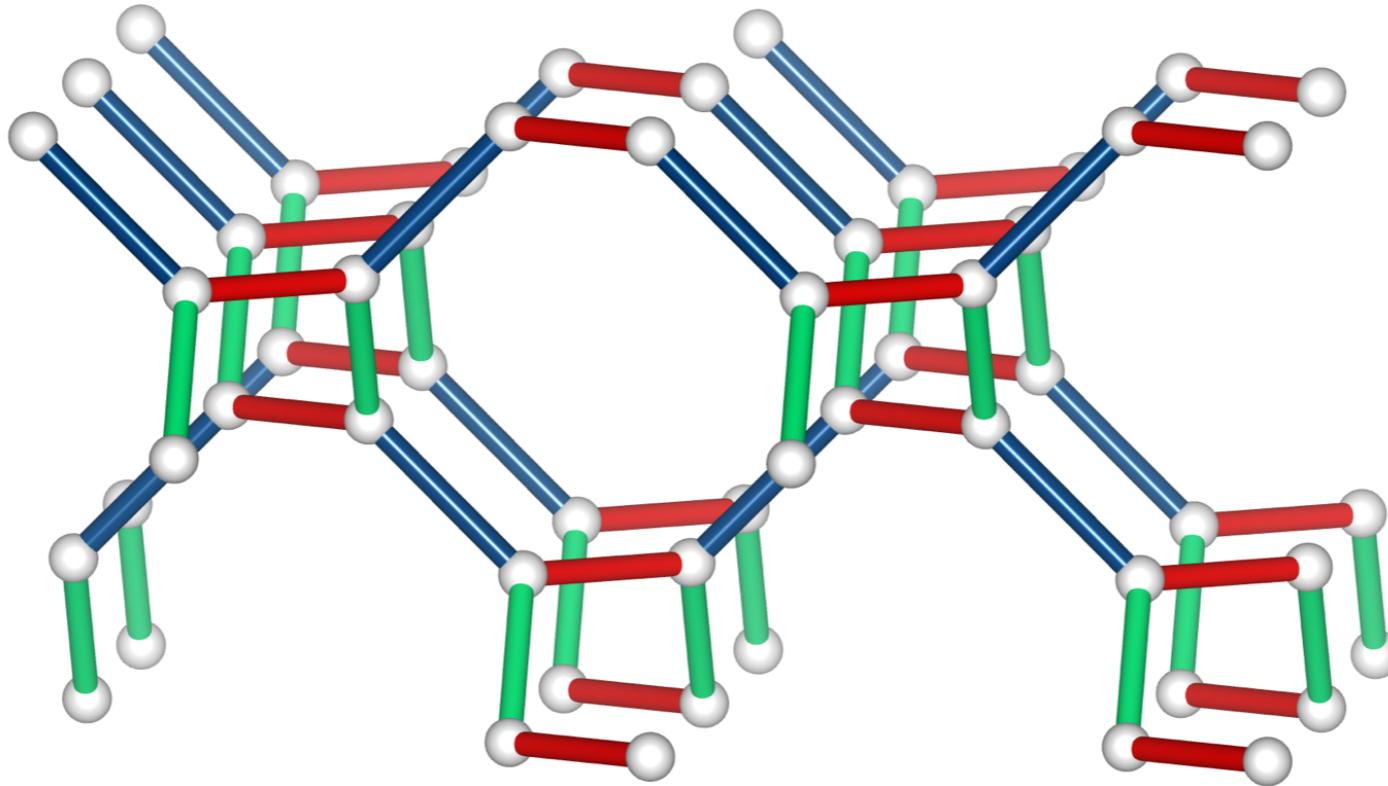


120° rotation



180° rotation

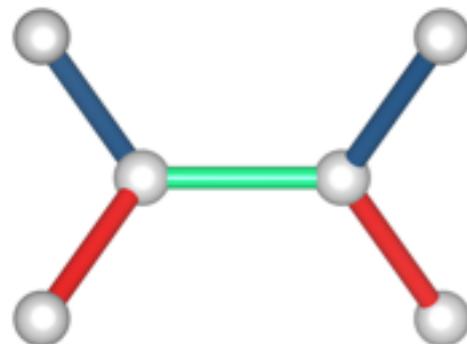
# Exchange frustration on the hyperoctagon lattice



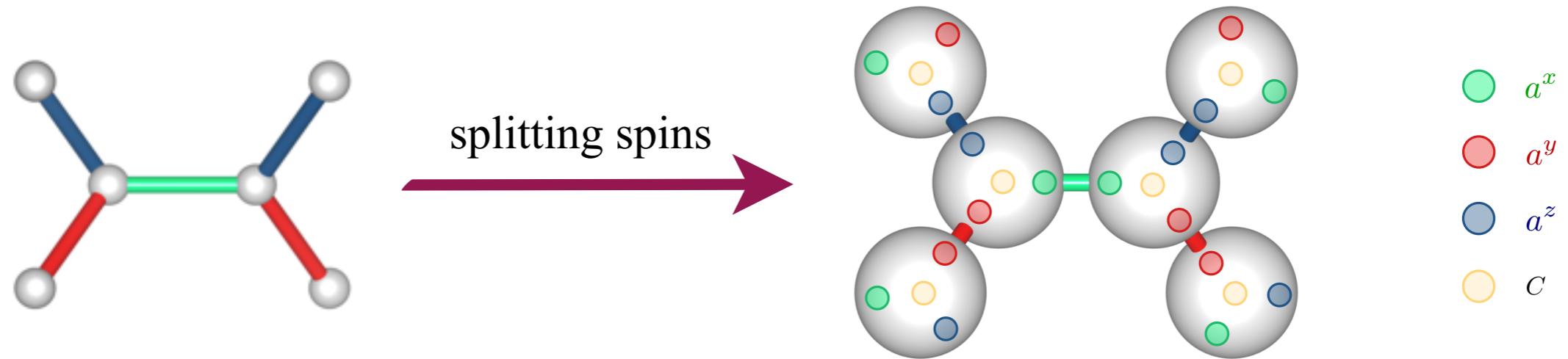
$$H = - \sum_{\text{x-bonds}} J_x \sigma_j^x \sigma_k^x - \sum_{\text{y-bonds}} J_y \sigma_j^y \sigma_k^y - \sum_{\text{z-bonds}} J_z \sigma_j^z \sigma_k^z$$

3D generalization of Kitaev's honeycomb model

# Spin fractionalization and Majorana fermions

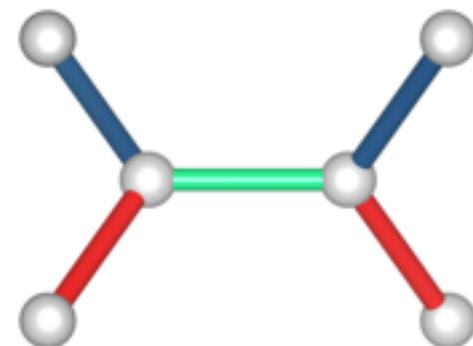


# Spin fractionalization and Majorana fermions

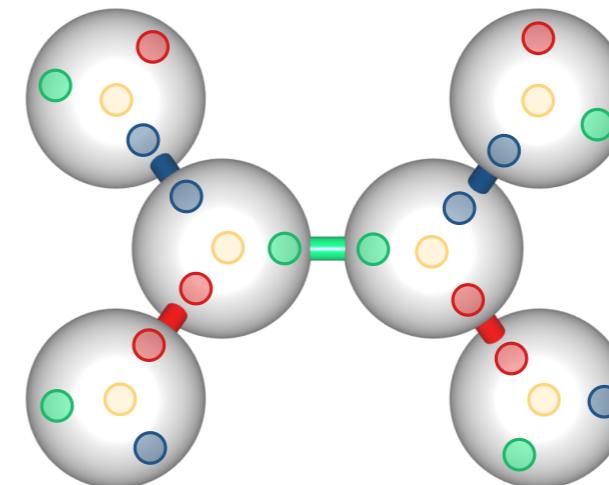


- Represent spins in terms of four **Majorana fermions**  $\sigma^\alpha = ia^\alpha c$

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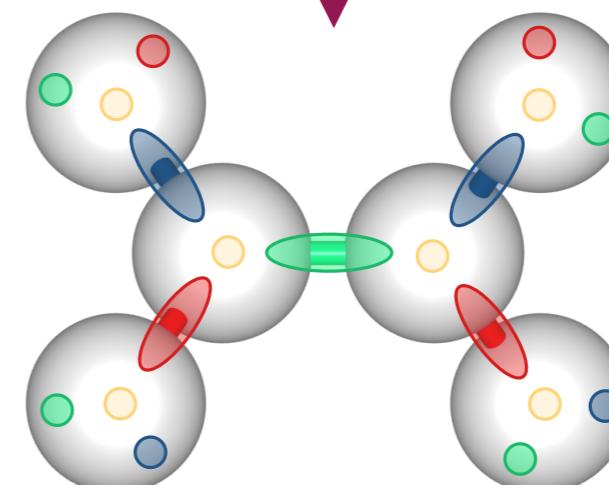


splitting spins →



Legend:  
●  $a^x$   
●  $a^y$   
●  $a^z$   
●  $C$

↓ joining Majoranas



Legend:  
●  $ia^x a^x$   
●  $ia^y a^y$   
●  $ia^z a^z$   
●  $C$

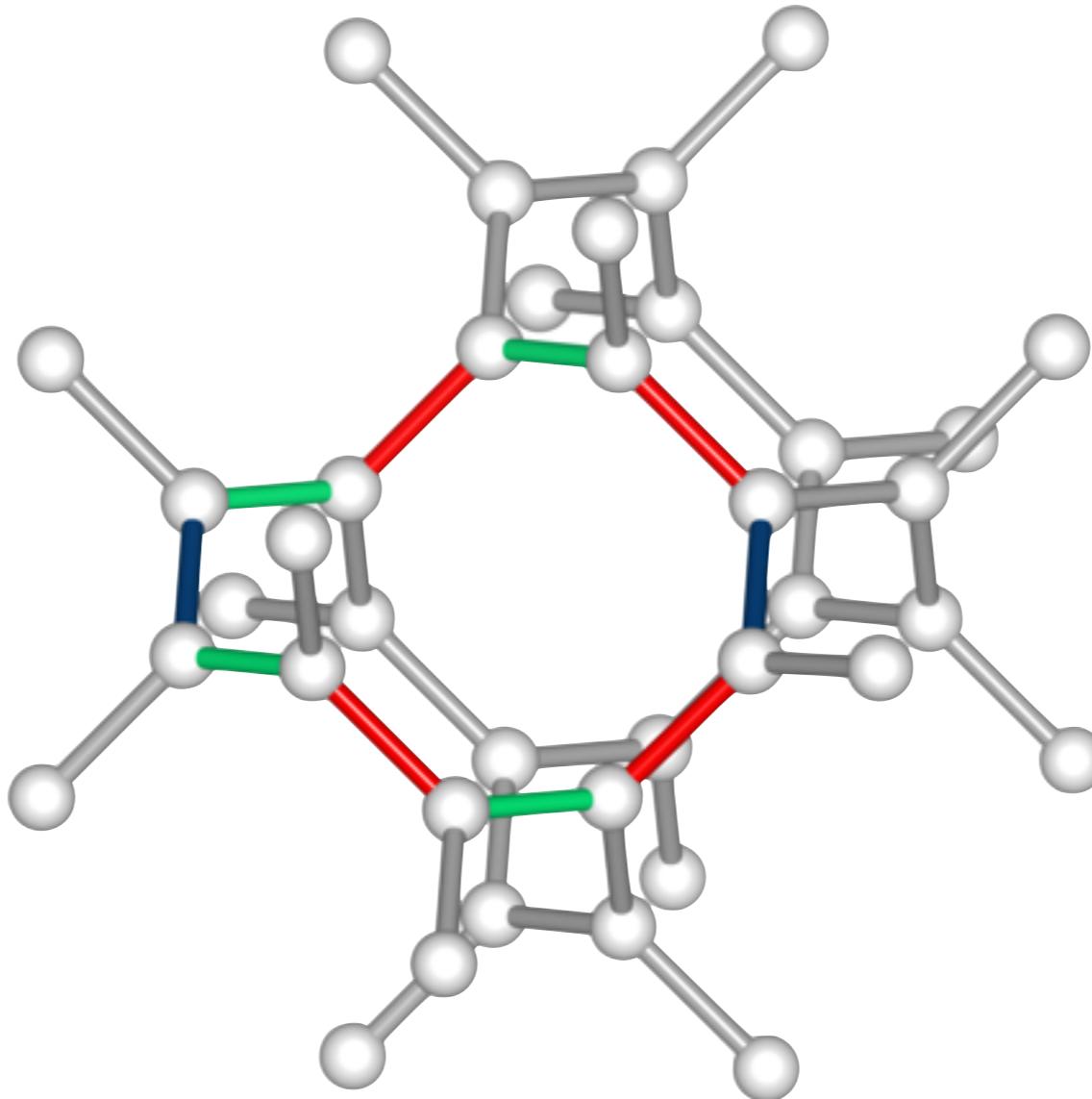
- Represent spins in terms of four **Majorana fermions**  $\sigma^\alpha = ia^\alpha c$
- Bond operators  $\hat{u}_{jk} = ia_j^\alpha a_k^\alpha$  realize an emergent **Z<sub>2</sub> gauge field**

# Physics of the $Z_2$ gauge field

$Z_2$  gauge field is **static** due to presence of additional conserved quantities

Six fundamental **loop operators** (per unit cell)  $W_l = \prod_{\langle\alpha,\beta\rangle \in l} \sigma_\alpha^{\gamma_{\alpha\beta}} \sigma_\beta^{\gamma_{\alpha\beta}}$

**conserved quantities**

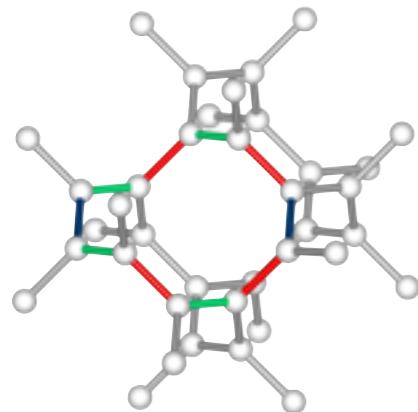


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↓  
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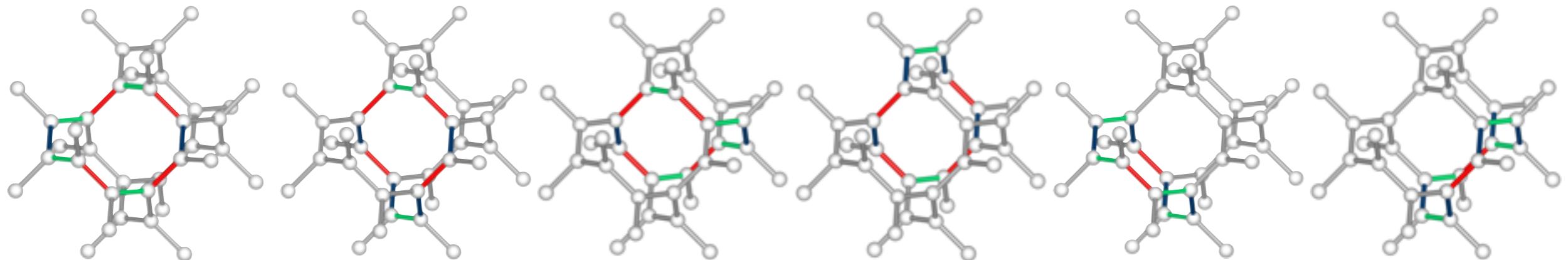


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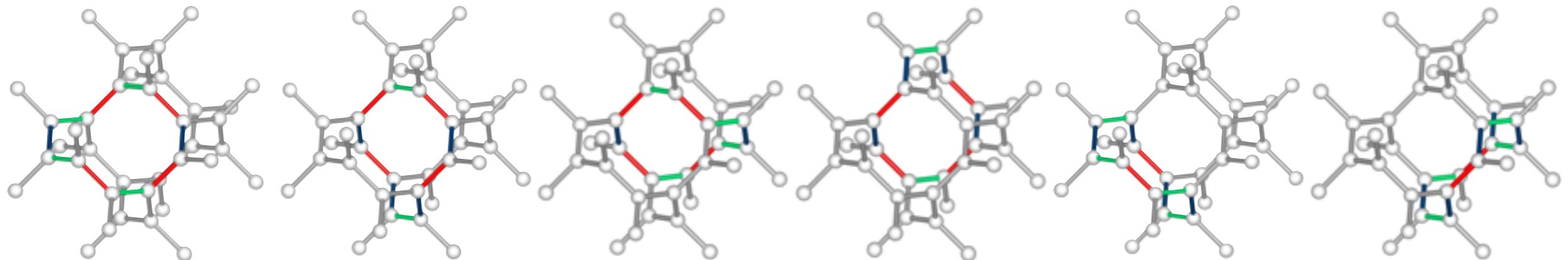


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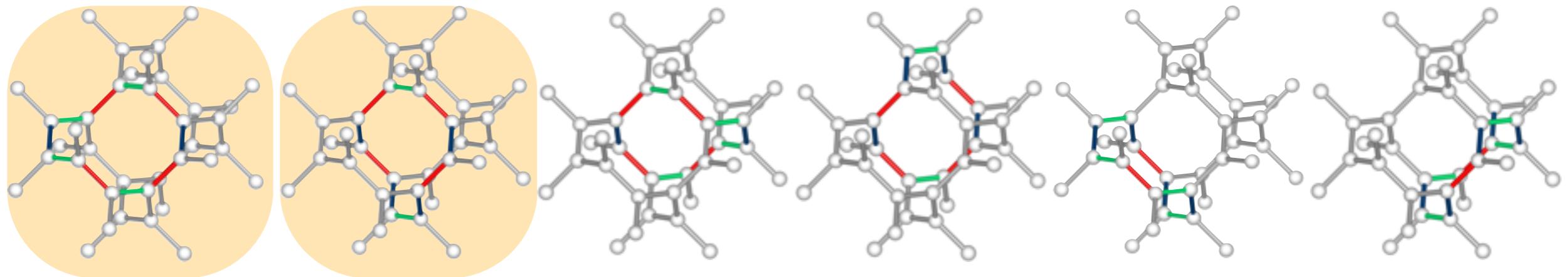
loop operators define closed  $Z_2$  flux loops – **no monopoles**

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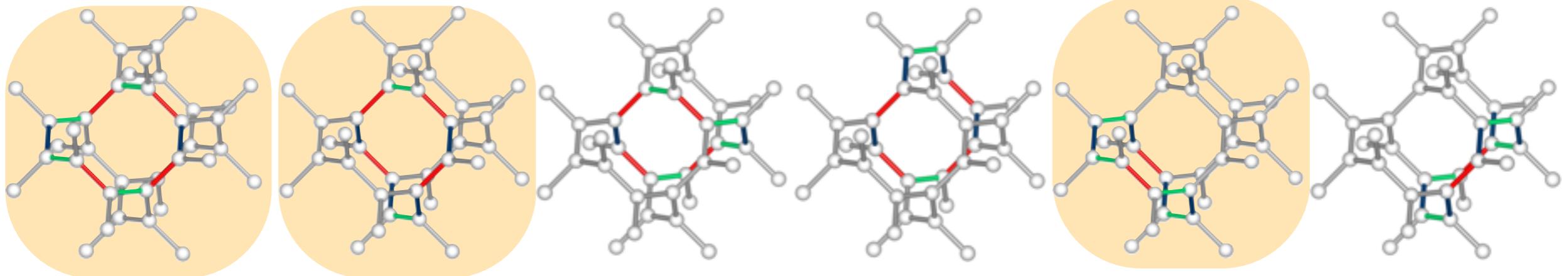
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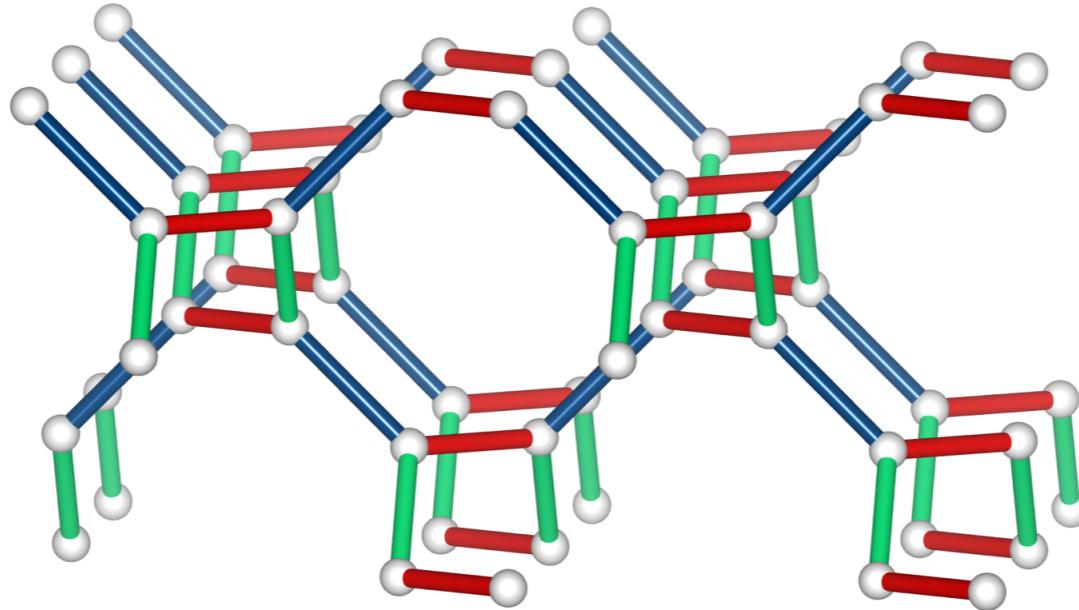
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↓  
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loop operators define closed  $Z_2$  flux loops – **no monopoles**  
only two loop operators per unit cell are linearly independent

