Surface Defects, Symmetries and Dualities

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Plan:

<u>1. Topological defects in quantum field theories</u>
- Relation to symmetries and dualities
- Some applications of surface defects in 3d TFT

<u>2. Defects in 3d topological field theories: Dijkgraaf-Witten theories</u>
- Defects from relative bundles
- Relation to (categorified) representation theory
- Brauer-Picard groups as symmetry groups

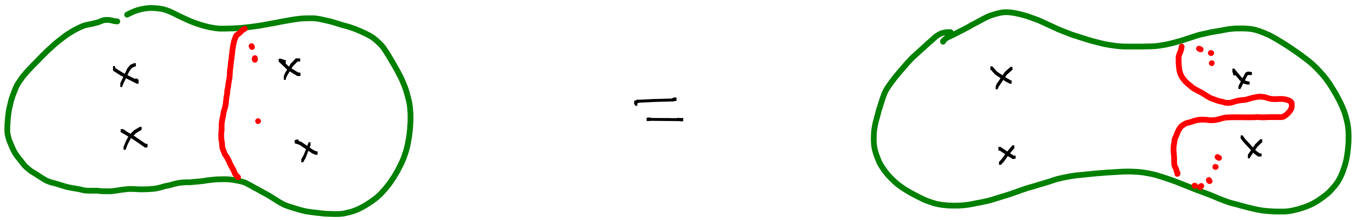
1. Topological defects in quantum field theories

Central insight:

Defects and boundaries are important parts of the structure of a quantum field theory

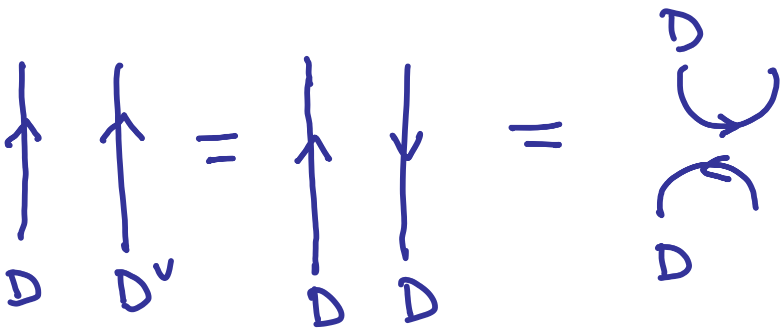
Particularly important subclass of defects: topological defects

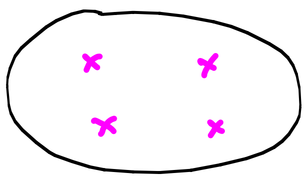
topological = correlators do not change under small deformations of the defect



1.1 Symmetries from invertible topological defects (2d RCFT [FFRS '04])

Invertible Defects





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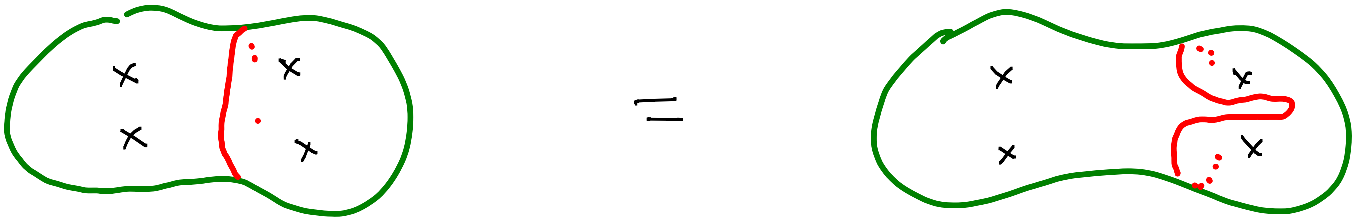

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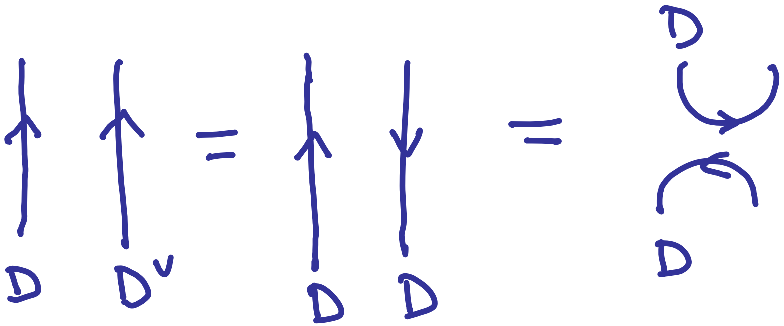

