Projected entangled-pair states for chiral topological phases

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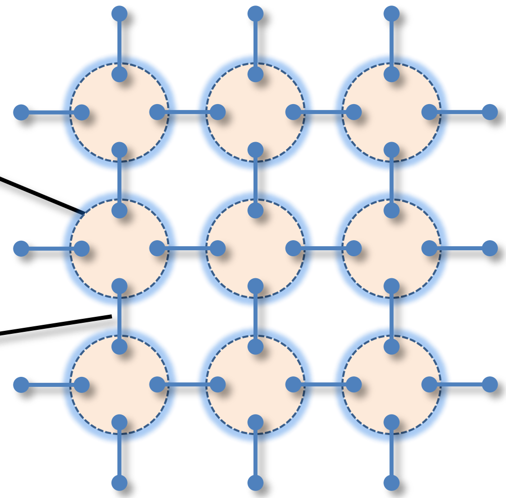
ESI-programme on Topological Phases of Quantum Matter,
Vienna, August 28th, 2014

Projected entangled-pair state (PEPS)

$$|P\rangle_i = \sum_{s_i=1}^{d} \sum_{\alpha_i,\beta_i,\gamma_i,\delta_i=1}^{D} A^{s_i}_{\alpha_i\beta_i\gamma_i\delta_i} |s_i\rangle|\alpha_i,\beta_i,\gamma_i,\delta_i\rangle$$

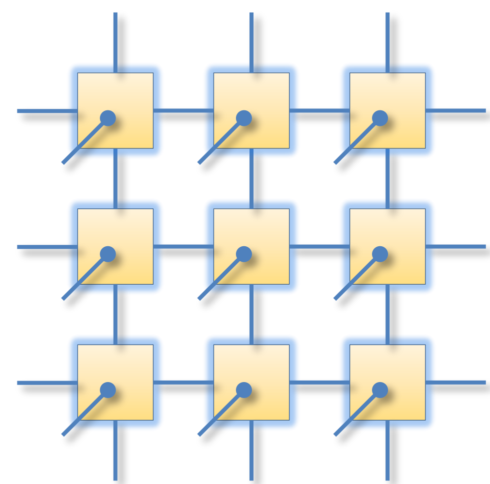


$$|I\rangle_{i,j} = \sum_{\alpha=1}^{D} |\alpha_i,\alpha_j\rangle$$

D: bond dimension

$$|\psi\rangle = \left(\bigotimes_{\langle ij\rangle}\langle I|_{i,j}\right)\left(\bigotimes_{i=1}^{N}|P\rangle_i\right)$$

$$= \sum_{\{s\}}\left(\sum_{\{\alpha\beta\gamma\delta\}} \cdots A^{s_i}_{\alpha_i\beta_i\gamma_i\delta_i}\cdots\right)|\ldots s_i \ldots\rangle$$



Verstraete & Cirac, cond-mat/0407066

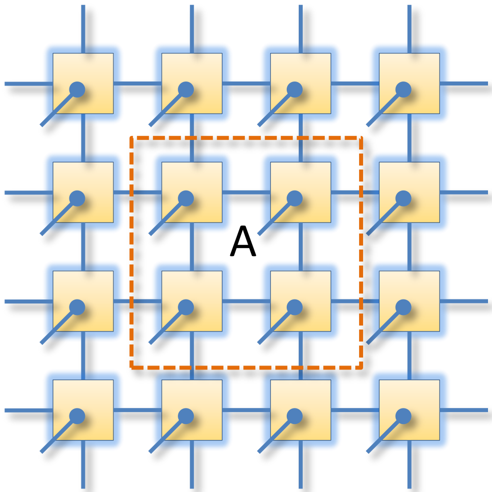
Parent Hamiltonian

Existence of null space if $d^{N_A} > D^{N_{\partial A}}$

$$H = \sum_i h_i$$

$$h_i \geq 0$$

$$h_i|\psi\rangle = 0$$



$$\mathrm{Rank}(\rho_A) \leq D^{N_{\partial A}}$$

- PEPS satisfy the entanglement area law

$$S(A) \sim N_{\partial A}$$

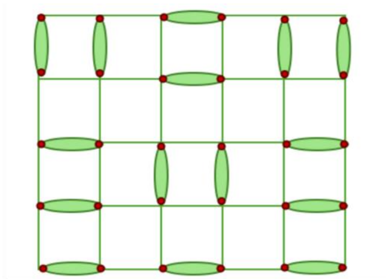
➢ Conjecture: area law holds for all gapped ground states of local Hamiltonians

Verstraete, Wolf, Perez-Garcia & Cirac, PRL (2006)