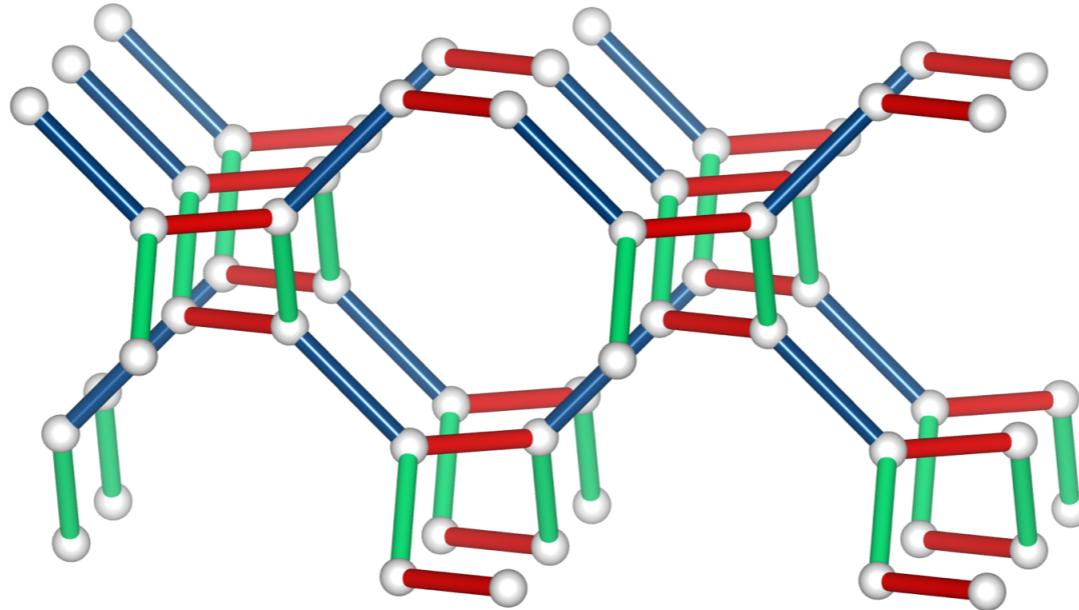
# Exchange frustration on the hyperoctagon model



$$H = - \sum_{\text{x-bonds}} J_x \sigma_j^x \sigma_k^x - \sum_{\text{y-bonds}} J_y \sigma_j^y \sigma_k^y - \sum_{\text{z-bonds}} J_z \sigma_j^z \sigma_k^z$$

Hilbert space split into two separate sectors:  $2^N = 2^{N/2} \times 2^{N/2}$

# Exchange frustration on the hyperoctagon model



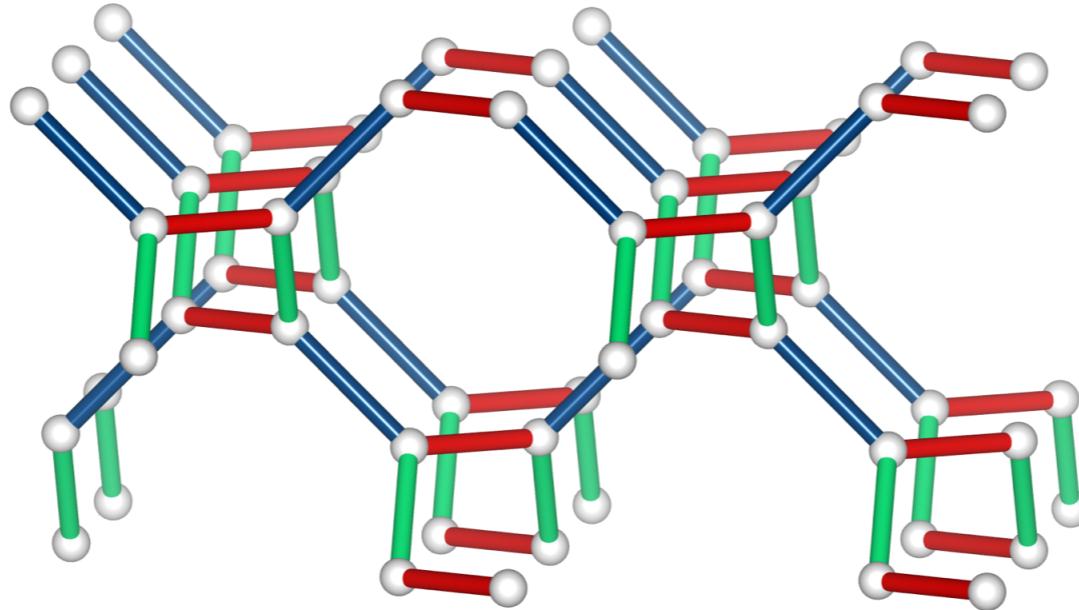
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Majorana fermions  $c_j$   
“spinons”

flux loops “visons”  
(static and gapped)

# Exchange frustration on the hyperoctagon model



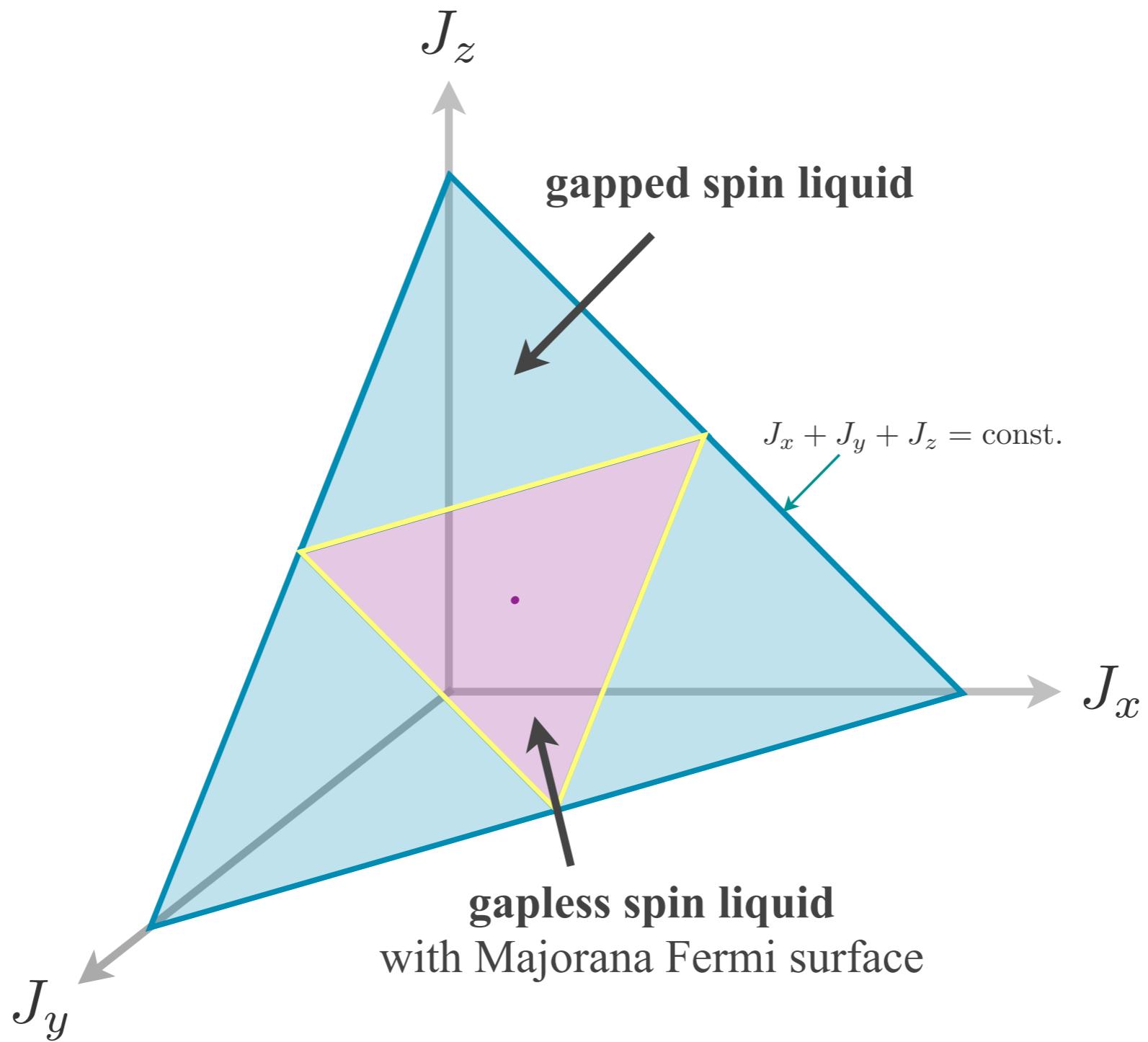
$$H = i \sum_{\gamma-\text{bond}} J_\gamma c_j c_k$$

Hilbert space split into two separate sectors:  $2^N = 2^{N/2} \times 2^{N/2}$

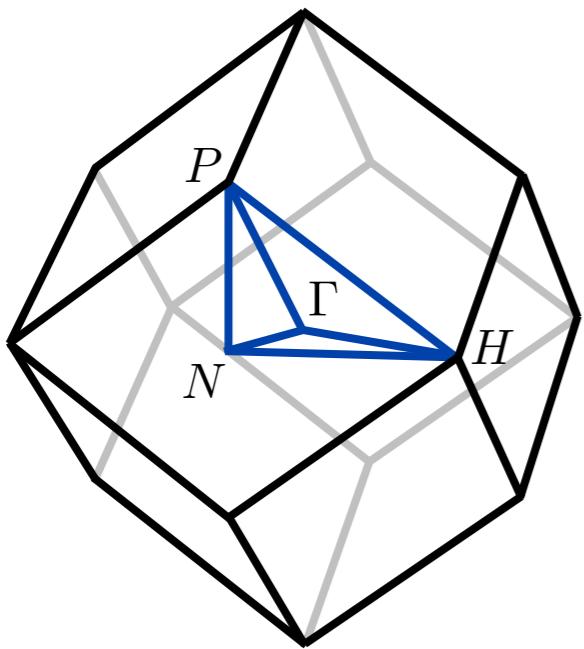
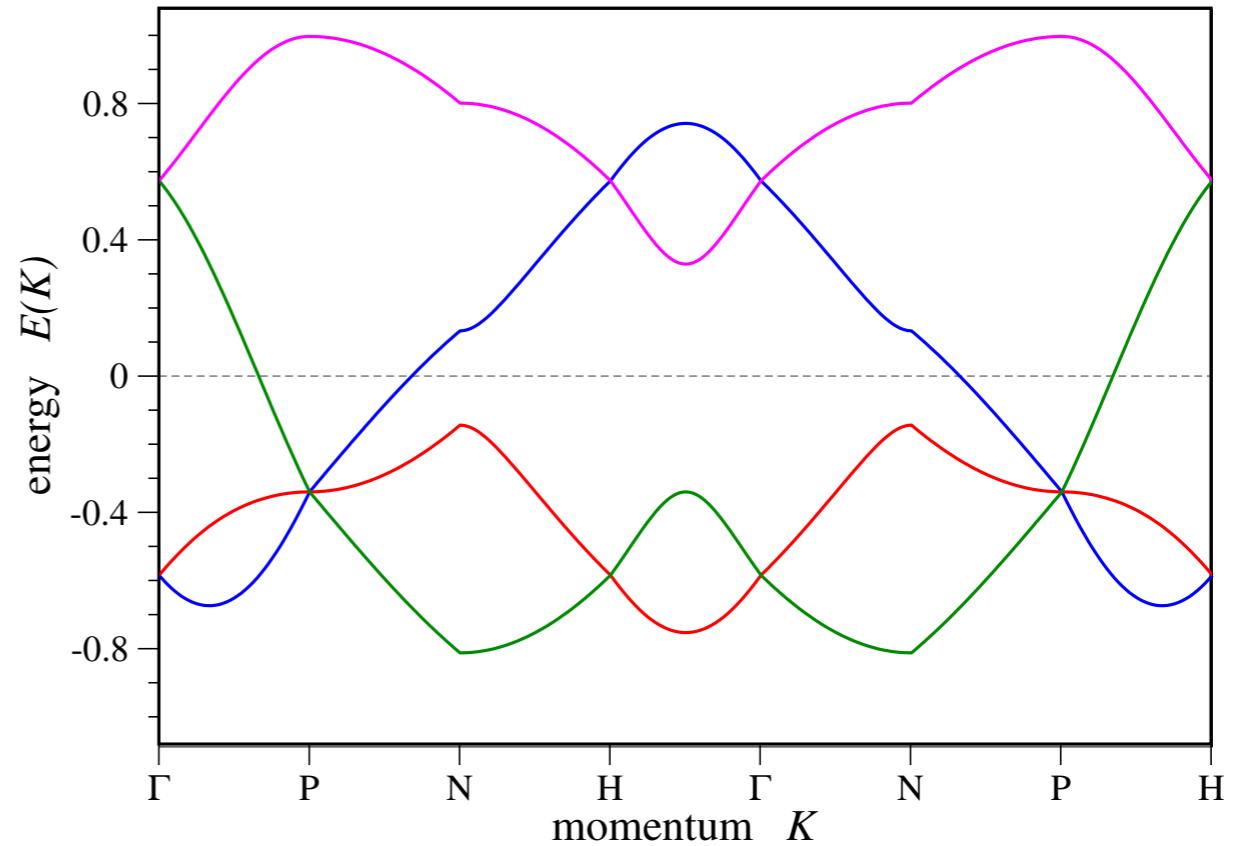
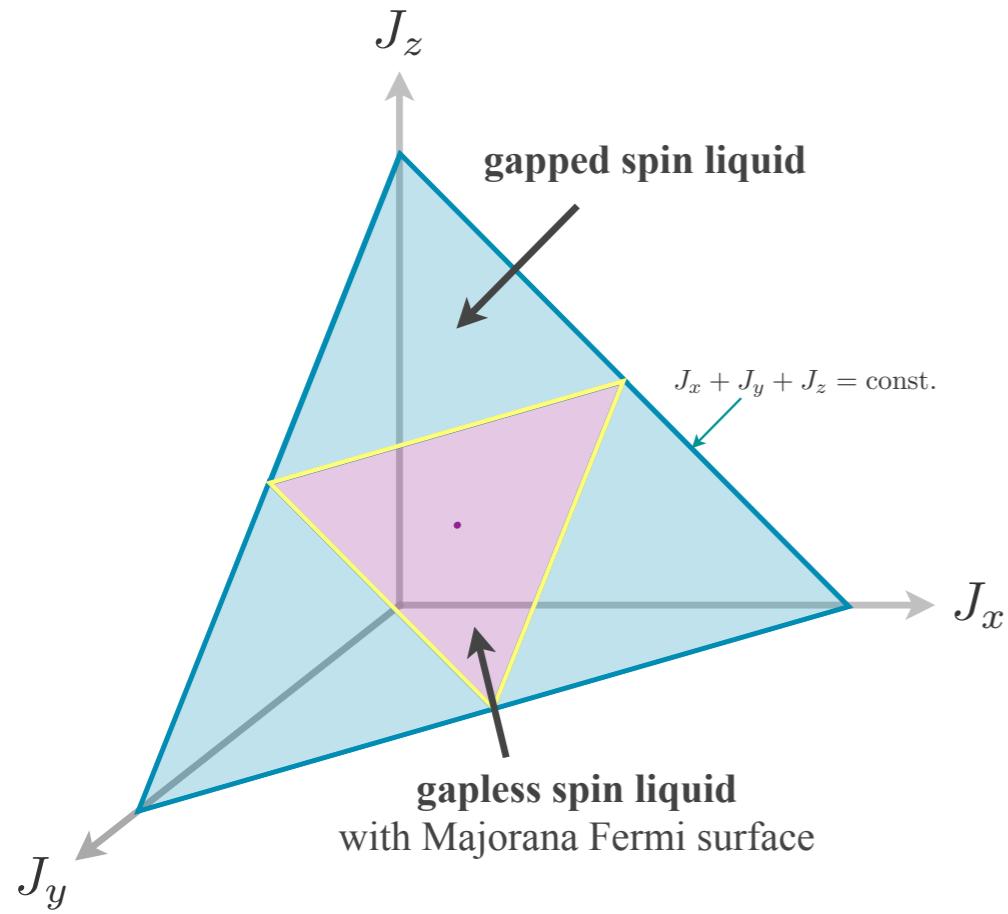
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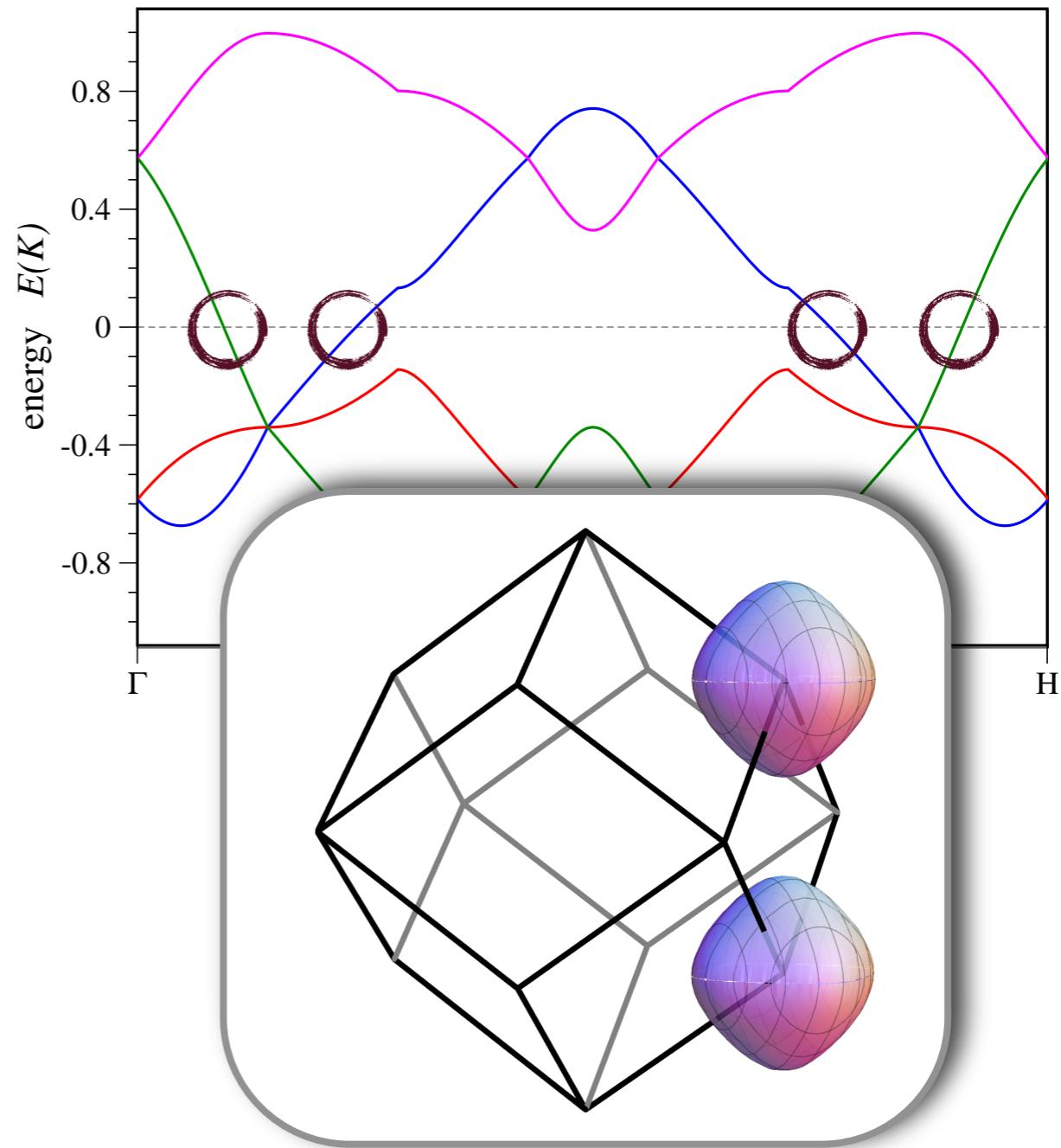
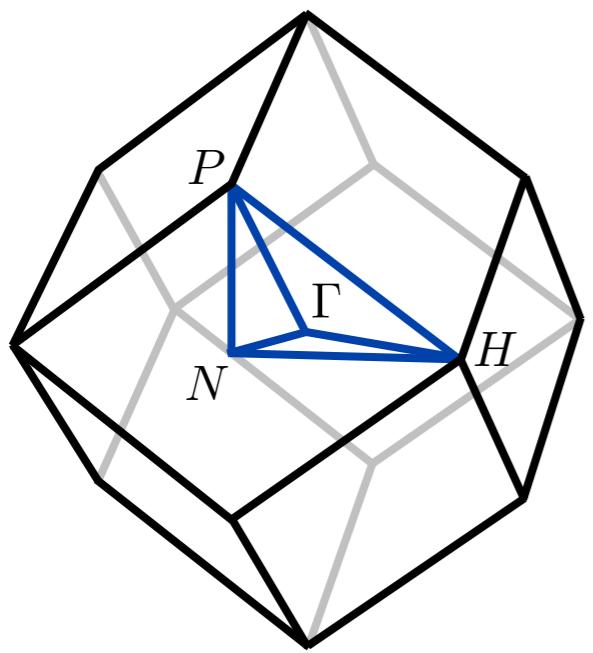
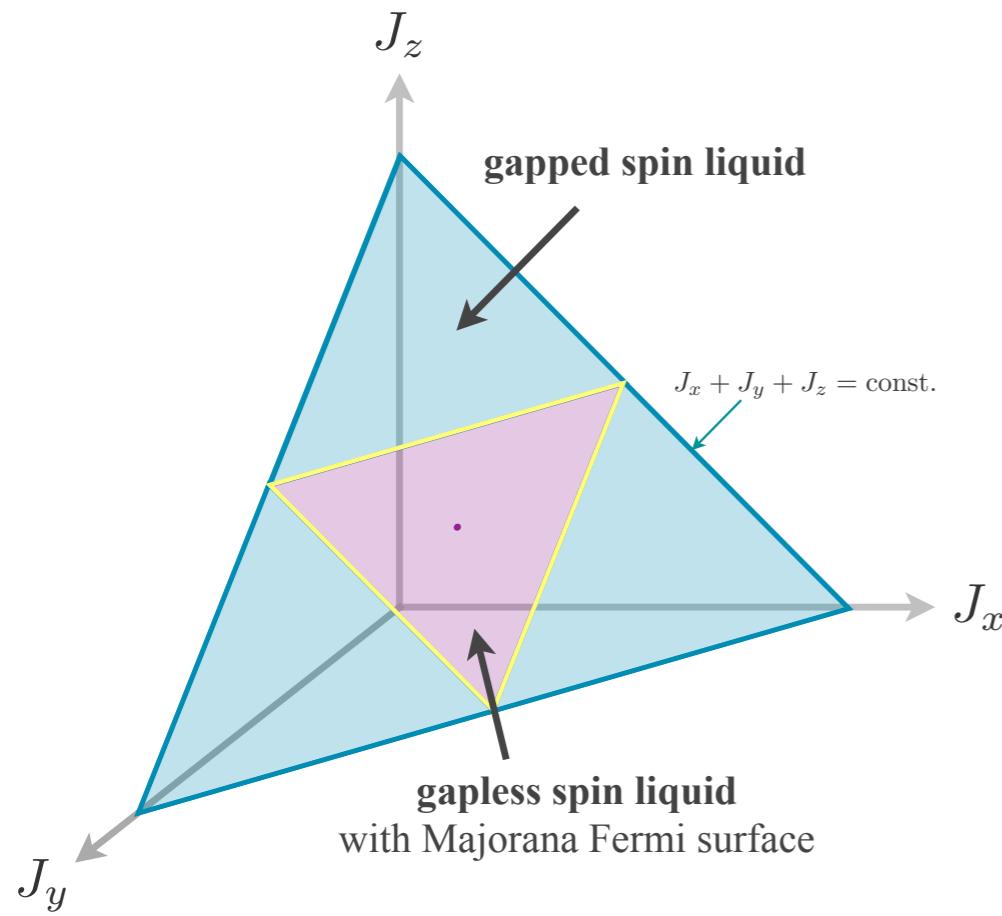
# The phase diagram