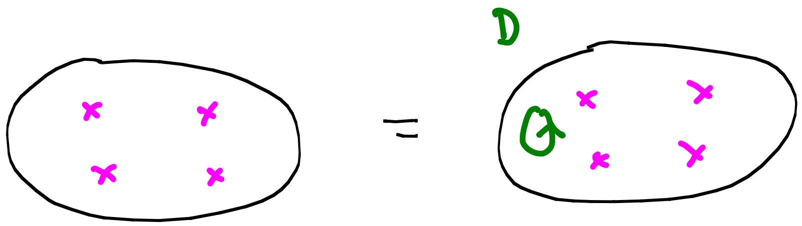

equality of
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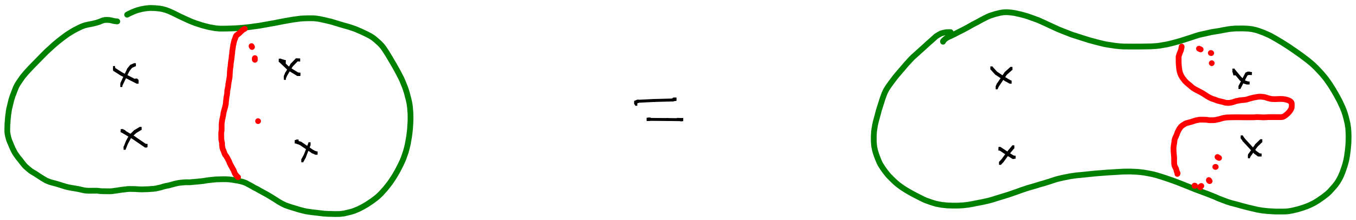

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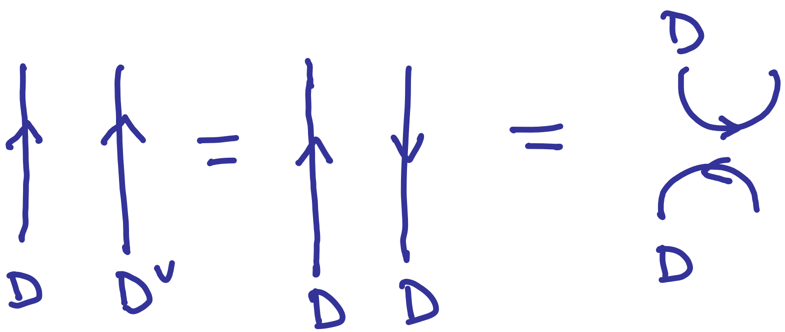

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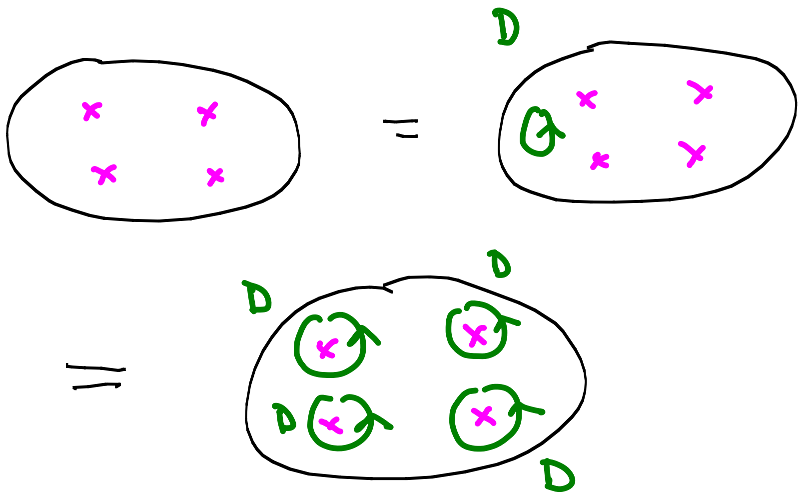