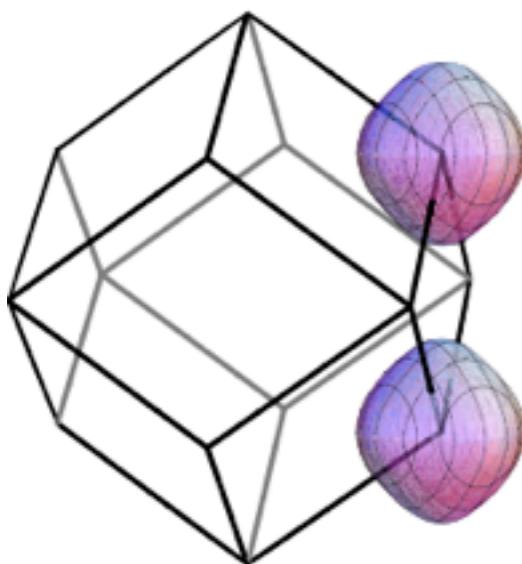
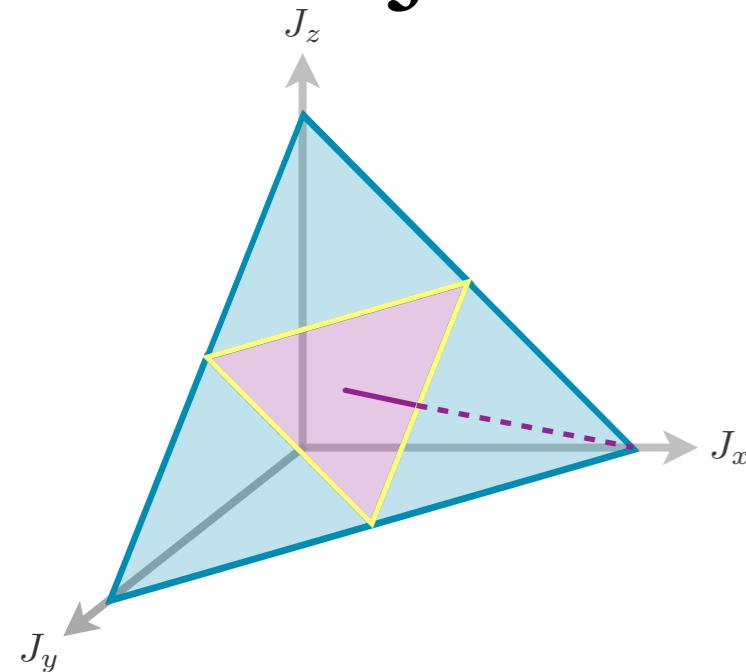
# Majorana Fermi surface



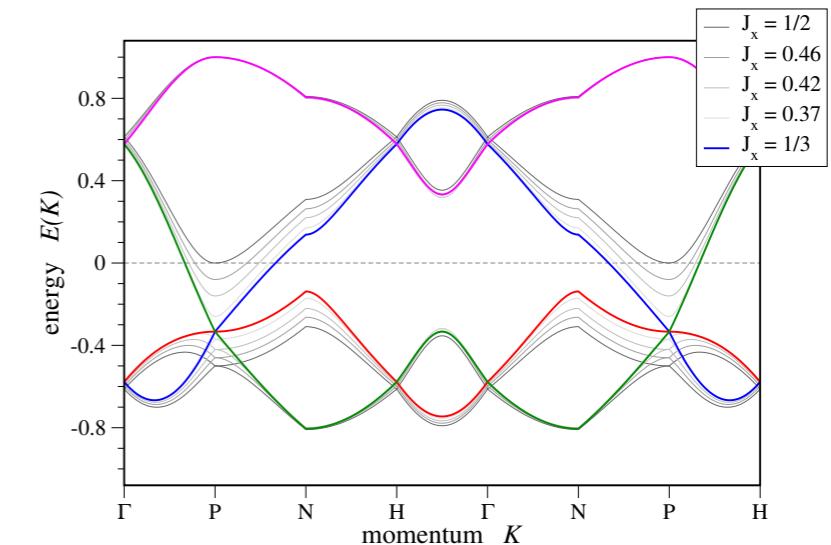
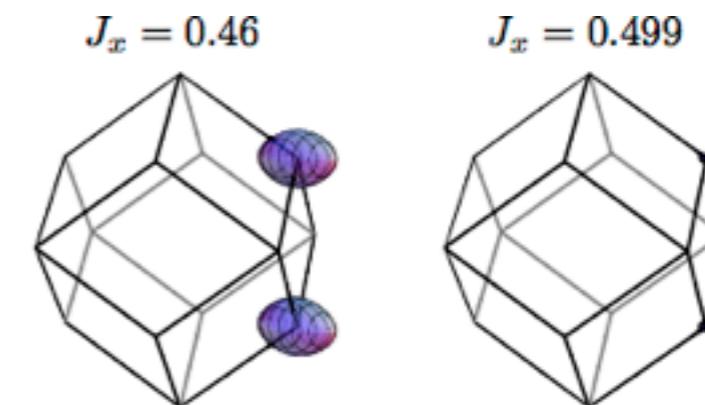
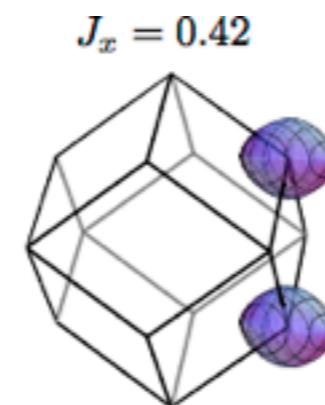
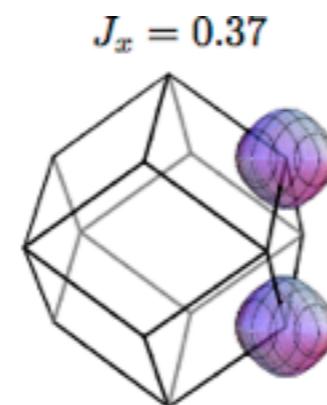
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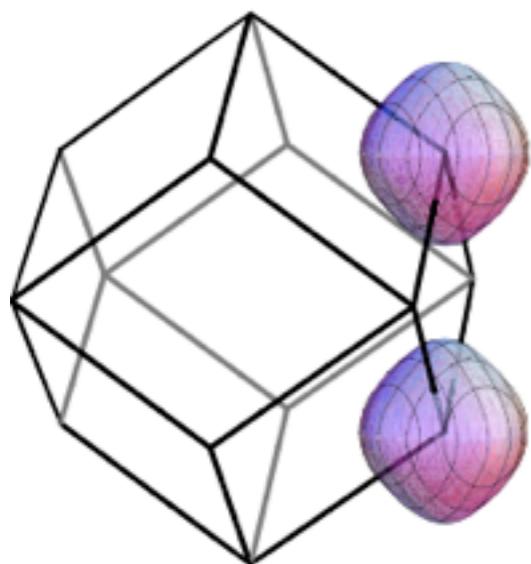
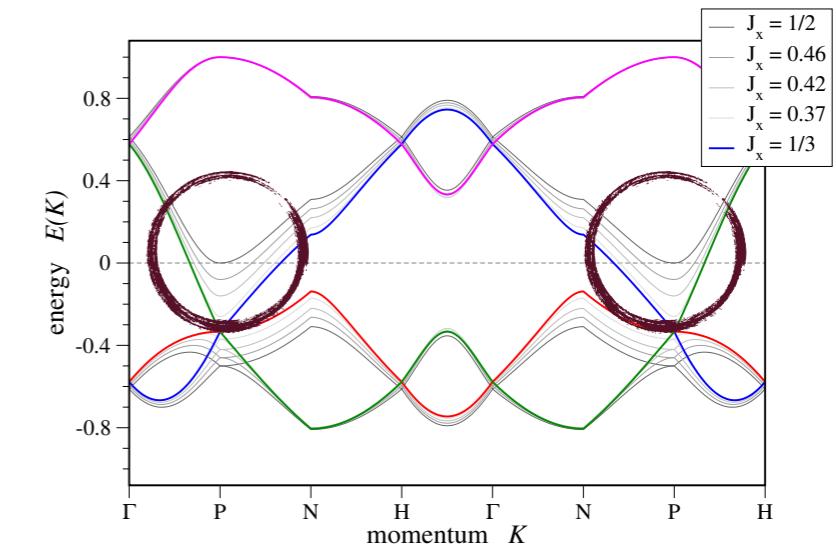
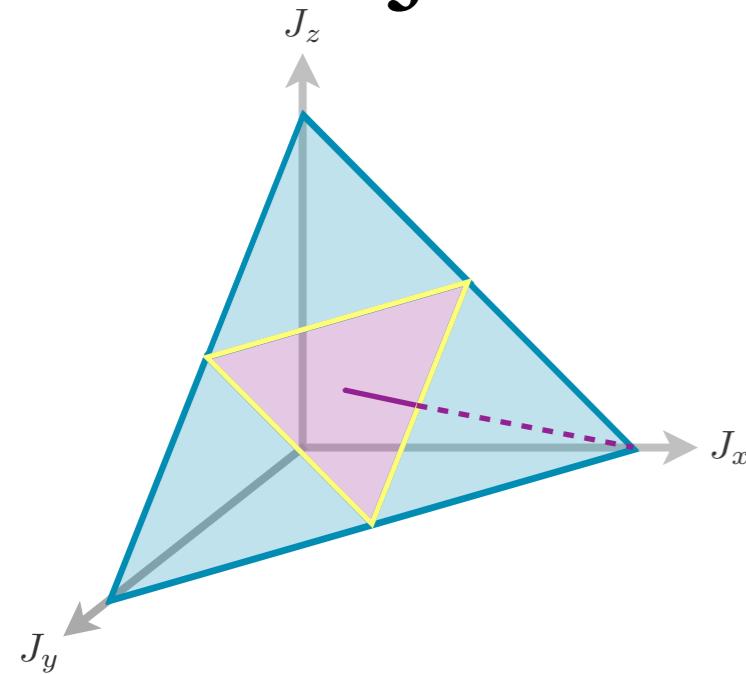
# Majorana Fermi surface in gapless region



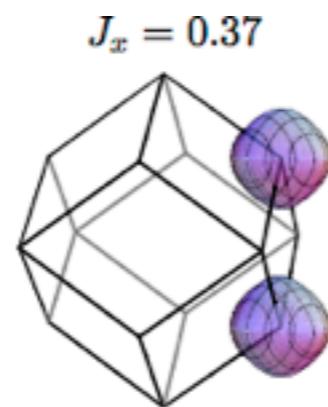
$J_x = 1/3$



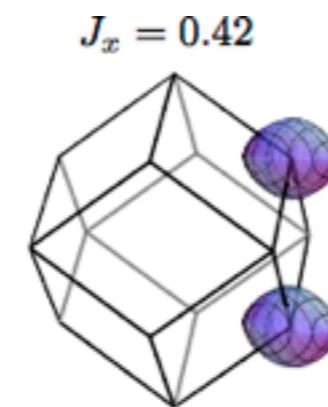
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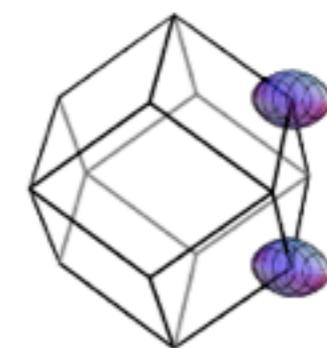
$J_x = 1/3$



$J_x = 0.37$



$J_x = 0.42$

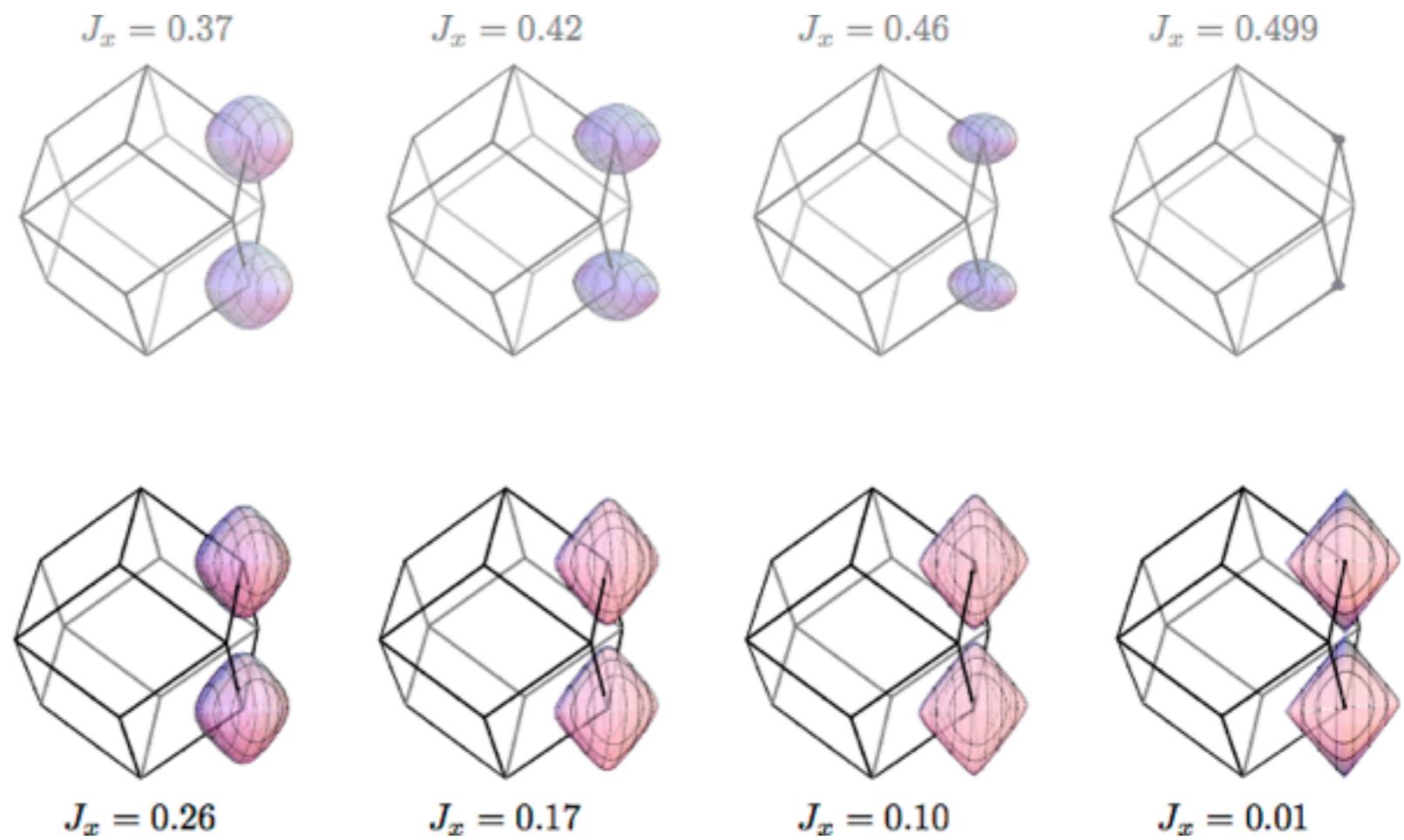
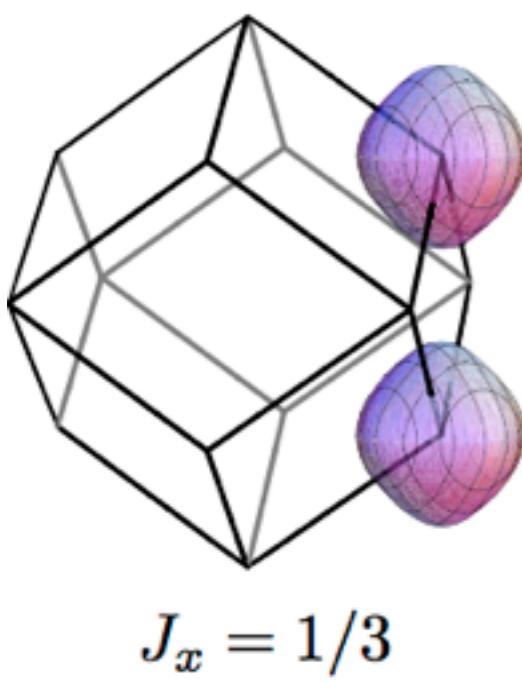
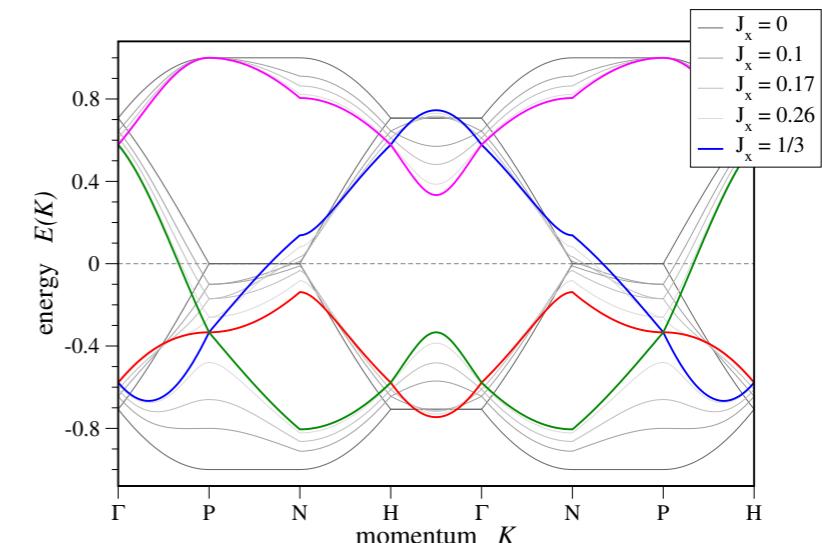
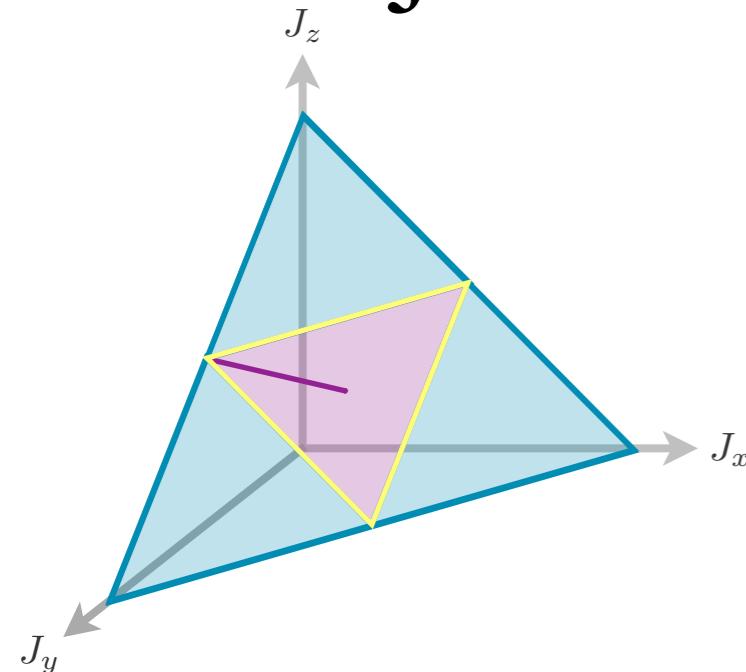