Example: critical Ising model

$\{ 1, \varepsilon, \sigma \}$

$0, \frac{1}{2}, \frac{1}{16}$

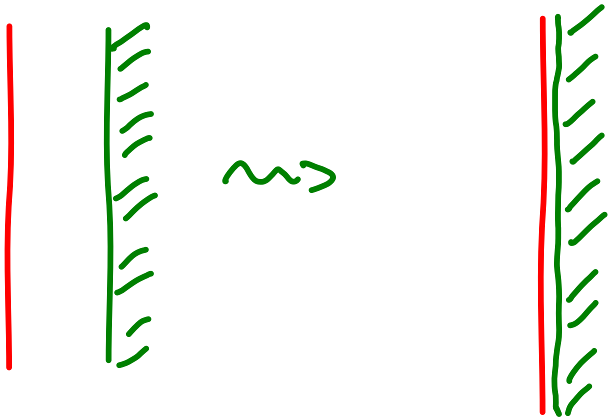
$\varepsilon \otimes \varepsilon = 1$

$\Rightarrow \quad \varepsilon$ is invertible defect

$\varepsilon \bigcirc 1 = 1 \qquad \varepsilon \bigcirc \varepsilon = \varepsilon \qquad \varepsilon \bigcirc \sigma = -\sigma$

symmetry: $\uparrow \rightsquigarrow \downarrow$

Action on boundaries:



defect
e.g. ε

boundary
e.g. free

composite boundary
e.g. fixed $\uparrow + \downarrow$

<u>Insight:</u>
In two-dimensional theories:
Group of invertible topological line defects acts as a symmetry group

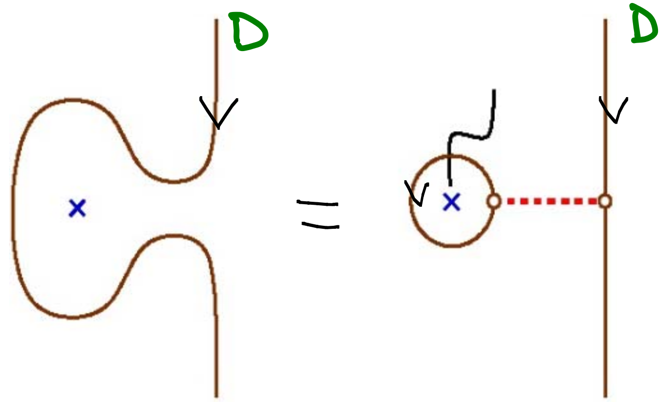
<u>Ex:</u>
critical Ising model $\quad \mathrm{Pic}(\mathcal{C}) = \mathbb{Z}_2$
3-state Potts model $\quad \mathrm{Pic}(\mathcal{C}^*) = S_3$

Important for this talk:

Similar statements apply to codimension-one topological defects
in higher dimensional field theories.

1.2 T-dualities and Kramers-Wannier dualities from topological defects

General situation:



Defect D creates a disorder field

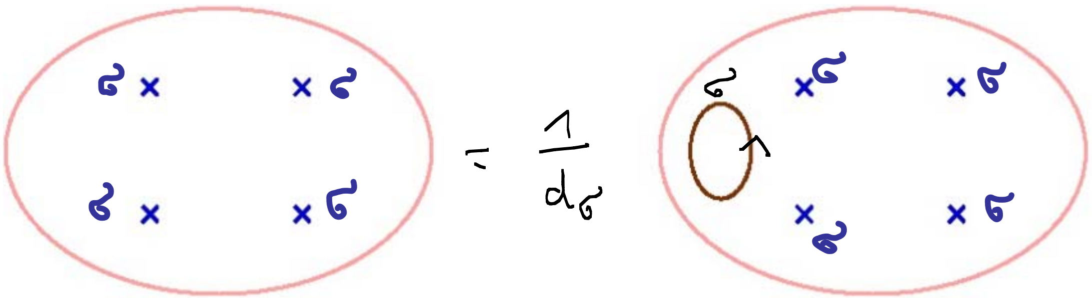
can be undone by $D^\vee$ if

$$D^\vee \otimes D = \bigoplus_{g \in G} D_g$$

invertible defect

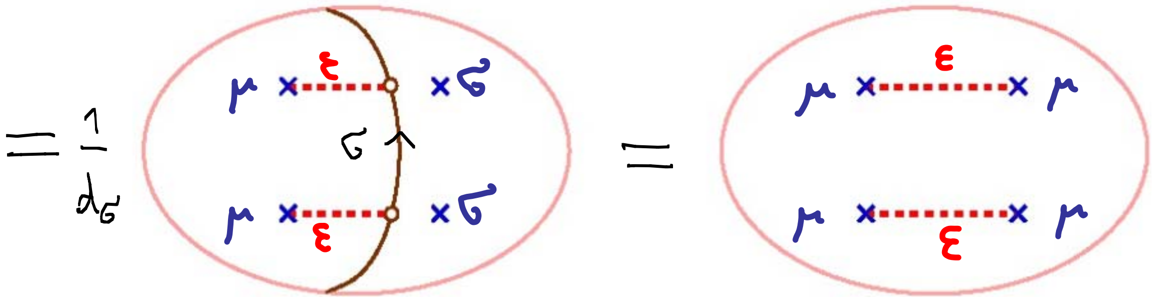
e.g. Ising $\sigma \otimes \sigma = 1 \oplus \varepsilon$

Action on correlators:



Order / disorder duality

For critical Ising model:
remnant of
Kramers-Wannier duality
at critical point



Defects for T-dualities
of free boson can be
constructed from twist fields

1.3 Defects in 3d TFTS

Topological codimension one defects also occur in other dimensions.

This talk: Topological surface defects in 3d TFT of Reshetikhin-Turaev type.
(Examples: abelian Chern-Simons, non-abelian Chern-Simons with compact gauge group, theories of Turaev-Viro type → toric code)

Motivation:

- A local two-dimensional rational conformal field theory can be described as a theory on a topological surface defect in a 3d TFT of Reshetikhin-Turaev type [FRS, Kapustin-Saulina].
  Example: construction of (rationally) compactified free boson using abelian Chern-Simons theory

- Topological phases

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- Topological phases

A general theory for such defects involving "categorified algebra" (e.g. fusion categories, (bi-)module categories) emerges.
This talk: rather a case study in the class of Dijkgraaf-Witten theories (example: ground states of toric code), including the relation between defects and symmetries

2. Defects and boundaries in topological field theories

2.1 Construction of Dijkgraaf-Witten theories from G-bundles

G finite group

M closed oriented 3-manifold

$$tft_G(M) := \int_{Bun_G(M)} \mathcal{D}A \, e^{-0} := |Bun_G(M)|$$

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Γ a finite groupoid

$$|\Gamma| = \sum_{\gamma \in \pi_0(\Gamma)} \frac{1}{|\mathrm{Aut}_\Gamma(\gamma)|}$$

Trivially, $\mathrm{tft}(-)$ is a 3-mfd invariant.
It is even a local invariant.

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Σ closed oriented 2-manifold

Choose M 3-mfd with $\partial M = \Sigma$

Consider for $P \in Bun_G(\Sigma)$ function defined by

$$\Psi_M : P \longmapsto |Bun_G(M, P)|$$

This is a (gauge invariant) function on $Bun_G(\Sigma)$

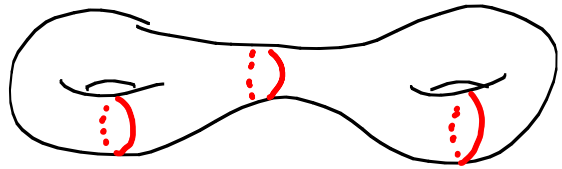
$$tft_G(\Sigma) := Fun^{gauge}(Bun_G(\Sigma))$$

* A 3d TFT assigns vector spaces to 2-mfds
* In DW theories, vector spaces are obtained by linearization from gauge equivalence classes of bundles

2.2 DW Theory as an extended TFT

Idea: implement even more locality by cutting surfaces along circles

S oriented 1-mfg.