$J_x = 0.46$

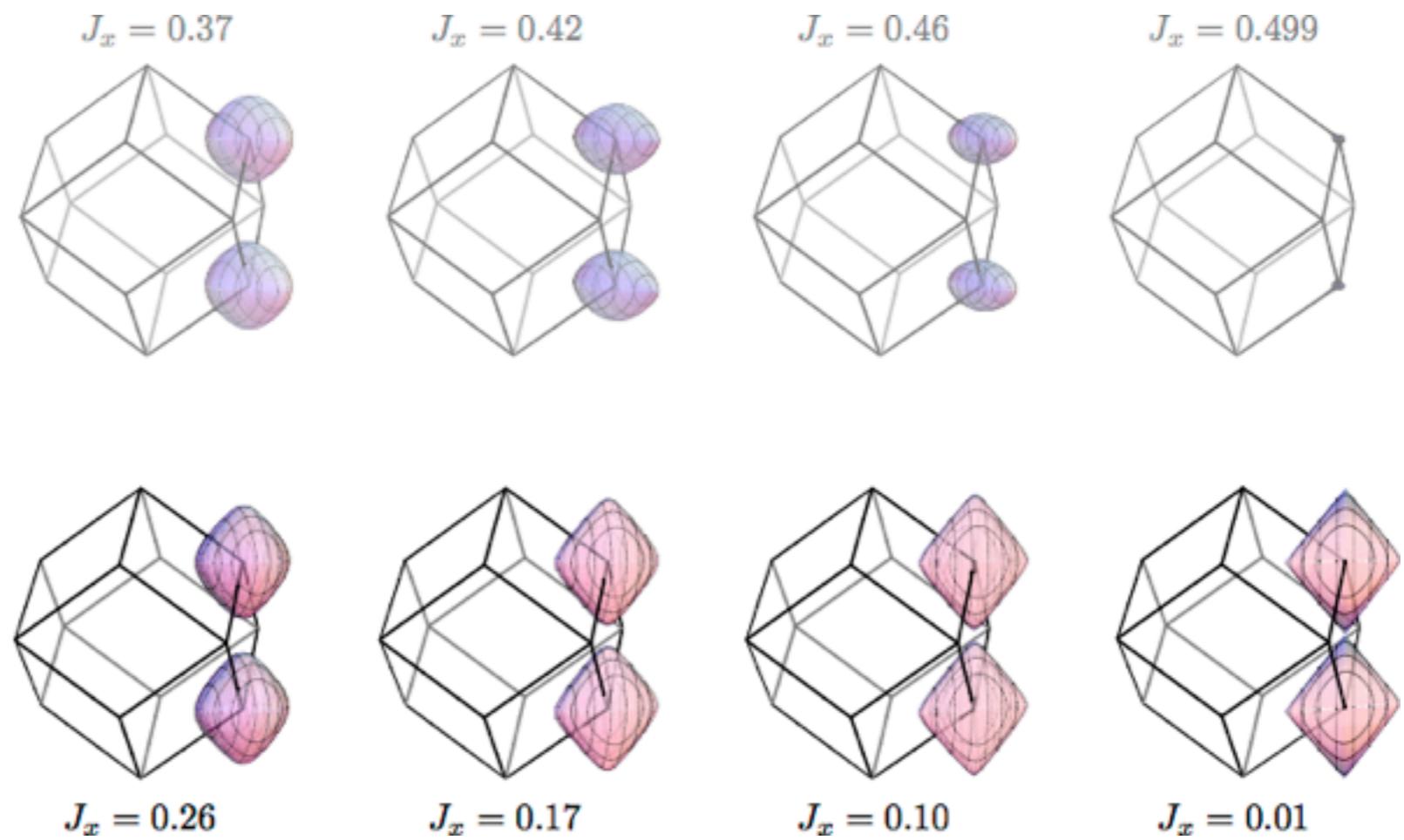
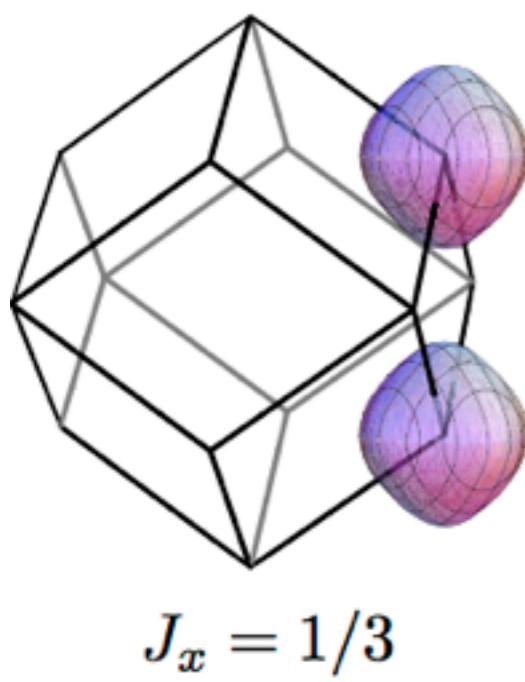
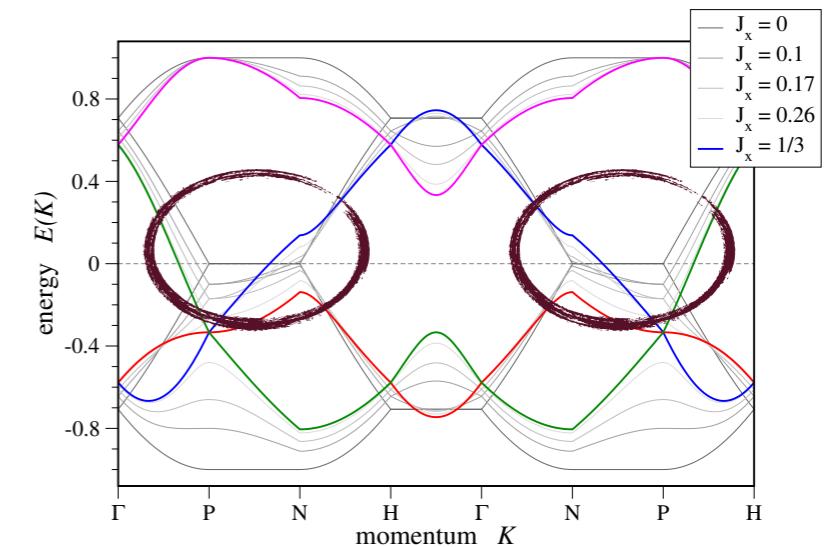
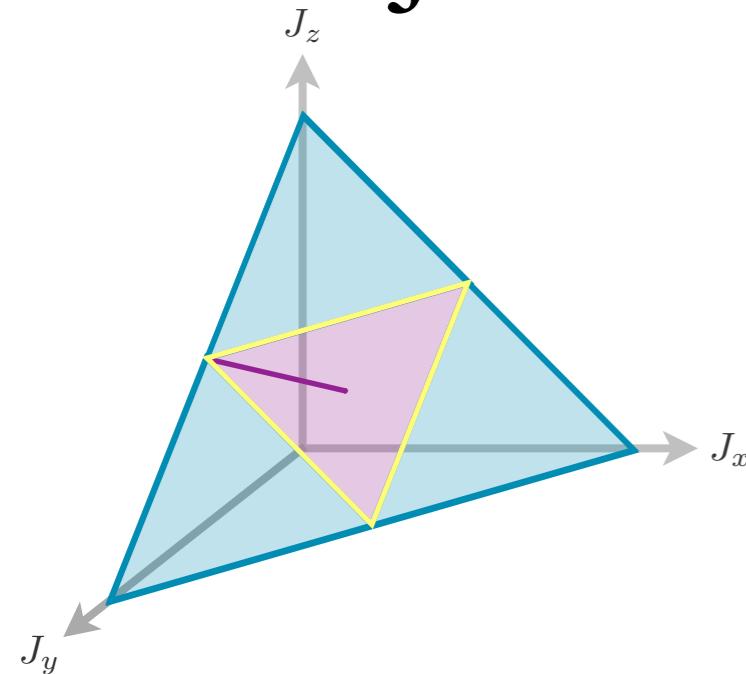


$J_x = 0.499$

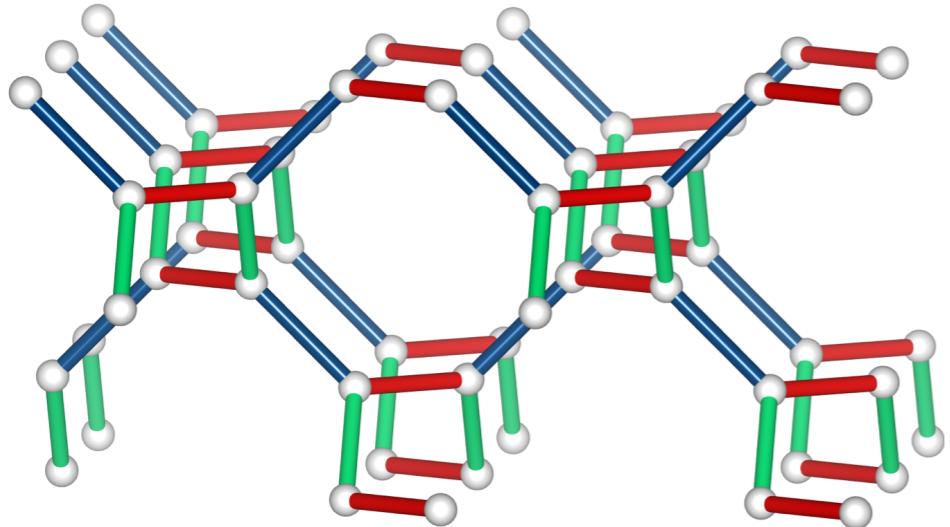
# Majorana Fermi surface in gapless region



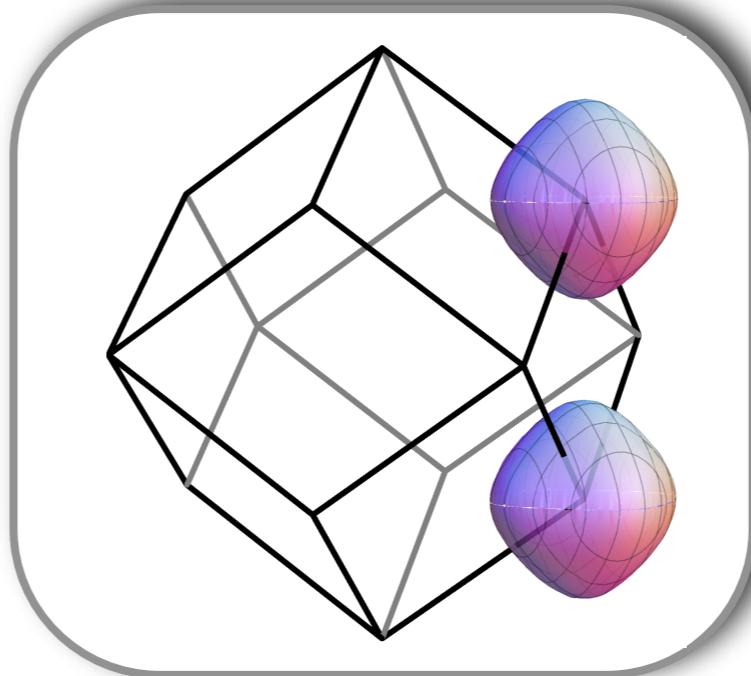
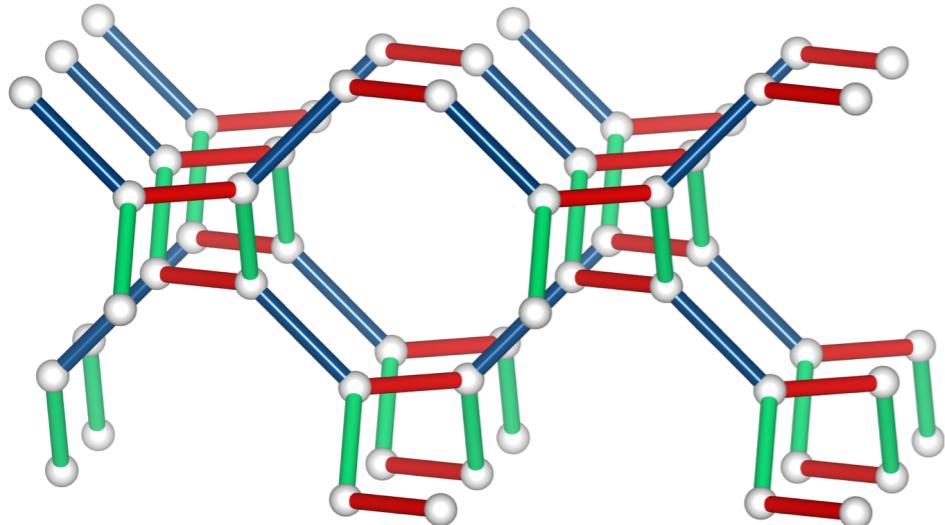
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# Zoo of gapless spin liquids

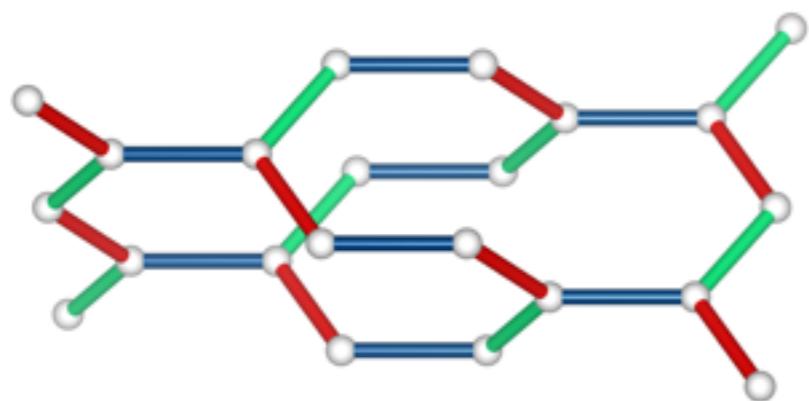
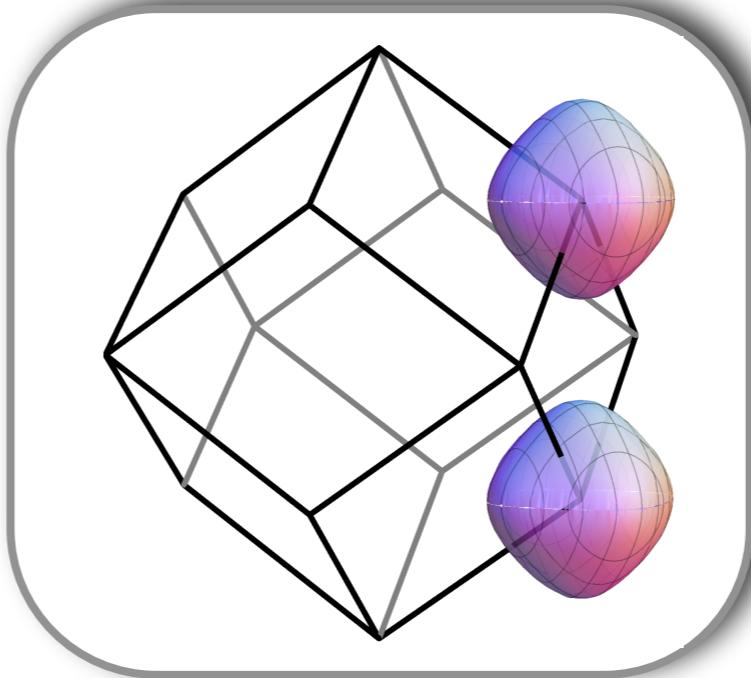
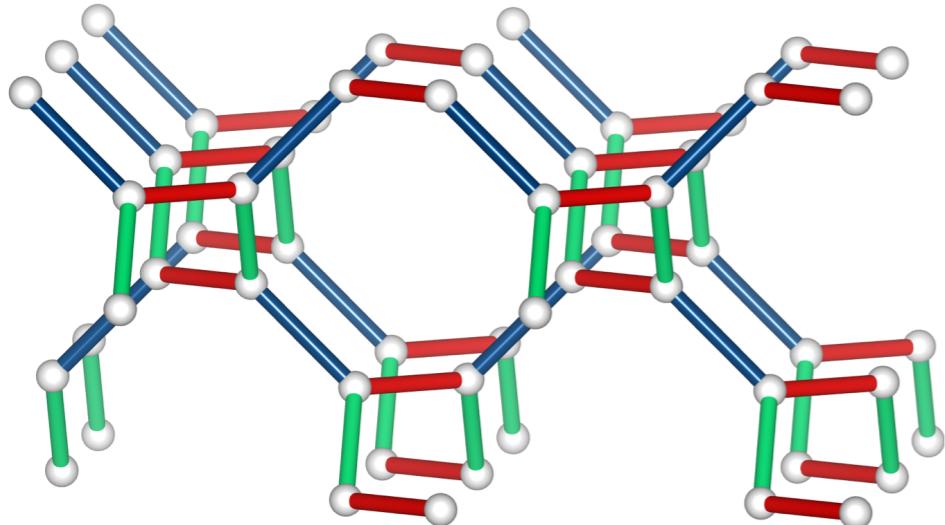


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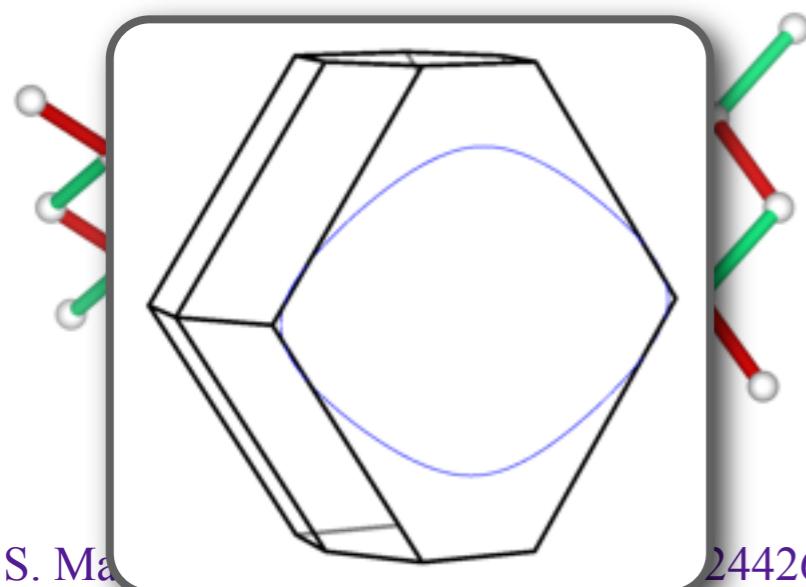
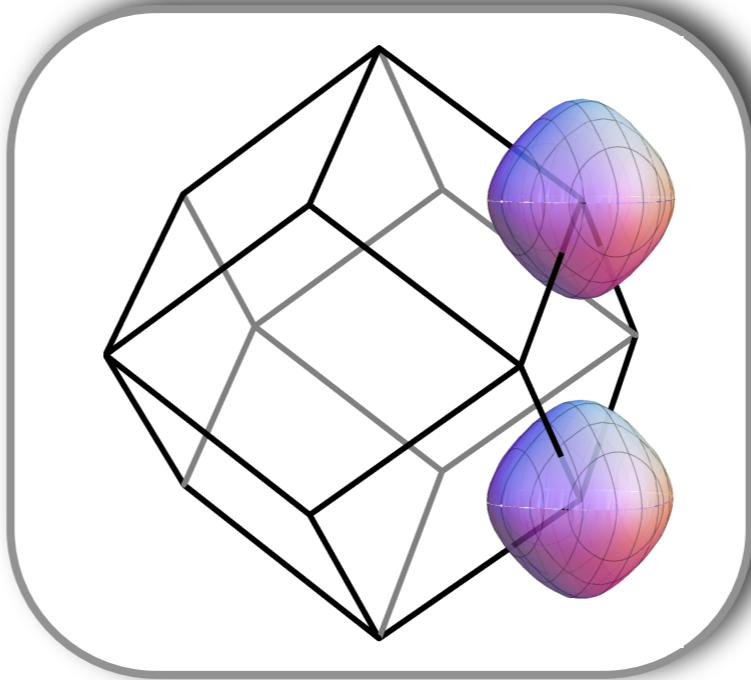
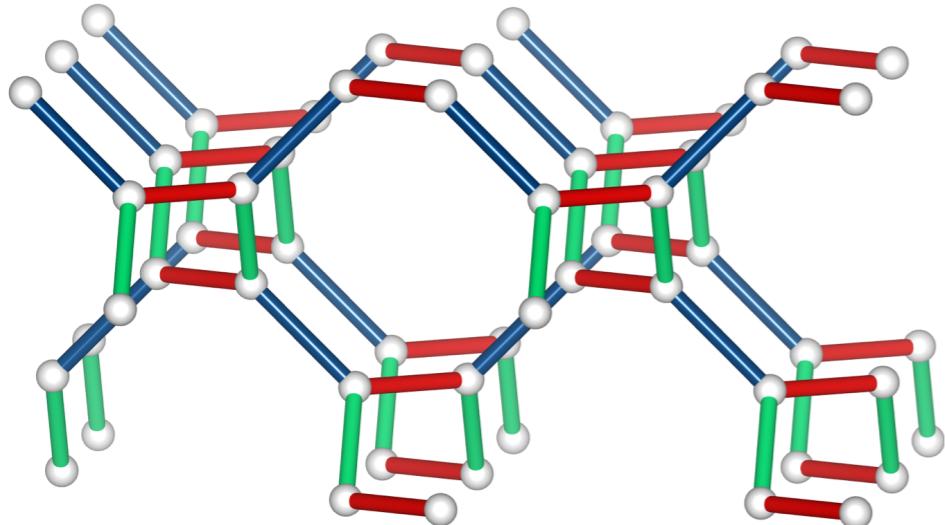