``Pair of pants decomposition''

For any $\partial\Sigma = S$:

$$\Psi_\Sigma : Bun_G(S) \longrightarrow Vect$$

$$P \mapsto Fun^{gauge}(Bun_G(\Sigma, P))$$

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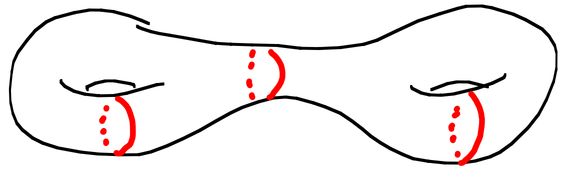

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$$\Psi_\Sigma : \mathrm{Bun}_G(S) \longrightarrow \mathrm{Vect}$$

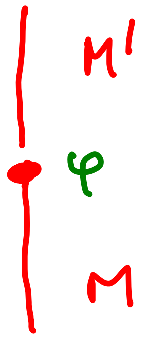
$$P \mapsto \mathrm{Fun}^{\mathrm{gauge}}(\mathrm{Bun}_G(\Sigma, P))$$

This is a vector bundle over the space of field configurations on S.
To a 1-mfd S associate thus the collection of vector bundles over the space of G-bundles on S.
Bundles come with gauge transformations. Keep them! $\Rightarrow$ Two-layered structure.

Two-layered structure: Bun_G is category. TFT associates to S the category

$$\mathrm{tft}_G(S) := [\mathrm{Bun}_G(S), \mathrm{vect}]$$

Interpretation: category of Wilson lines: for $S = S^1$



M, M' types of Wilson lines

φ pointlike insertions

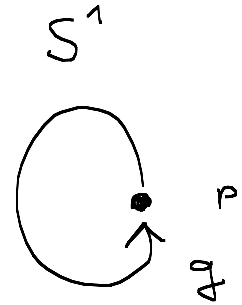
Exercise: compute this category!

- $\mathrm{Bun}_G(S^1) = G/\!/G$

  – bundle described by holonomy $g \in G$

  – gauge transformation in $p \in S^1$

  $$g \longmapsto h g h^{-1}$$



- Linearize

  $g \rightsquigarrow$ vector space $V_g \rightsquigarrow \bigoplus_{g \in G} V_g$

  $h \rightsquigarrow$ linear map $V_g \to V_{hgh^{-1}}$

- G- equivariant vector bundle on G
- Element in Drinfeld center $Z(\mathrm{vect}(G))$.

Example: toric code: $G = \mathbb{Z}_2 = \{1, \gamma\}$

4 simple Wilson lines $\mathbb{C}_1, \mathbb{C}_\gamma$ with h acting by ± 1.

2.3. Defects and boundaries in Dijkgraaf-Witten theories

Idea: keep the same 2-step procedure, but allow for more general field configurations

$$\mathrm{tft}_G : \mathrm{Cob}_{3,2,1} \xrightarrow[\mathrm{Bun}_G]{} \mathrm{Span\,Grp} \xrightarrow[\text{linearize}]{} \text{2-vect}$$

Idea: relative bundles Allow for a different gauge group on the defect or boundary

Given relative manifold $j: Y \to X$ and group homomorphism $i: H \to G$

(e.g. a subgroup)

$$\mathrm{Bun}_{H\to G}(Y \to X) = \left\{ \begin{matrix} P_G \\ \downarrow \\ X \end{matrix} , \begin{matrix} P_H \\ \downarrow \\ Y \end{matrix} , \alpha : \mathrm{Ind}_H^G P_H \xrightarrow{\sim} j^* P_G \right\}$$

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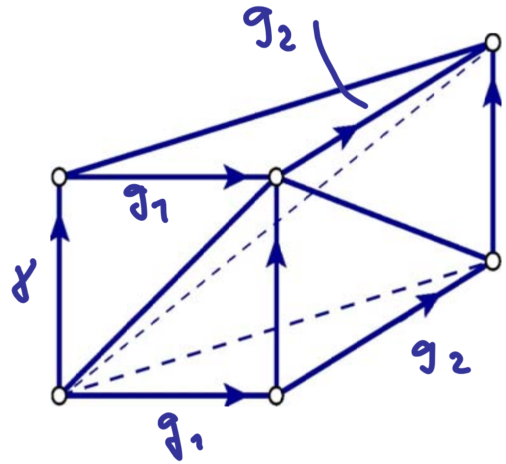
Additional datum in DW theories: twisted linearization from topolog. Lagrangian

$$\omega \in Z^3(G, \mathbb{C}^*) \cong Z^3(\mathrm{Bun}_G, \mathbb{C}^*)$$

Transgress to $\mathrm{Bun}_G(S^1) \simeq G//G$

$$\tau(\omega) \in Z^2(G//G, \mathbb{C}^*)$$

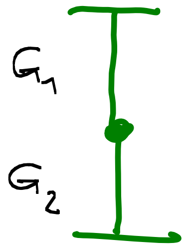
twisted linearization



$$\tau(\omega)(\gamma; g_1, g_2) = \omega(\gamma, g_1, g_2) \cdot \omega(g_1, {}^{g_1}\gamma, g_2)^{-1} \, \omega(g_1, g_2, {}^{g_1 g_2}\gamma)$$

2.4 Categories from 1-manifolds

Example: Interval $\mathcal{I}$



$i_1 : H_1 \to G_1$

$H_{12} \to G_1 \times G_2$

$i_2 : H_2 \to G_2$

$$\mathrm{Bun}(\mathcal{I}) \cong {}_{G_1 \times G_2}\backslash\!\backslash \; G_1 \times G_1 \times G_2 \times G_2 \;/\!/_{H_1 \times H_{12} \times H_2}$$

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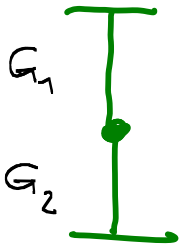

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Data:

$\omega_a \in Z^3(G_a, \mathbb{C}^*)$ bulk Lagrangian

$\theta_a \in C^2(H_a, \mathbb{C}^*)$ bdry Lagrangian

such that $d\theta_a = i^* \omega_a$

$\theta_{12} \in C^2(H_{12}, \mathbb{C}^*)$

such that $d\theta_{12} = \omega_1 (\omega_2)^{-1}$

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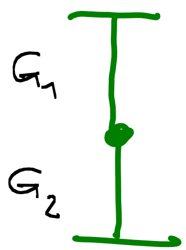

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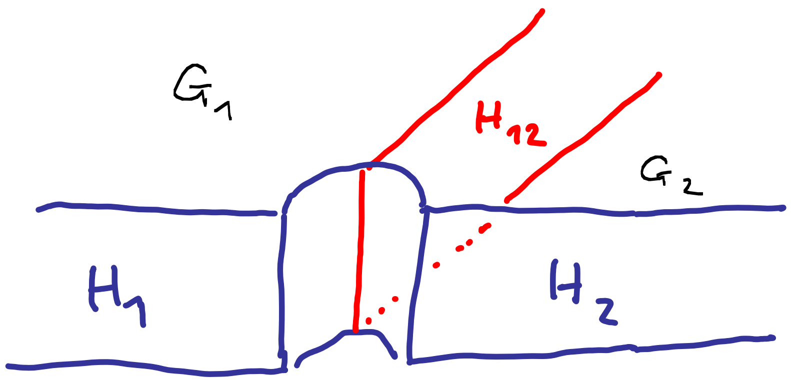
such that $d\,\theta_{12} = \omega_1 (\omega_2)^{-1}$

Transgress to 2-cocycle on $\mathrm{Bun}(\mathcal{I})$

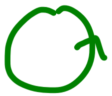
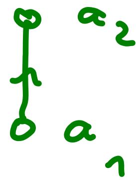
Twisted linearization gives $\mathbb{C}$-linear category
for generalized boundary Wilson lines.
Here:



2.5 A glimpse of the general theory

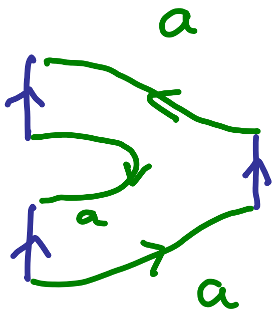
Warmup: Open / closed 2d TFT [Lauda-Pfeiffer, Moore-Segal]