S. Mandal, N. Surendran, PRB 79, 024426 (2009)

E. K.-H. Lee et al., PRB 89, 045117 (2014)

I. Kimchi, J.G. Analytis, A. Vishwanath, arXiv:1309.1171

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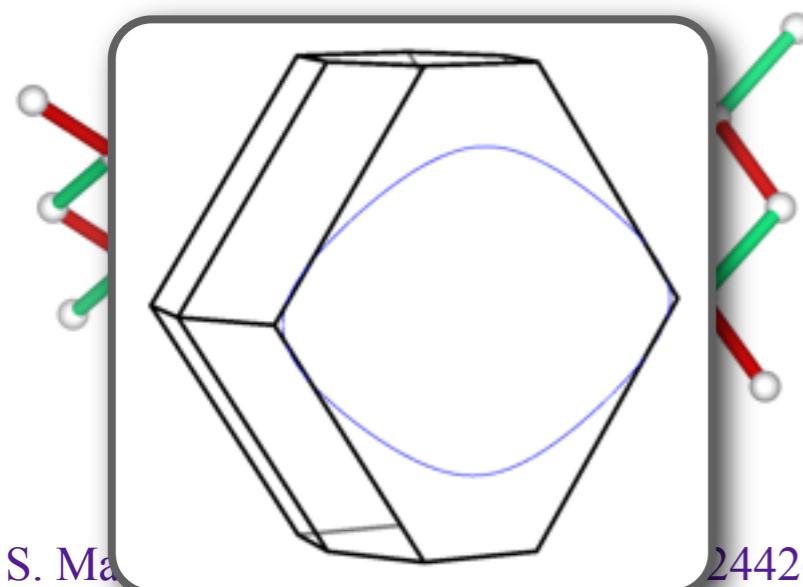
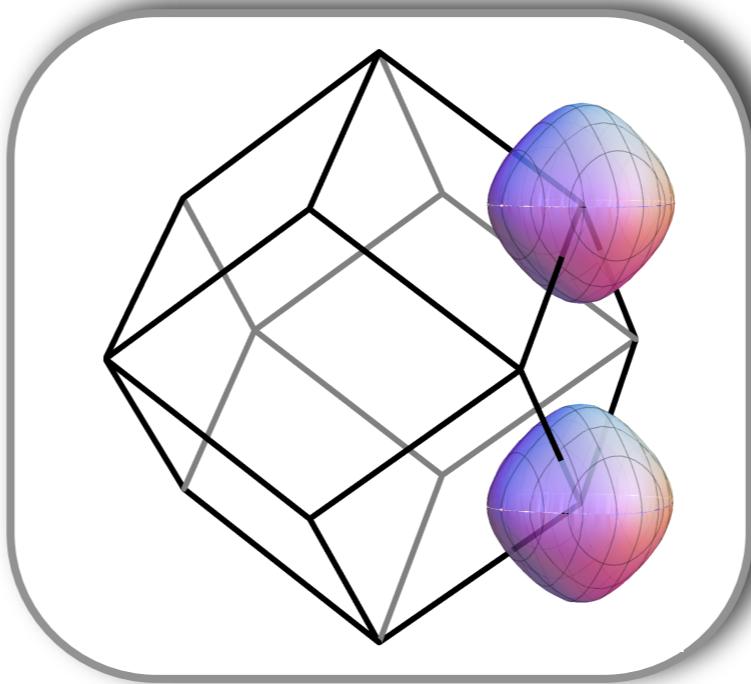
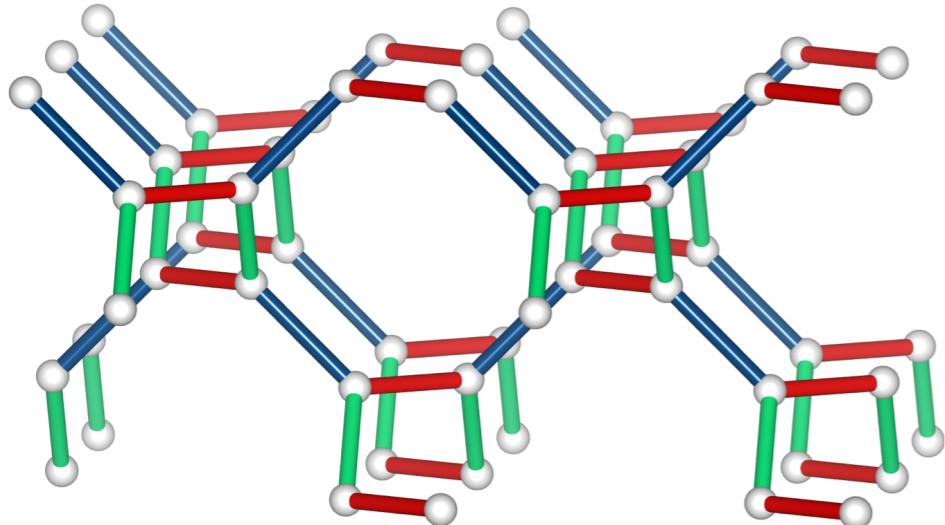
S. Ma

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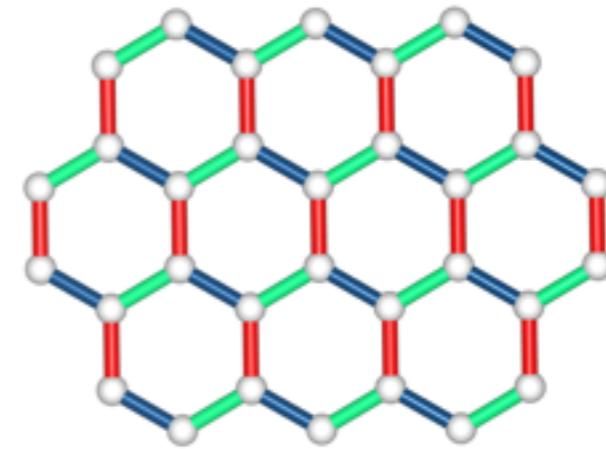
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S. Ma et al., arXiv:0905.24426 (2009)

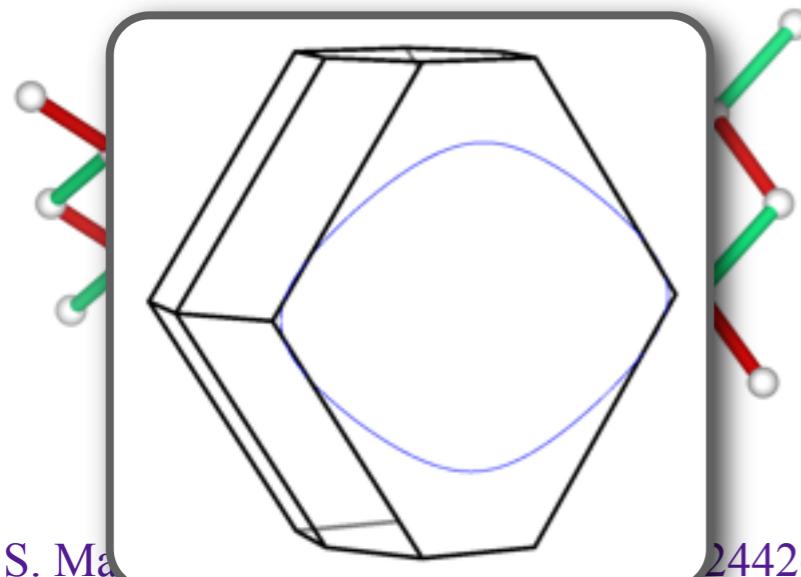
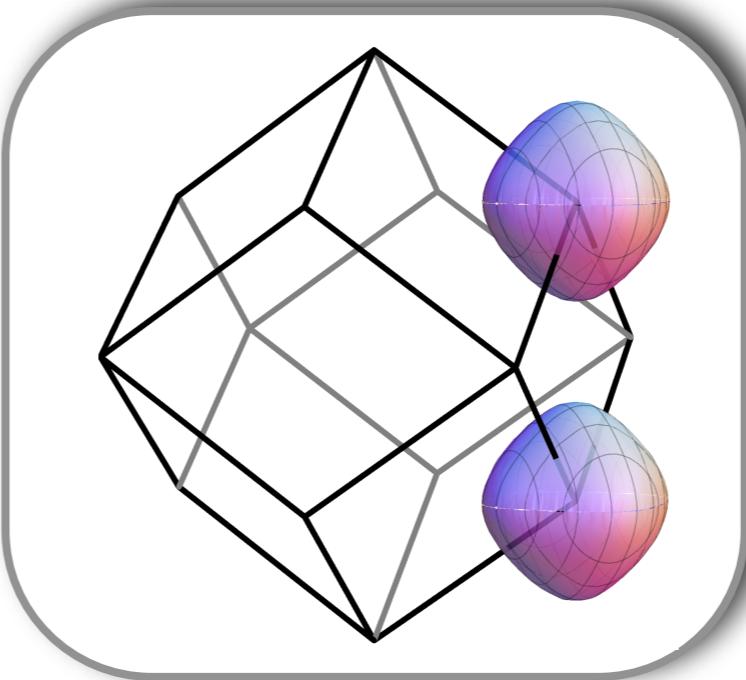
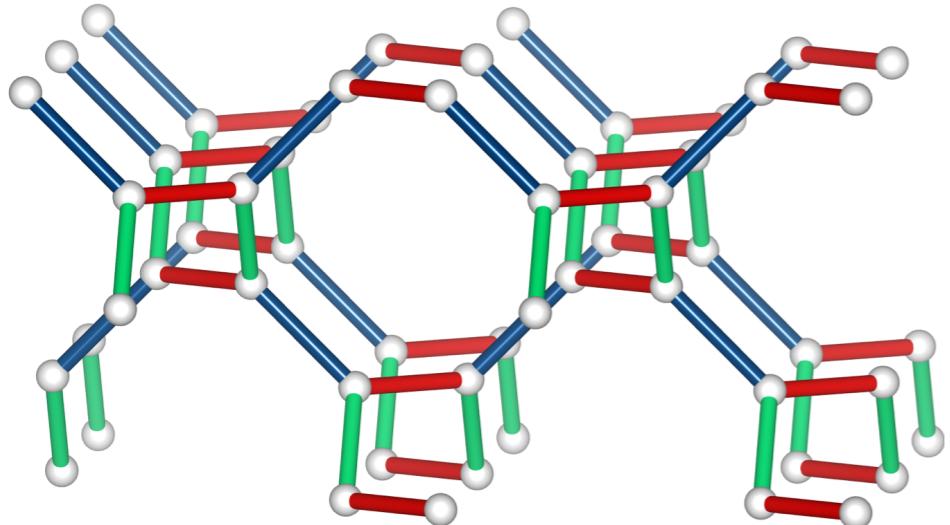
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A. Kitaev, Annals of Physics 321, 2 (2006)

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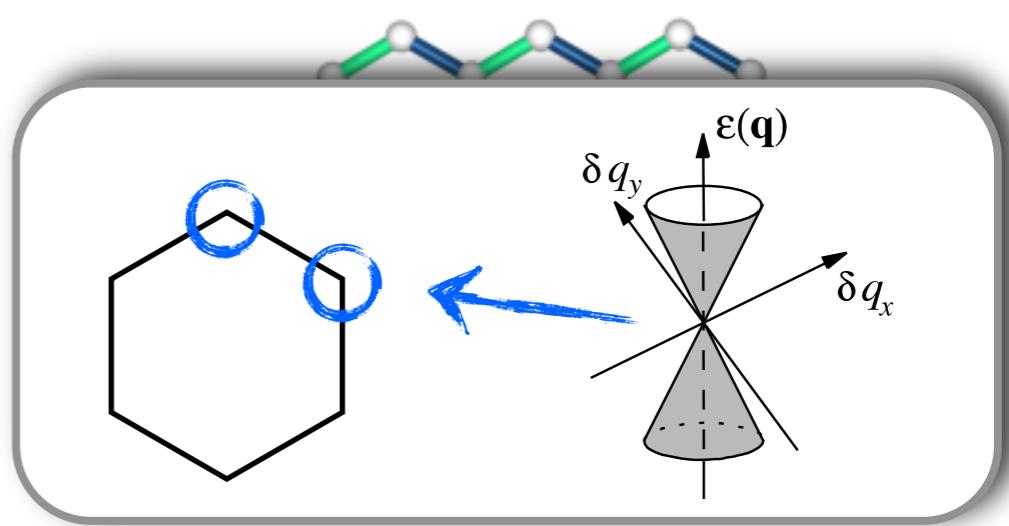


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# Thermodynamics

static spin correlation functions  $\langle \sigma_i^\alpha(\vec{r})\sigma_j^\beta(0) \rangle$  decay exponentially

Bond-energy correlation functions  $\langle \mathcal{B}_\gamma(0)\mathcal{B}_\gamma(\vec{r}) \rangle - \langle \mathcal{B}_\gamma(0) \rangle \langle \mathcal{B}_\gamma(\vec{r}) \rangle$

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Different 3D gapless spin liquids can be **experimentally distinguished** by measuring the **specific heat coefficient**  $C(T)/T$

$Z_2$  spin liquid with spinon Fermi line  
(hyperhoneycomb)

$$C(T) \sim T^2$$

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$Z_2$  spin liquid with spinon Fermi surface  
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Majorana metal!

# Symmetries in Majorana Systems

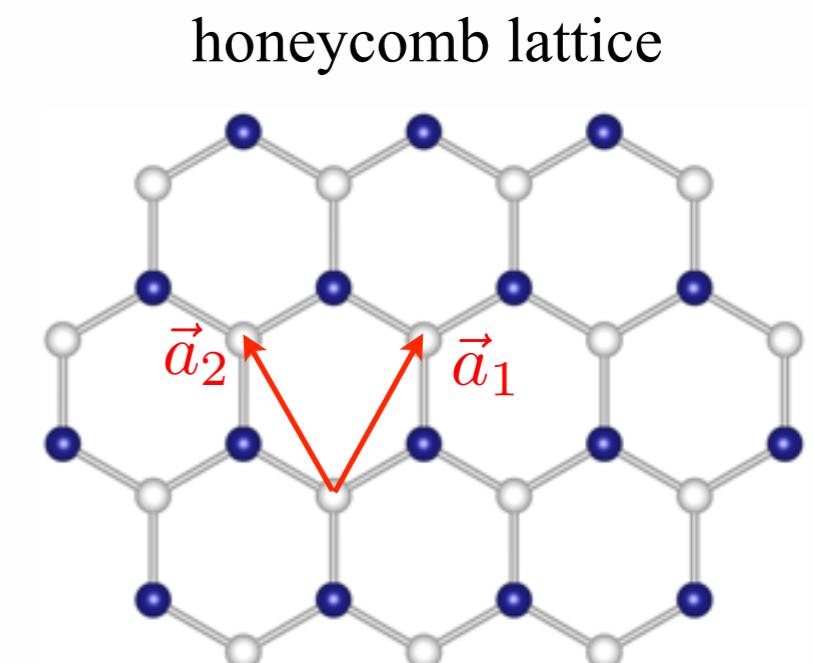
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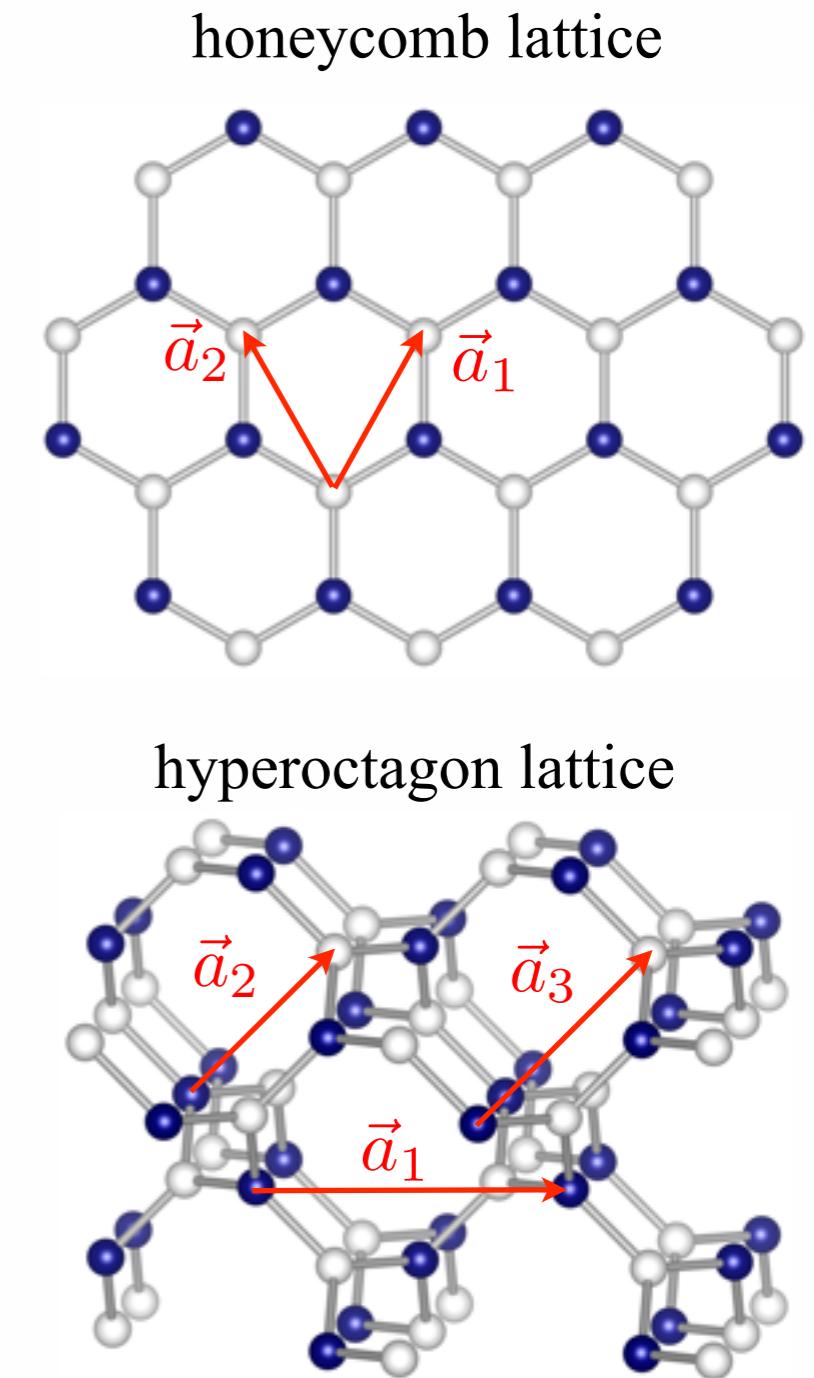
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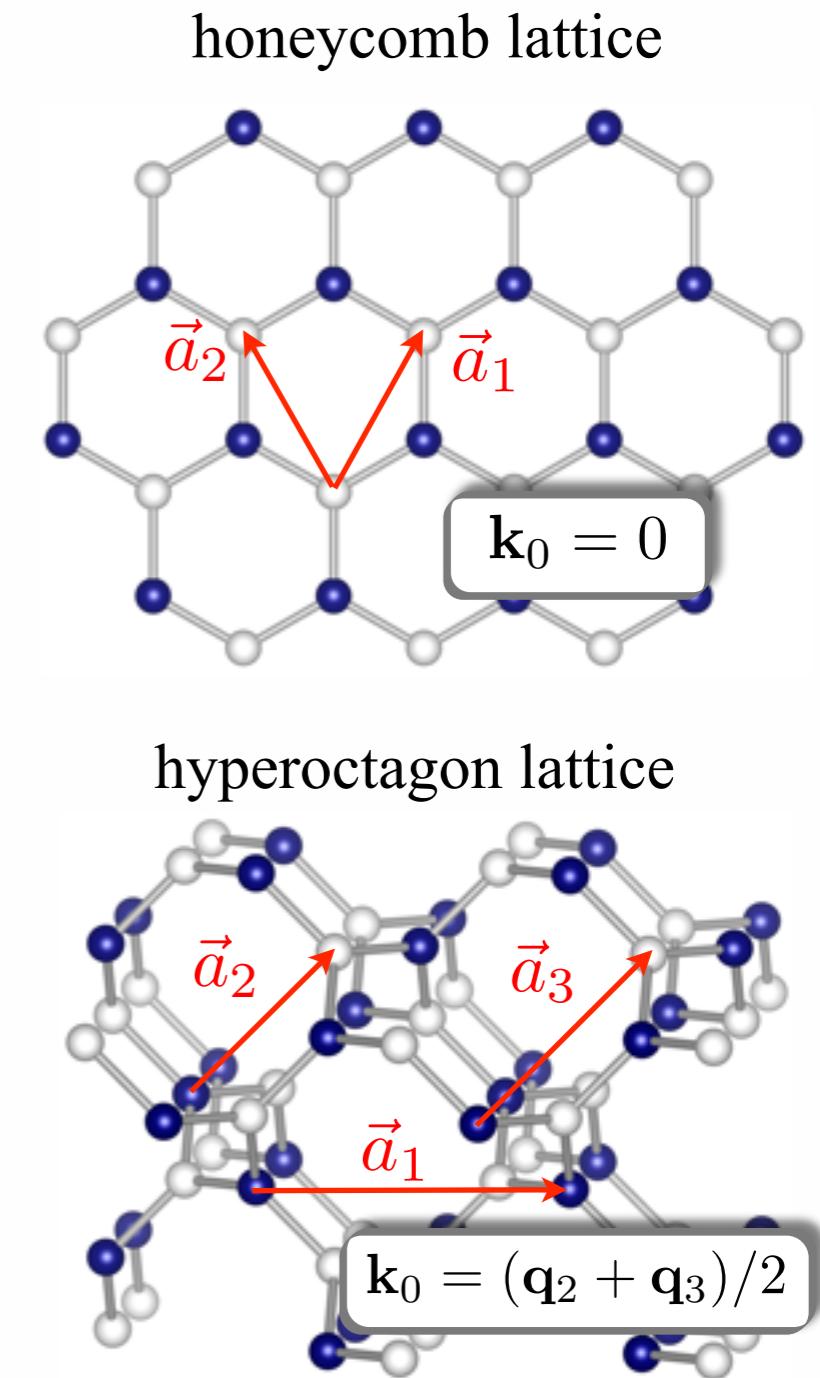
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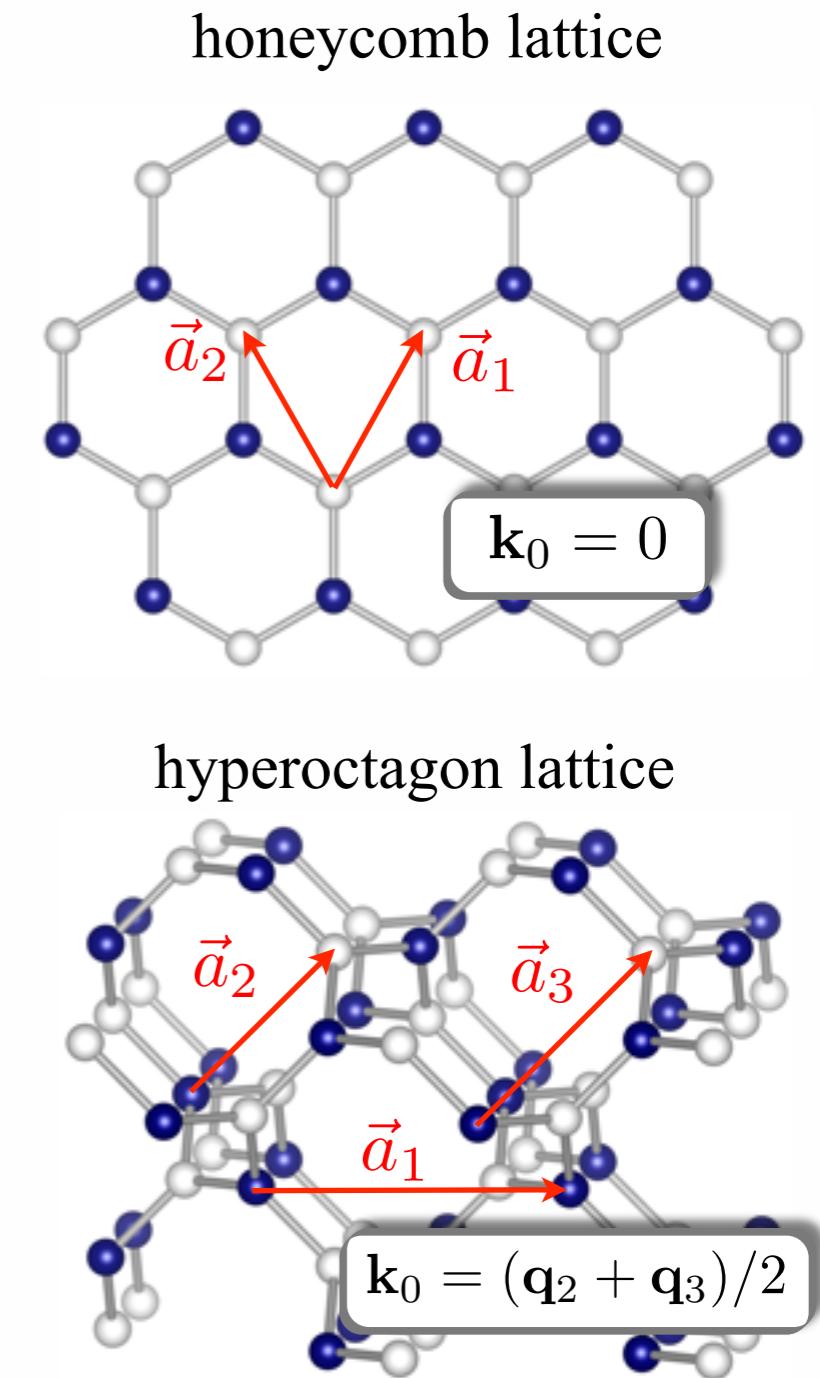
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Inversion symmetry	$\epsilon(\mathbf{k}) = \epsilon(-\mathbf{k} + \mathbf{k}_0)$