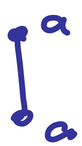
Idea: associate vector spaces not only to circles, but also to intervals

 $= S^1$ and 

boundary conditions

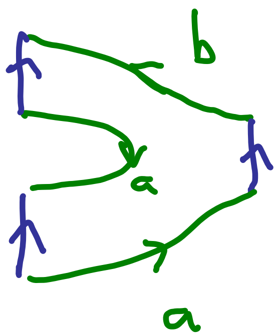
$a_i \in \mathcal{B}$ with $\mathcal{B}$ to be determined



tft() $= A_a$

Frobenius algebra

(not necessarily commutative; assume semisimple)



tft() is an A_a- module

- Boundary conditions correspond to A_a - modules; moreover

$$\mathrm{tft}\left(\Big|_{b_1}^{b_2} \right) = \mathrm{Hom}_{A_a\text{-mod}}(b_1, b_2)$$

(intertwiners)

- Algebra $\mathrm{tft}\left(\Big|_{b}^{b} \right) = \mathrm{End}_{A_a\text{-mod}}(b)$

  is Morita-equivalent to algebra A_a

- $$C = \mathrm{tft}\left(\circlearrowleft \right) = Z\left(\mathrm{tft}\left(\Big|_{a}^{a} \right) \right) = Z(A_a)$$

  for all $a \in B$. C is the center of A_a.

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Dictionary	2d TFT	
one bc	(semisimple) algebra A_a	
other bc defects bulk	A_a-modules bimodules center $Z(A_a)$	

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Dictionary	2d TFT	3d TFT of Turaev Viro type
one bc	(semisimple) algebra A_a	fusion category (bdry Wilson lines) $\mathcal{A}$ for DW theories: $\mathcal{A} = \mathrm{vect}(G)^\omega$
other bc	A_a-modules	$\mathcal{A}$ - module categories
defects	bimodules	bimodule categories
bulk	center $Z(A_a)$	Drinfeld center $Z(\mathcal{A})$

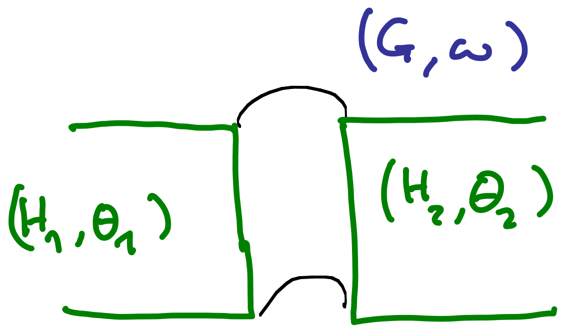
2.6 An example from Dijkgraaf-Witten theories

- Fusion category is $\mathcal{A} = \mathrm{vect}(G)^{\omega}$, with G finite group, $\omega \in Z^3(G, \mathbb{C}^*)$

- Known: indecomposable $\mathcal{A}$-module categories $\leftrightarrow$

$$H \leq G \qquad \theta \in C^2(H, \mathbb{C}^*) \text{ such that } d\theta = \omega|_H$$

Module category $\mathcal{M}(H, \theta)$ over $\mathrm{vect}(G)^{\omega}$



The two ways to compute boundary Wilson lines agree:

$$\mathrm{tft}\left(I_{G\;H_1}^{\;\;\;H_2} \right) = \mathrm{Fun}_{\mathrm{vect}(G)^{\omega}}\left(\mathcal{M}(H_1, \theta_1), \mathcal{M}(H_2, \theta_2) \right)$$

(computed from twisted bundles) (computed from module categories over fusion categories)

Twisted linearization of relative bundles exactly reproduces representation theoretic results.

2.7 Symmetries from defects

Recall: Symmetries $\leftrightarrow$ invertible topological defects

For 3d TFT of Turaev-Viro type (e.g. Dijkgraaf-Witten theories) based on fusion category $\mathcal{A}$
these are invertible $\mathcal{A}$-bimodule categories
(e.g. for DW-theories invertible $\mathrm{vect}(G)^\omega$ -bimodule categories)

Bicategory ("categorical 2-group") $\mathrm{BrPic}(\mathcal{A})$, the Brauer-Picard group

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Important tool: Transmission of bulk Wilson lines

Braided equivalence,

$$T_{\mathcal{D}}: \quad Z(\mathcal{A}) \longrightarrow Z(\mathcal{A})$$

if $\mathcal{D}$ invertible

$$\mathrm{BrPic}(\mathcal{A}) \xrightarrow{\sim} \mathrm{BraidedAut}(Z(\mathcal{A}))$$

[Etingof-Nikshych-Ostrik]

"Symmetries can be detected
from action on bulk Wilson lines"

2.7 Symmetries from defects

Recall: Symmetries $\longleftrightarrow$ invertible topological defects

For 3d TFT of Turaev-Viro type (e.g. Dijkgraaf-Witten theories) based on fusion category $\mathcal{A}$
these are invertible $\mathcal{A}$-bimodule categories
(e.g. for DW-theories invertible $\mathrm{vect}(G)^{\omega}$-bimodule categories)

Bicategory ("categorical 2-group") $\mathrm{BrPic}(\mathcal{A})$, the Brauer-Picard group

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[Etingof-Nikshych-Ostrik]

"Symmetries can be detected
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Explicitly computable for DW theories:

$$\mathcal{C} = \mathcal{D}(G)\text{-mod} = \mathcal{Z}(G\text{-vect})$$

$\mathcal{D}$ described by

$$H \leq G \times G$$

Linearize span of action groupoids

$$\begin{tikzcd} & H/\!\!/H \arrow[dl] \arrow[dr] & \\ G/\!\!/G & & G/\!\!/G \end{tikzcd}$$

2.8 Symmetries for abelian Dijkgraaf-Witten theories

Special case: : $G = A$ abelian , $\omega = 1$

$$\mathrm{BrPic}(A\text{-vect}) = O_q(A \oplus A^*)$$

with $q(g, \chi) = \chi(g)$

quadratic form

Relation to lattices

Choose lattice $(\tilde{L}, \tilde{K})$ s.t. $\tilde{L}^* / \tilde{L} = A$.

Then consider

$$L = \tilde{L} \oplus \tilde{L} \text{ with } K = \begin{pmatrix} 0 & \tilde{K} \\ \tilde{K} & 0 \end{pmatrix}$$

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Obvious symmetries:

1) Symmetries of Bun_A

$$\varphi \in \mathrm{Aut}(\mathrm{Bun}_A) = \mathrm{Aut}(A)$$

Subgroup:

$$H_\varphi = \mathrm{graph}\,\varphi \subset A \oplus A, \quad \Theta = 1$$

Braided equivalence:

$$\varphi \oplus (\varphi^*)^{-1} : A \oplus A^* \longrightarrow A \oplus A^*$$

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2) Automorphisms of CS 2-gerbe

1-gerbe on Bun_A "B-field"

$$H^2(A, \mathbb{C}^*) \xrightarrow{\sim} \underset{\beta}{\mathrm{AltBiHom}(A, \mathbb{C}^*)}$$

(transgression)

Subgroup: $A_{diag} \subset A \oplus A \quad \Theta = \beta$

Braided equivalence:

$$A \oplus A^* \longrightarrow A \oplus A^*$$

$$(g, \chi) \longmapsto (g, \chi + \beta(g, -))$$

3) Partial e-m dualities:

Example: A cyclic, fix $\delta : A \xrightarrow{\sim} A^*$

Braided equivalence:

$$A \oplus A^* \longrightarrow A \oplus A^*$$
$$(g, \chi) \mapsto (\delta^{-1}\chi, \delta g)$$

Subgroup: $A_{diag} \subset A \oplus A$

$$\beta(a_1, a_2) = \frac{\delta(a_1)(a_2)}{\delta(a_2)(a_1)}$$

Theorem [FPSV]

These symmetries form a set of generators for $\mathrm{BrPic}(A\text{-vect})$.

3. Conclusions

Topological defects are important structures in quantum field theories

- Topological defects implement symmetries and dualities

- Applications to relative field theories on defects and topological phases

Defects in 3d topological field theories: Dijkgraaf-Witten theories

- Defects from relative bundles

- Relation to (categorified) representation theory:
  module categories over monoidal categories

- Brauer-Picard groups as symmetry groups