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# Symmetries $\leftrightarrow$ Fermi surface

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particle-hole symmetry at every  $\mathbf{k}$  point  
doubly degenerate zero-modes at same  $\mathbf{k}$

$$h(\mathbf{k}) = \begin{pmatrix} 0 & \mathbf{A} \\ \mathbf{A}^\dagger & 0 \end{pmatrix}$$

protected by time-reversal

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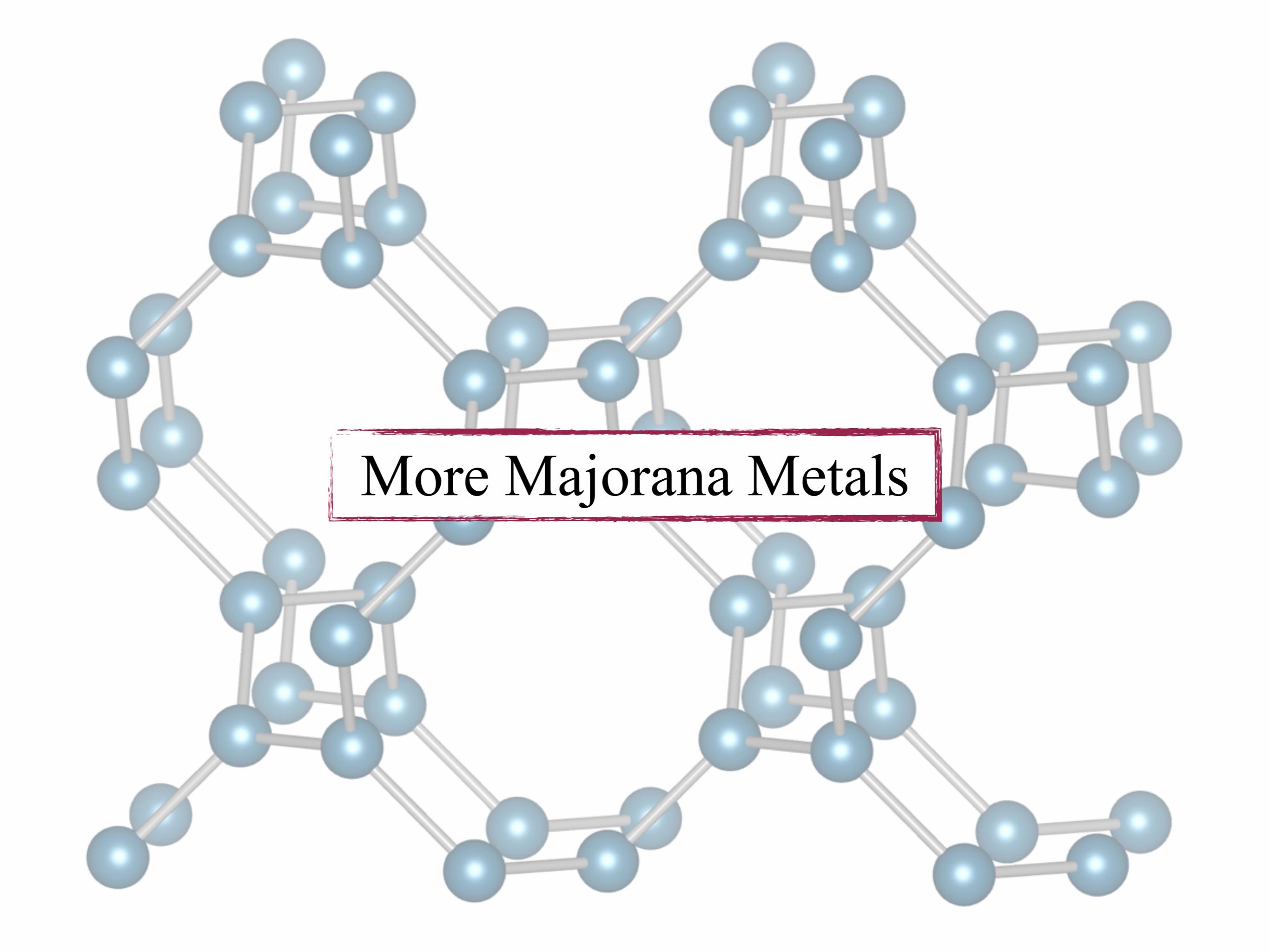
stable zero-mode manifolds are **separated points** (2D) and **lines** (3D)

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generic band hamiltonian at given  $\mathbf{k}$   
zero-modes are at different  $\mathbf{k}$

$$h(\mathbf{k}) = \begin{pmatrix} 0 & & \mathbf{A} \\ & \ddots & \\ \mathbf{A}^\dagger & & 0 \end{pmatrix}$$

stable zero-mode manifolds are **lines** (2D) and **surfaces** (3D)



More Majorana Metals

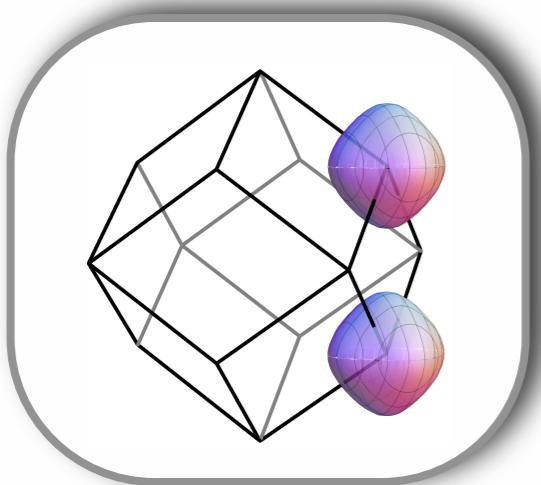
# P-wave pairing instability on the hyperoctagon lattice



Center of Majorana Fermi surfaces are  
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Time-reversal symmetry:  $\epsilon(\mathbf{q} + \mathbf{K}_j) = \epsilon(-\mathbf{q} + \mathbf{K}_j)$



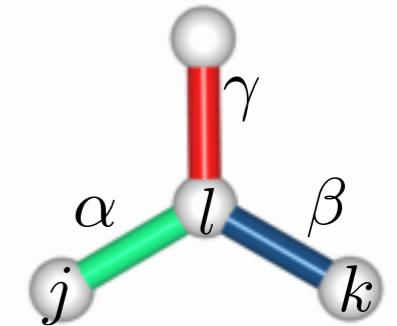
**p-wave** pairing instability in effective spinless Fermion model  
(broken translation symmetry)

relevant for additional nearest-neighbor Heisenberg coupling

# Breaking time-reversal symmetry

external **magnetic field** in (1,1,1)-direction

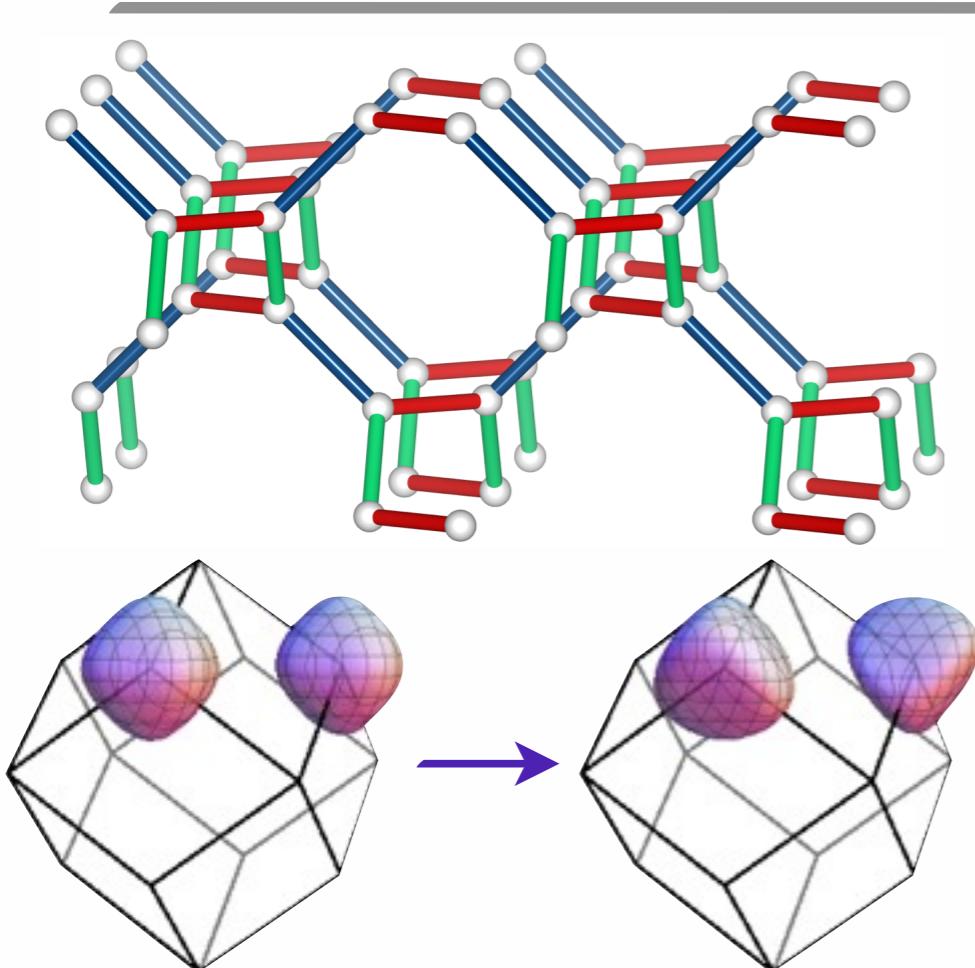
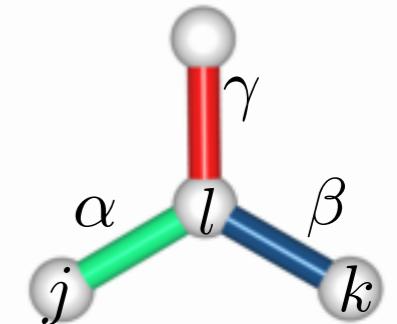
$$H_{eff} = -J \sum_{\gamma-bond} \sigma_j^\gamma \sigma_k^\gamma - \kappa \sum_{\langle j,l,k \rangle} \sigma_j^\alpha \sigma_k^\beta \sigma_l^\gamma$$



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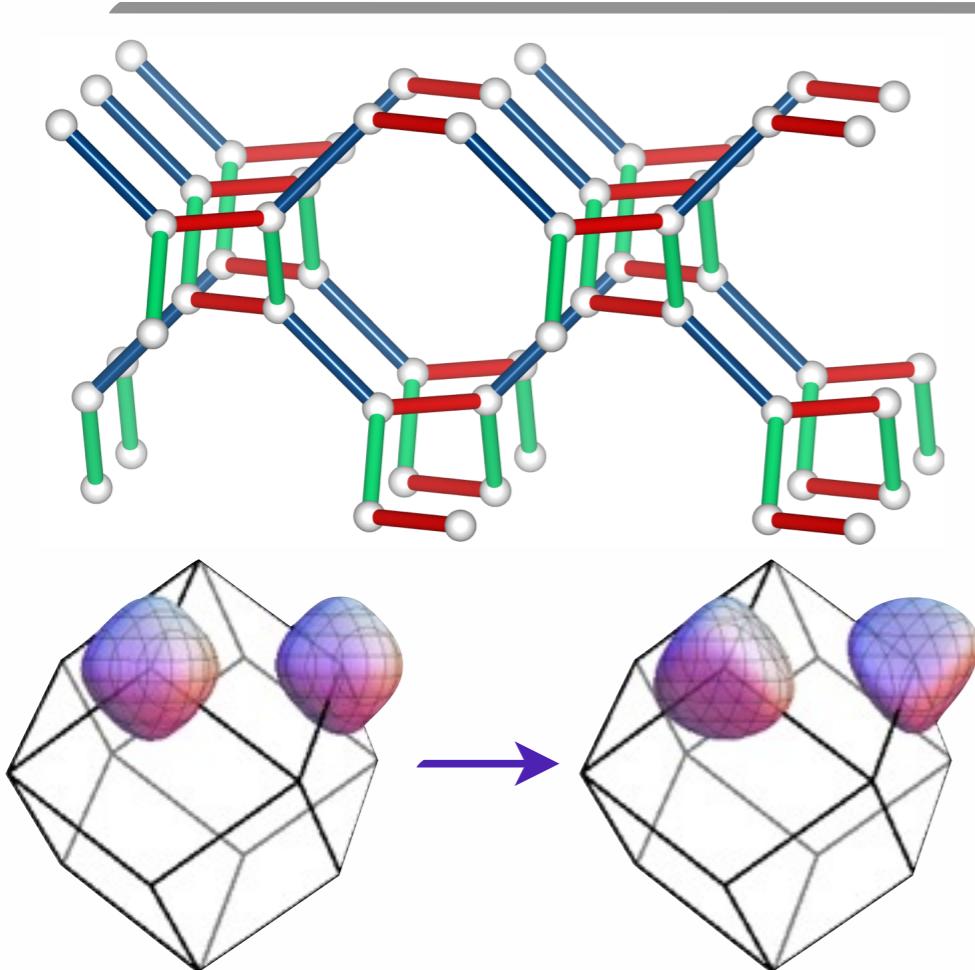
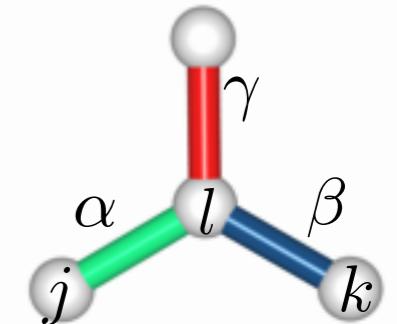
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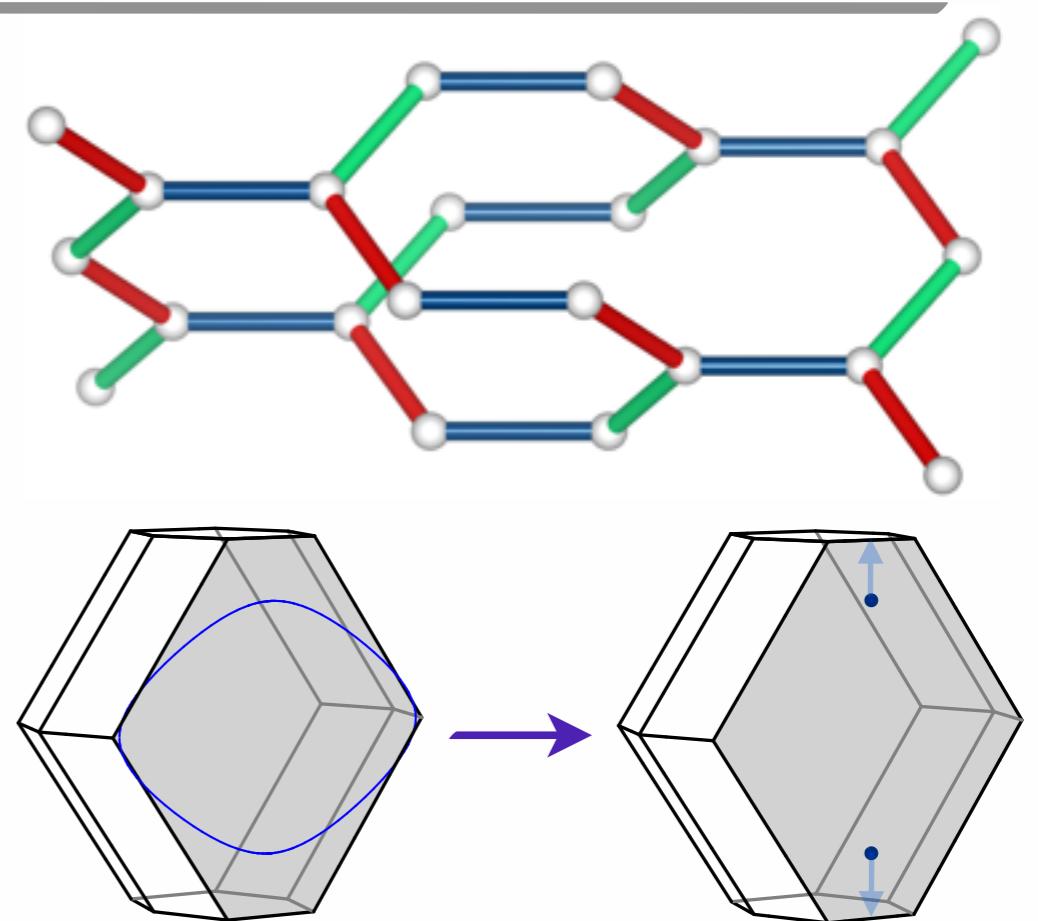
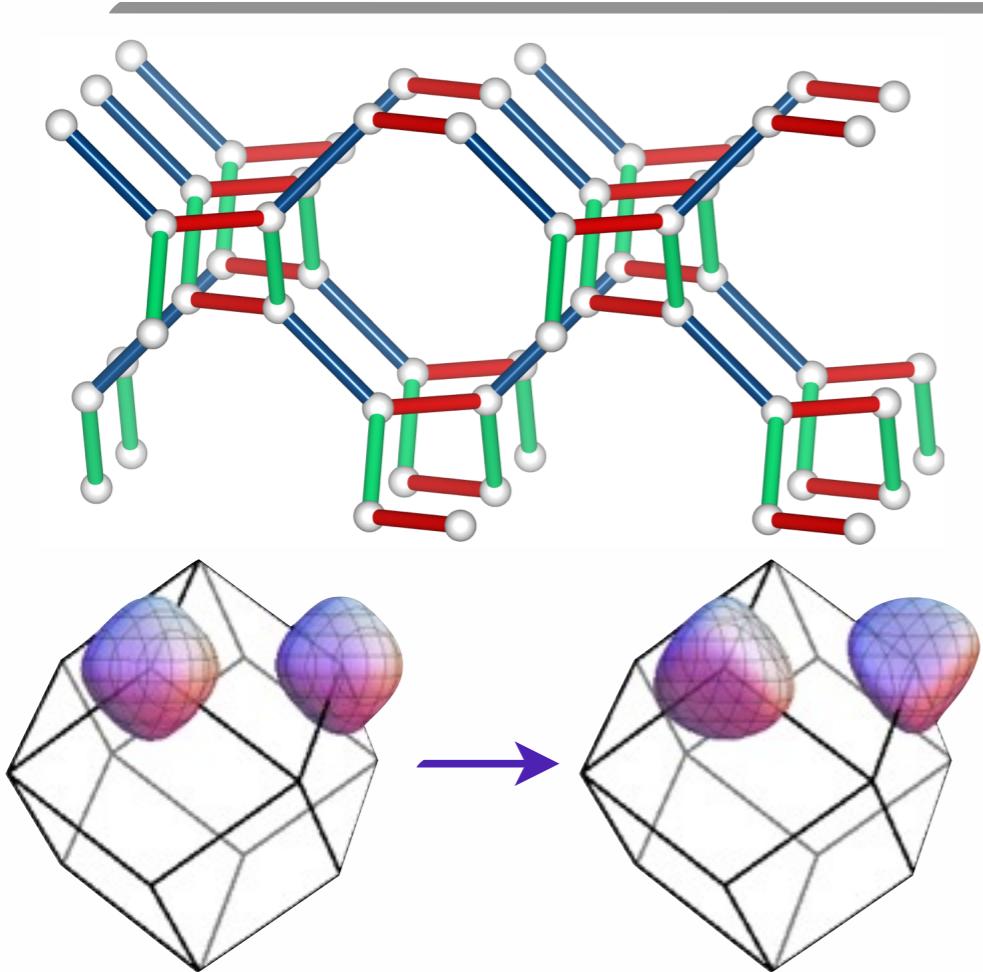
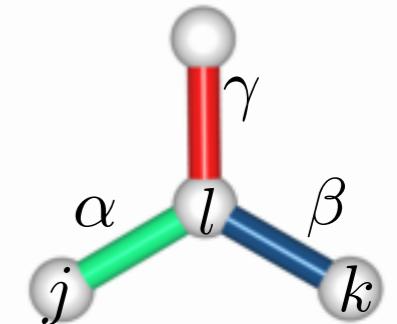


Breaking TR stabilizes Fermi surface

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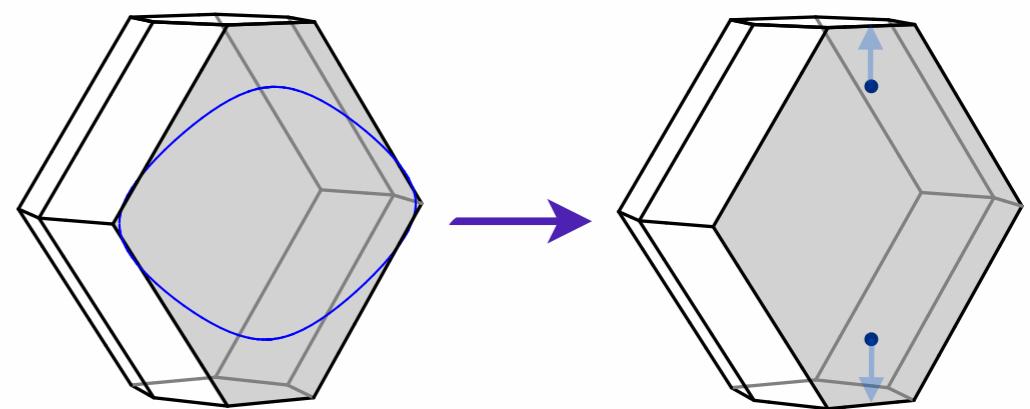
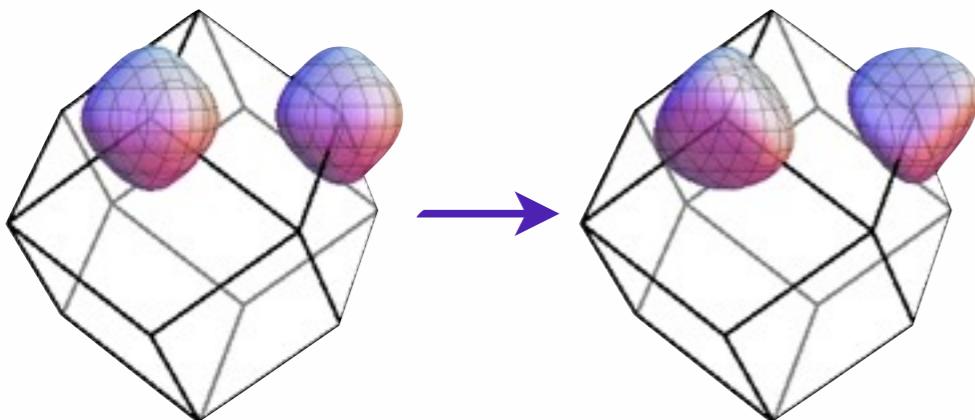
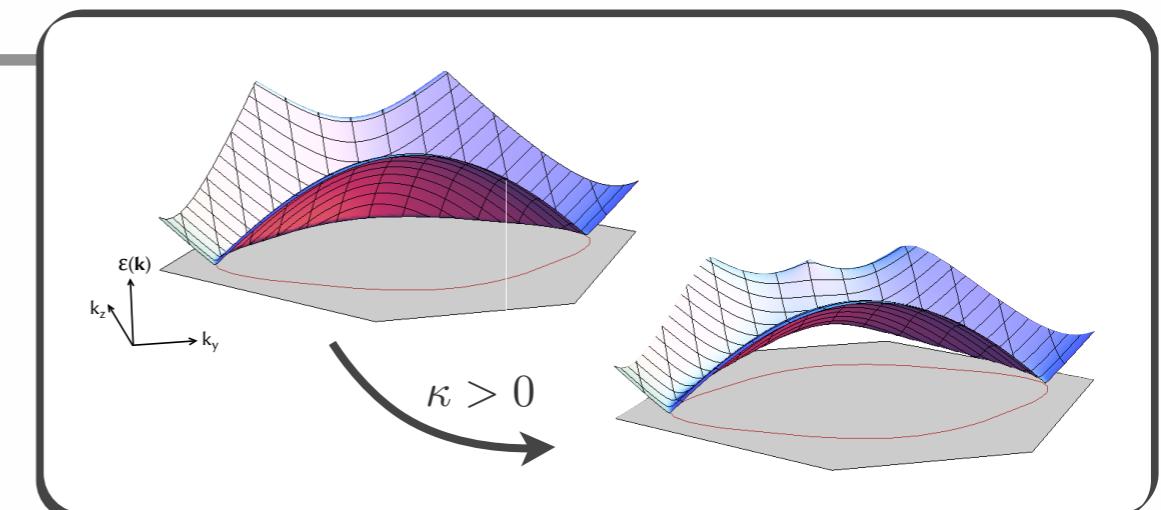
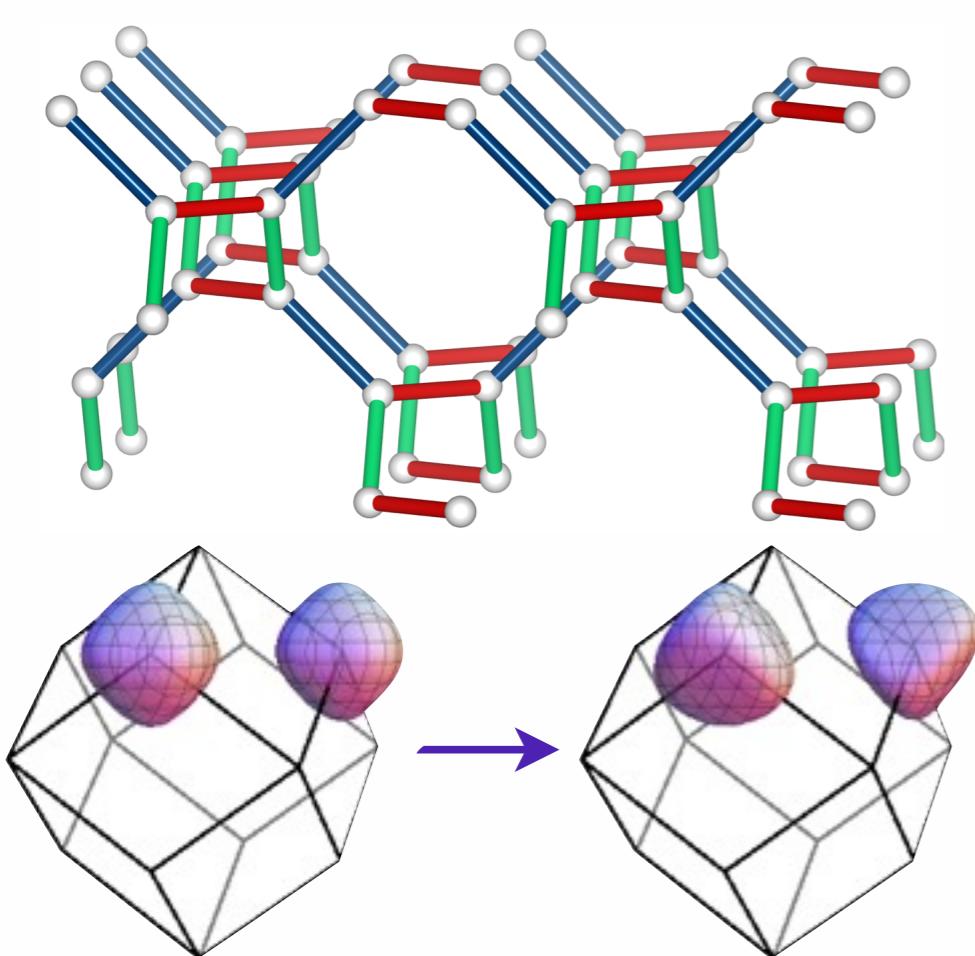
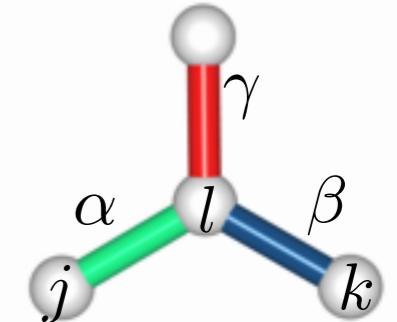


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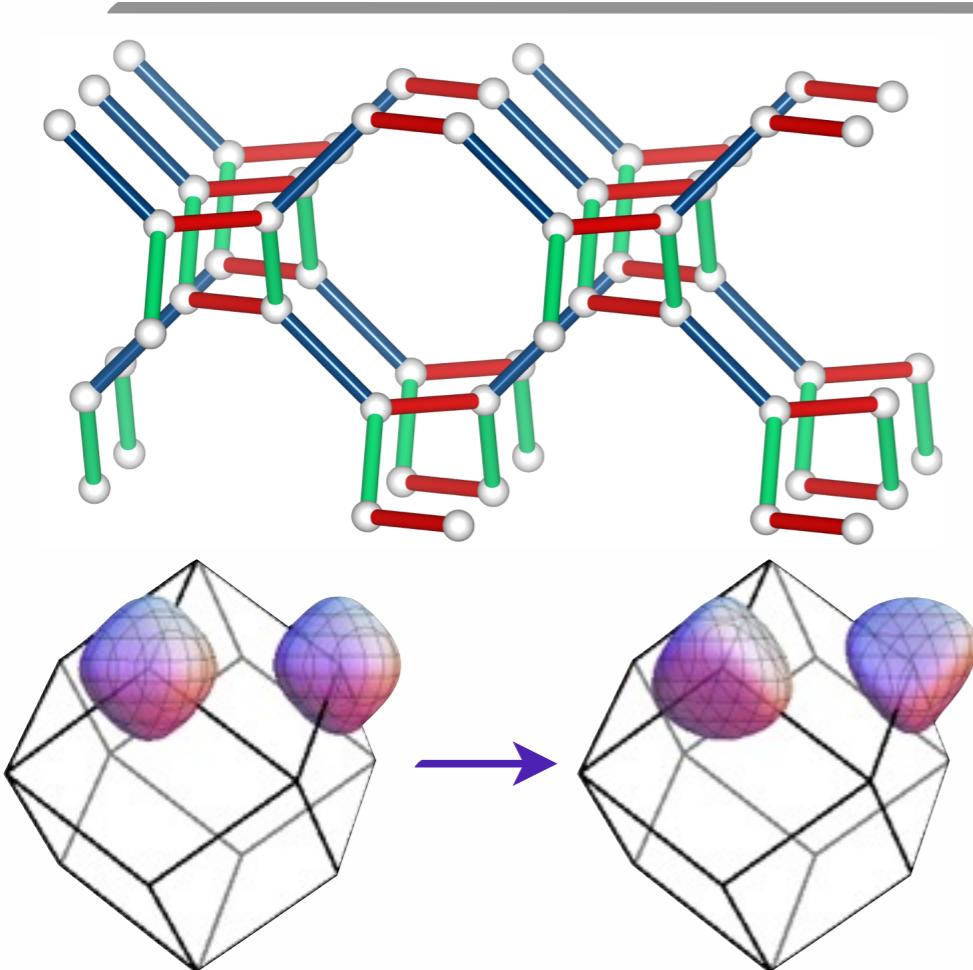
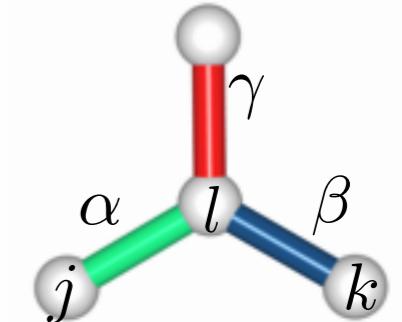


Breaking TR stabilizes Fermi surface

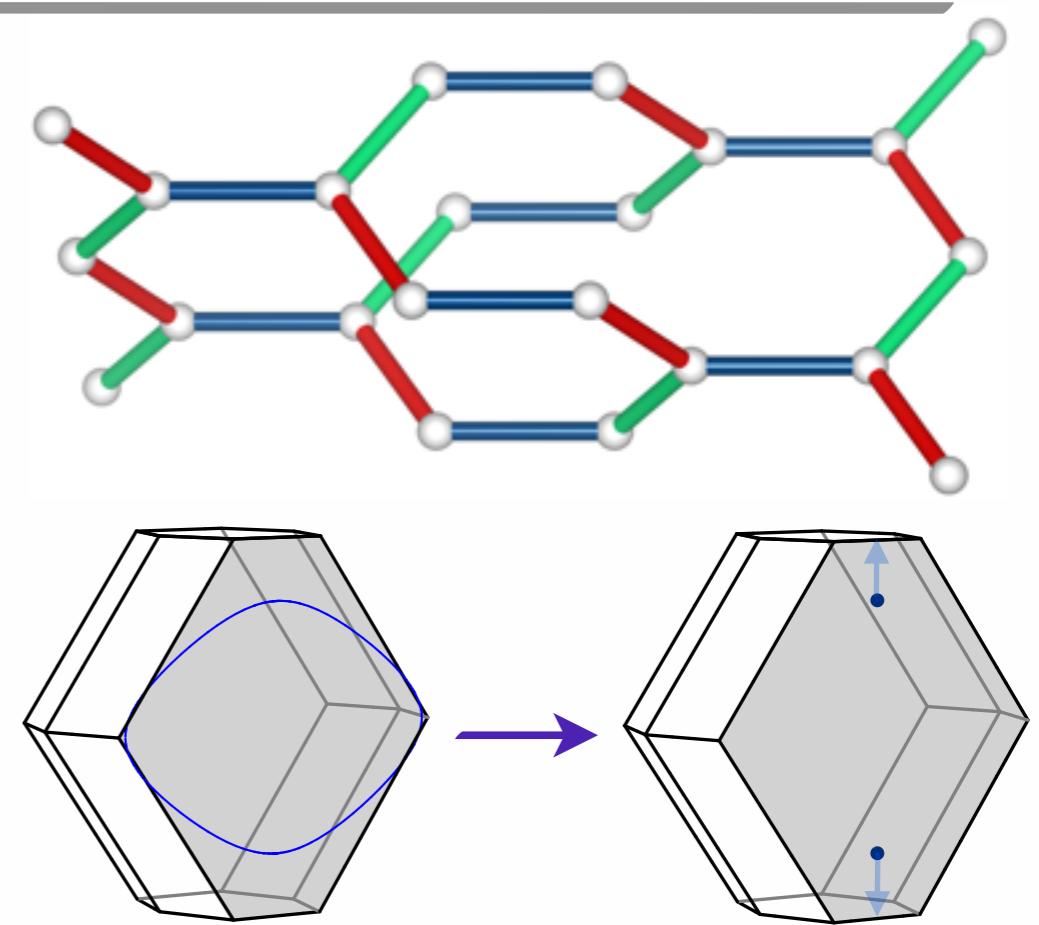
# Breaking time-reversal symmetry

external **magnetic field** in (1,1,1)-direction

$$H_{eff} = -J \sum_{\gamma-bond} \sigma_j^\gamma \sigma_k^\gamma - \kappa \sum_{\langle j,l,k \rangle} \sigma_j^\alpha \sigma_k^\beta \sigma_l^\gamma$$



Breaking TR stabilizes Fermi surface



Breaking TR reduces line to **pair of Weyl nodes**

# Weyl nodes and Weyl semi-metals

Touching of two bands in 3D is generically **linear**

$$\hat{H} = \vec{v}_0 \cdot \vec{q} \mathbb{1} + \sum_{i=1}^3 \vec{v}_j \cdot \vec{q} \sigma_j \quad \text{Weyl nodes}$$

# Weyl nodes and Weyl semi-metals

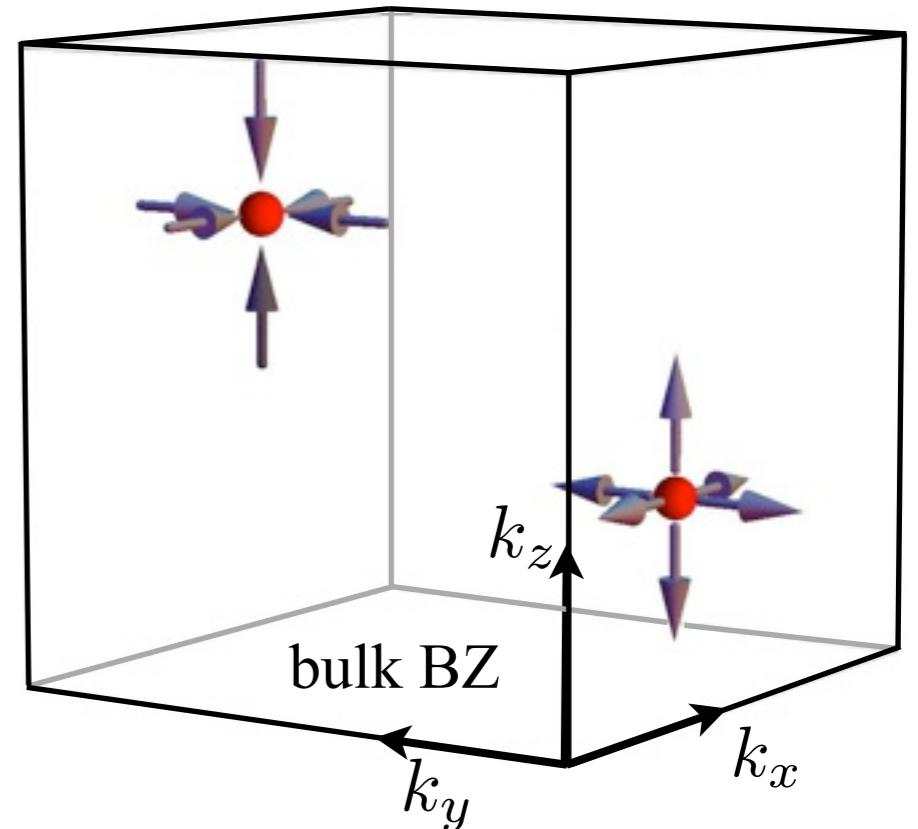
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Weyl nodes are **sources/sinks of Berry flux**

$$\vec{B}_n(\vec{k}) = \nabla_{\vec{k}} \times \left( i \langle n(\vec{k}) | \nabla_{\vec{k}} | n(\vec{k}) \rangle \right)$$

with chirality  $\text{sign}[\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)]$



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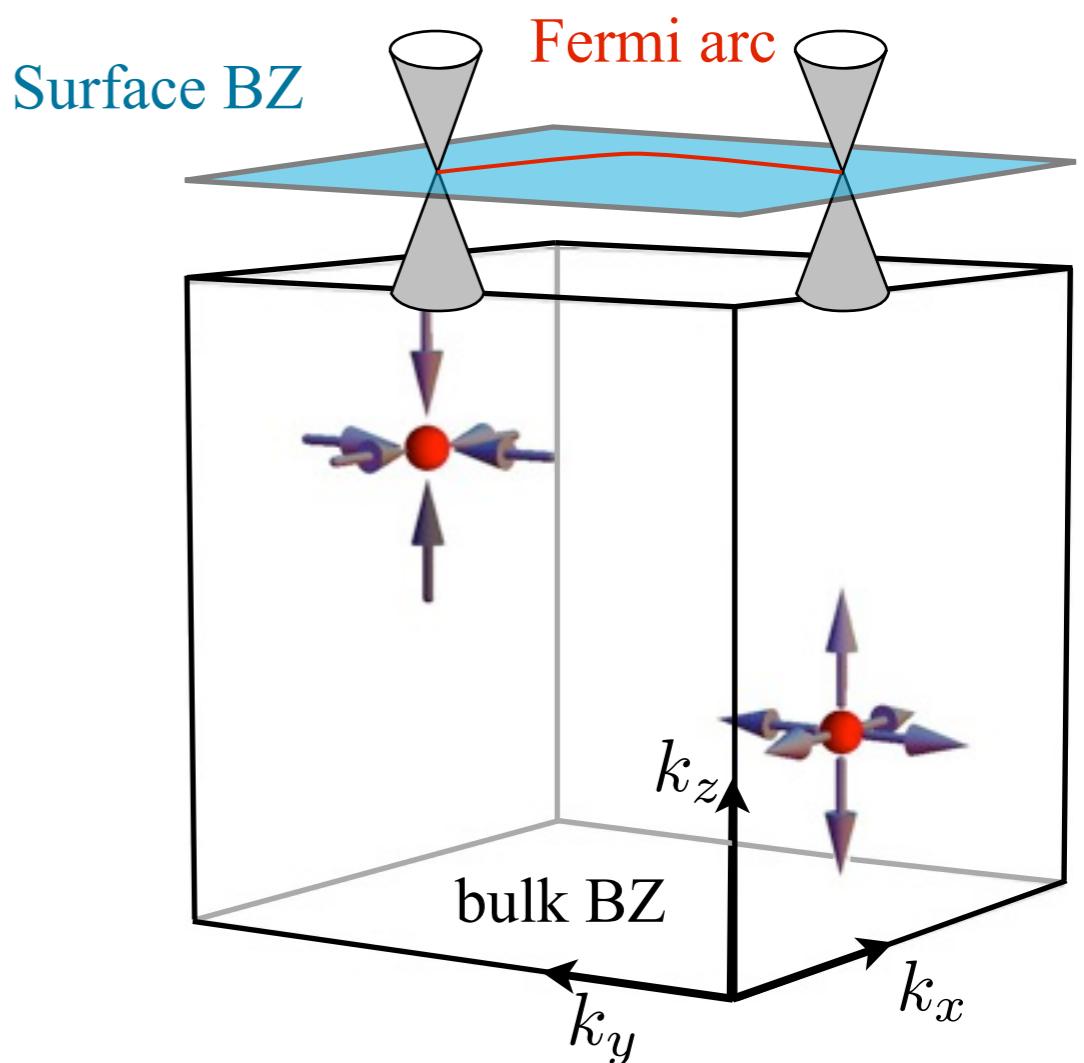
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here: Weyl nodes pinned at zero energy!



unusual surface states: **Fermi arcs**

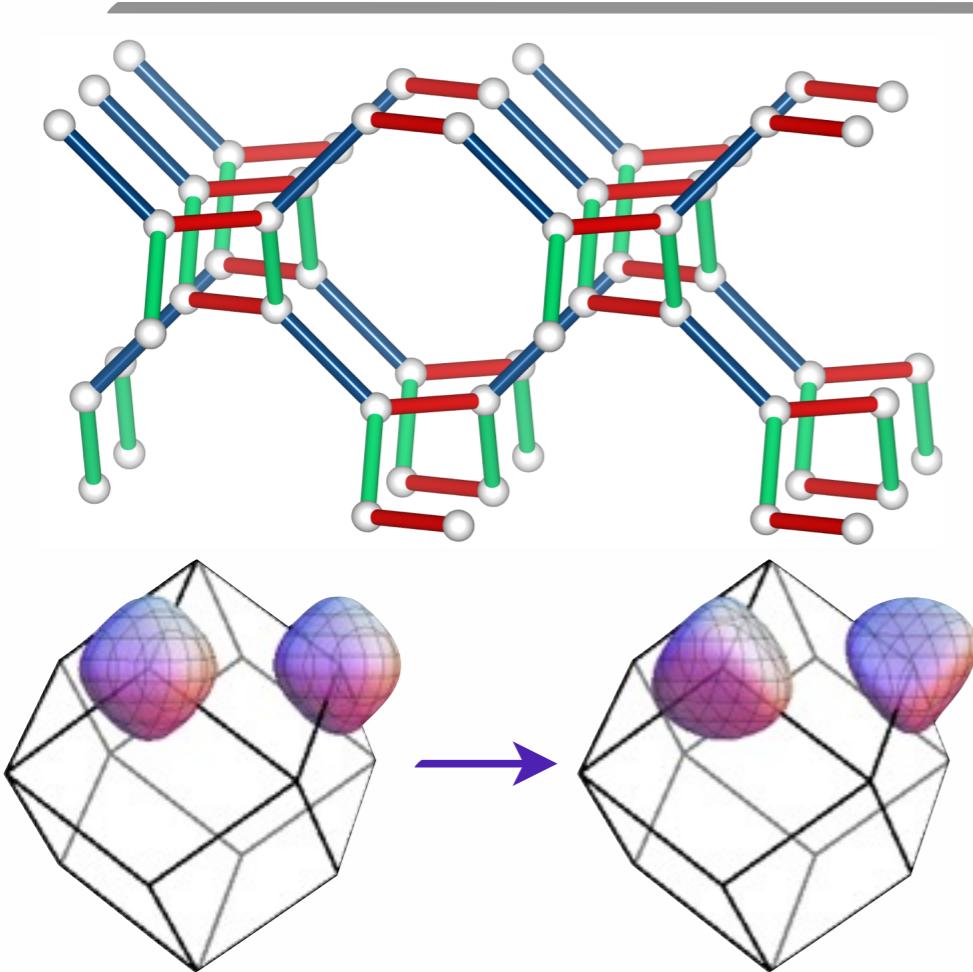
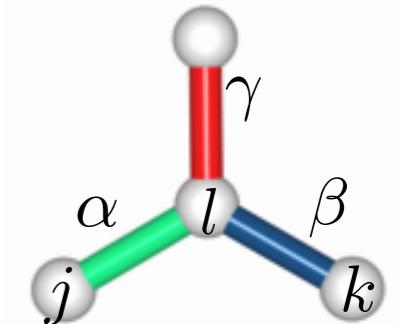
**topological semi-metal** with **protected** surface states  
(metallic cousin of the topological insulator)



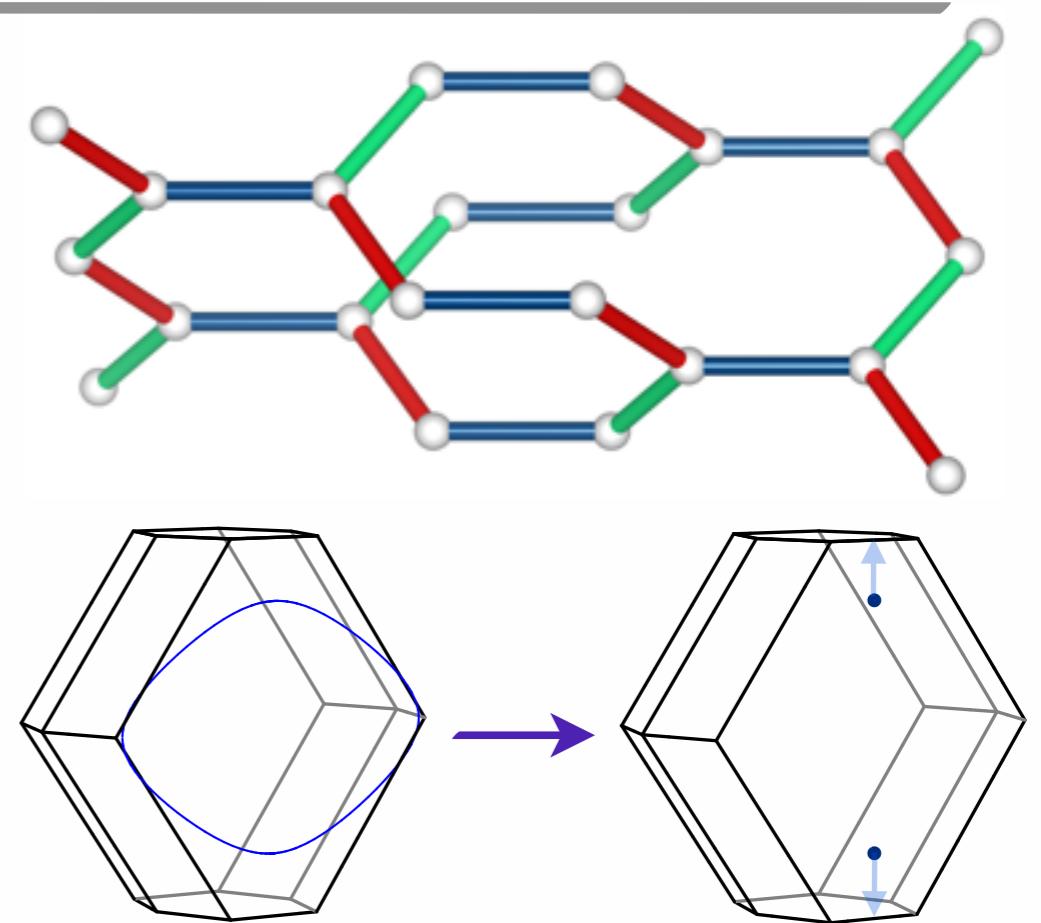
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Breaking TR stabilizes Fermi surface

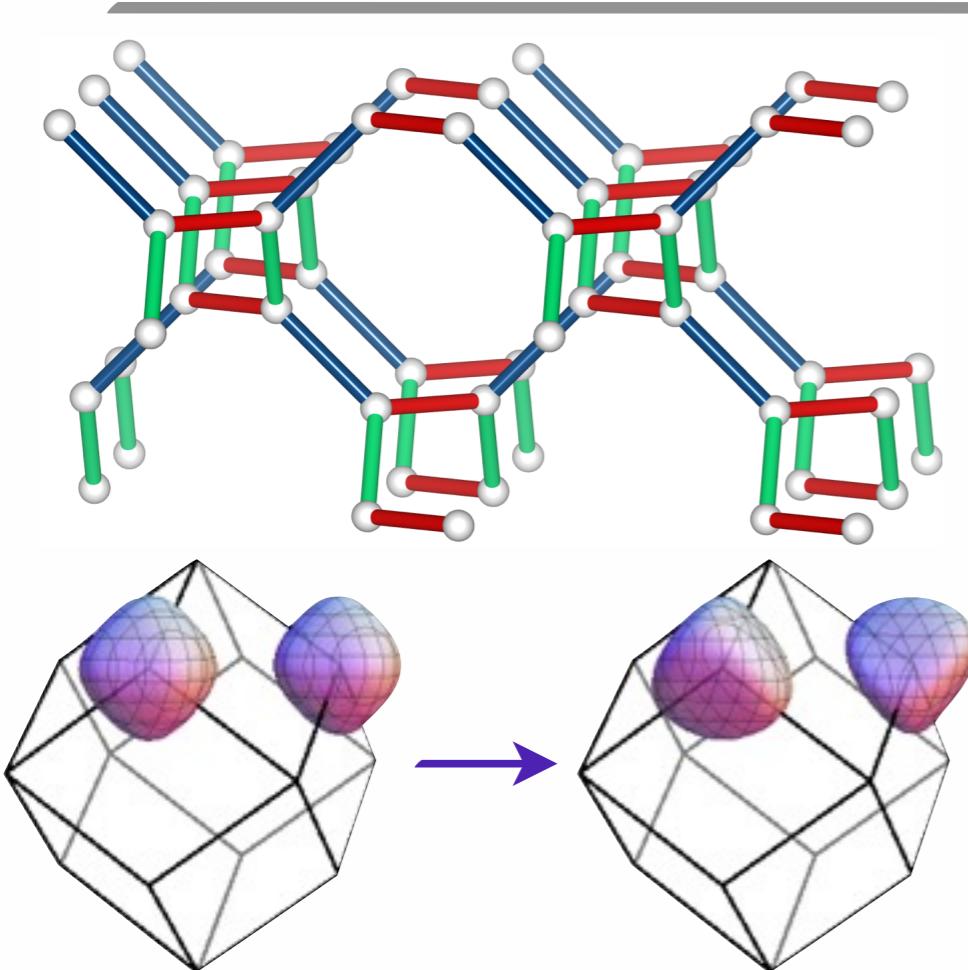
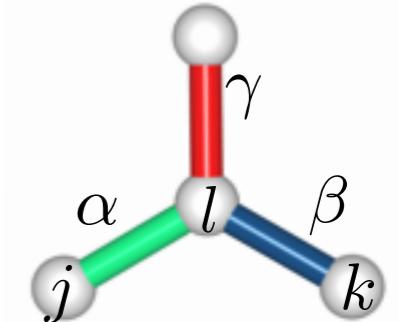


Breaking TR reduces line to **pair of Weyl nodes**

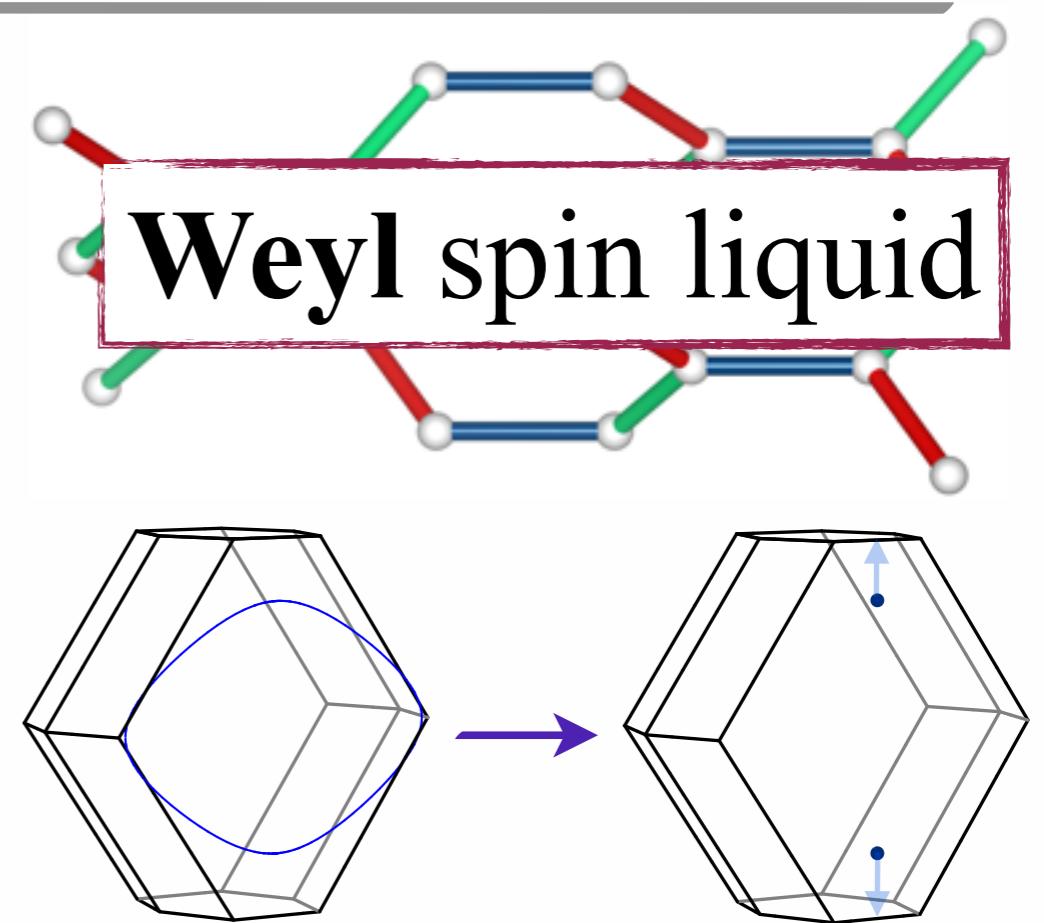
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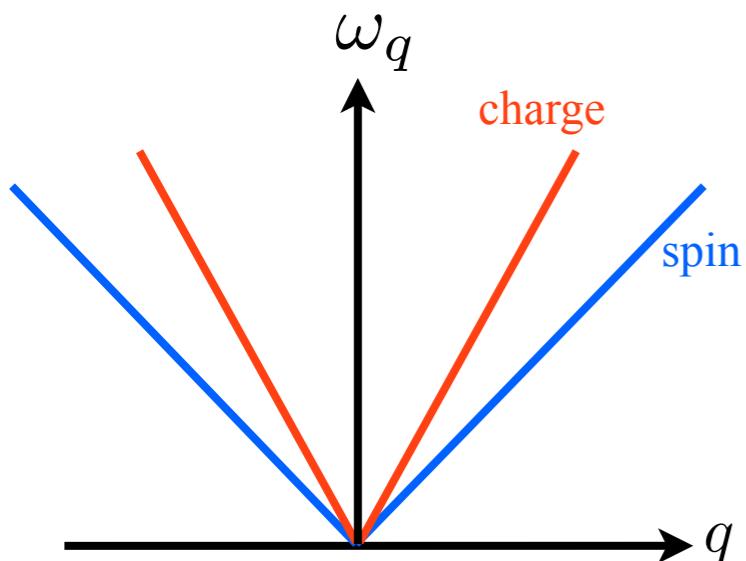
Breaking TR reduces line to **pair of Weyl nodes**

# Summary

(Strong) interactions can lead to emergent quasiparticles with ‘fractional’ quantum numbers

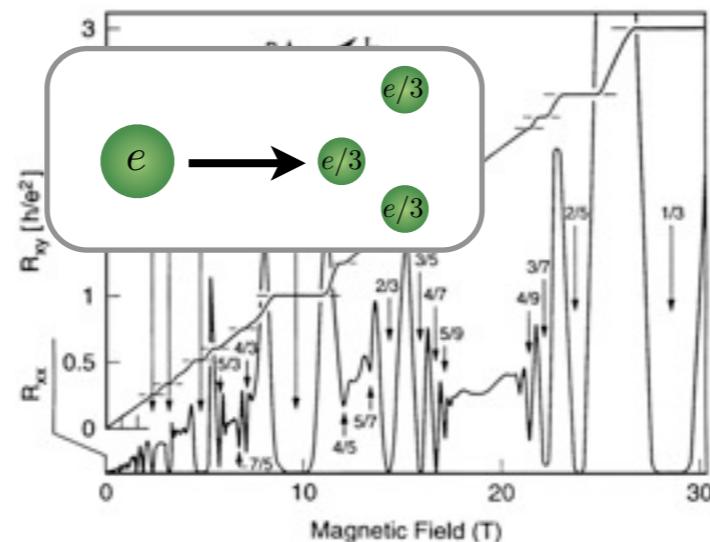
## Spin-charge separation

Fermi gas in 1D



## Electron fractionalization

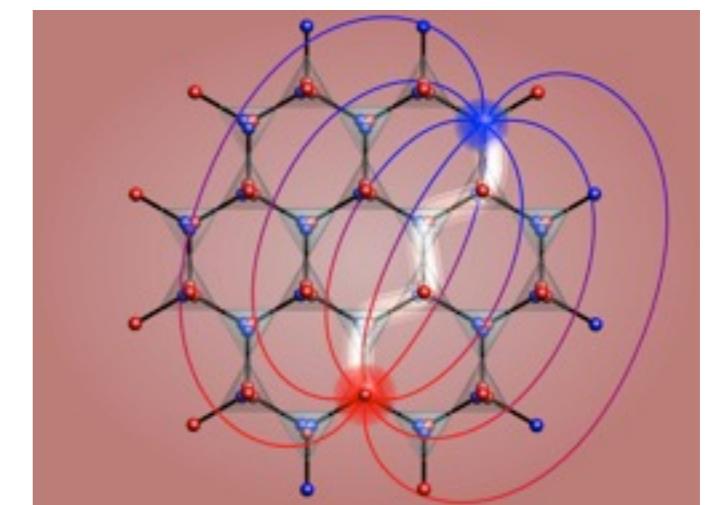
Quantum Hall liquids



Tsui, Störmer, Gossard, PRL 48, 1559 (1982)

## Magnetic monopoles

Spin ice

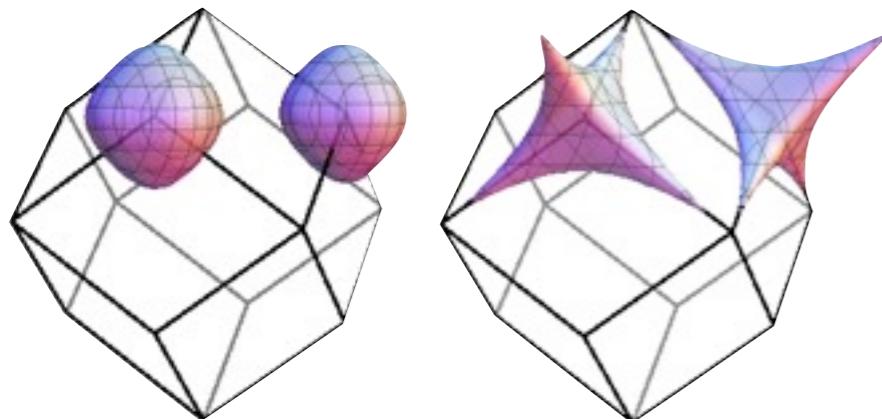


Castelnovo et al, Nature 451, 22-23 (2007)

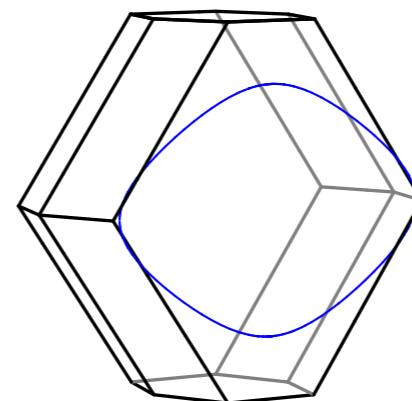
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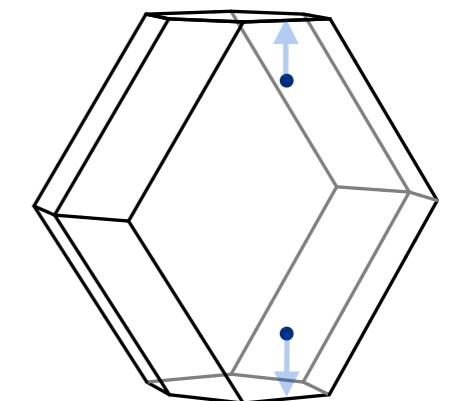
**Spin liquid with emergent spinon Fermi surface**



**Spin liquid with emergent spinon Fermi line**



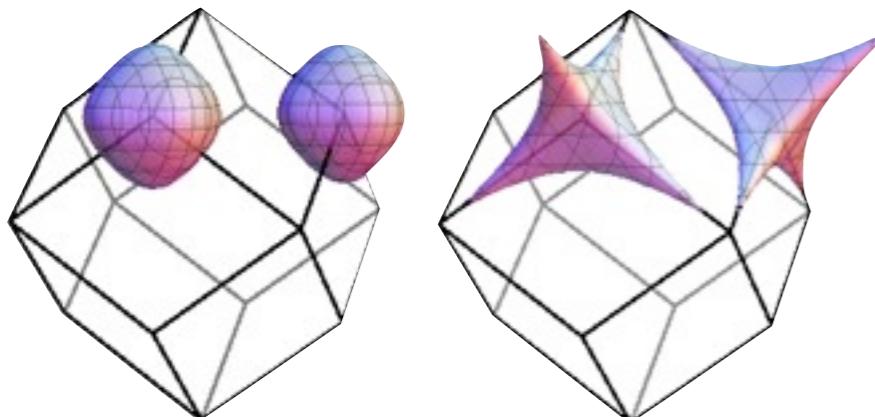
**Weyl spin liquid**



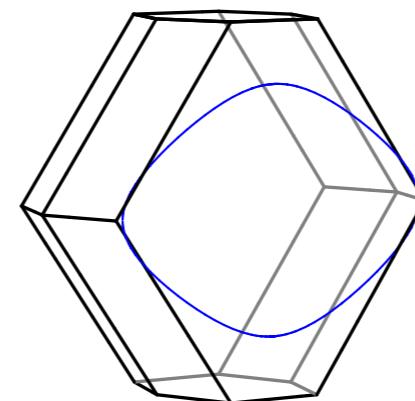
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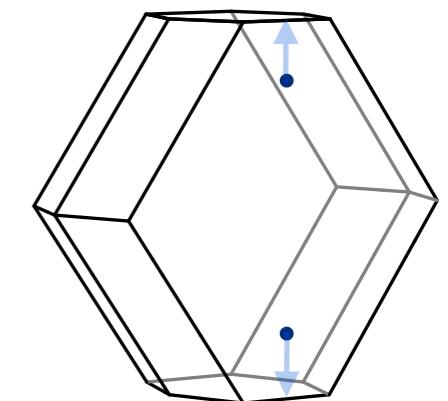
**Spin liquid with emergent spinon Fermi surface**



**Spin liquid with emergent spinon Fermi line**



**Weyl spin liquid**



**Majorana Metal**



**Majorana semi-metal**

How big is the zoo of Majorana metals?

M. Hermanns and S. Trebst, PRB 89, 235102 (2014)

M. Hermanns, K. O’Brien, and S. Trebst, in preparation