Topological PEPS

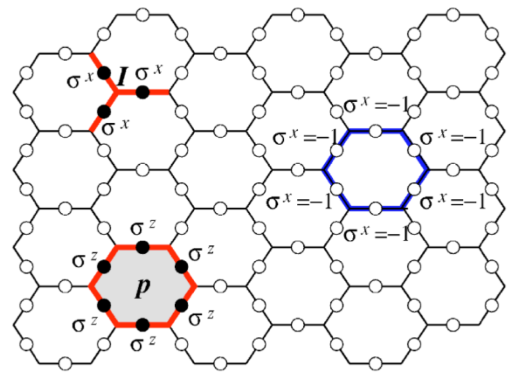
Resonating valence bonds

Anderson, Mater. Res. Bull. (1973)

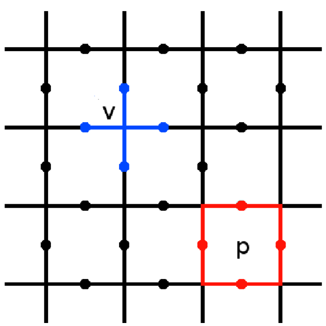


Levin-Wen string nets

Levin & Wen, PRB (2005)



Toric code Kitaev, Ann. Phys. (2003)



Q: What about chiral topological states?

Verstraete, Wolf, Perez-Garcia & Cirac, PRL (2006); Buerschaper & Aguado, PRB (2009); Gu, Levin, Swingle & Wen, PRB (2009)

PEPS for free fermionic chiral topological states

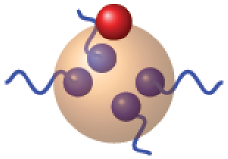
T.B. Wahl, HHT, N. Schuch & J.I. Cirac, PRL (2013);

T.B. Wahl, S.T. Hassler, HHT, N. Schuch & J.I. Cirac, arXiv:1405.0447.

See also J. Dubail & N. Read, arXiv: 1307.7726.

Fermionic Gaussian state

- PEPS projector $|P\rangle$ is a Gaussian state (ground/thermal states of quadratic Hamiltonians)



- Gaussian states are characterized by the convariance matrix

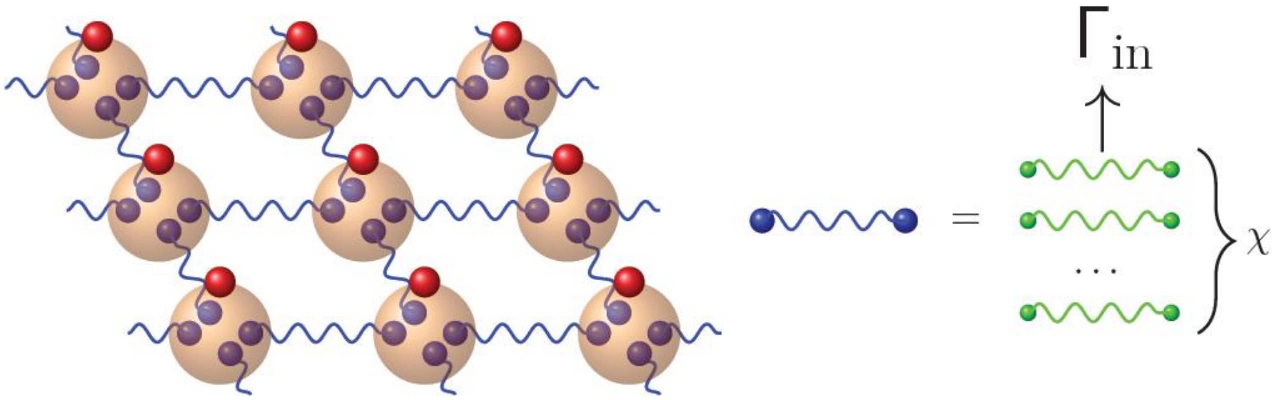
$$\Gamma_{jl} = \frac{i}{2}\mathrm{tr}(\rho[c_j, c_l]) \qquad \Gamma \leftrightarrow \rho$$

Majorana

Pure state: $\Gamma\Gamma^\top = \mathbb{I}$

Mixed state: $\Gamma\Gamma^\top < \mathbb{I}$

Gaussian Fermionic PEPS (GFPEPS)



- GFPEPS projector characterized by a convariance matrix:

$$M = \begin{pmatrix} A & B \\ -B^\top & D \end{pmatrix}$$

Pure to pure: $MM^\top = \mathbb{I}$

Pure to mix: $MM^\top < \mathbb{I}$

- Covariance matrix for GFPEPS

$$\Gamma_{\rm out}(\mathbf{k}) = B[D - \Gamma_{\rm in}(\mathbf{k})]^{-1}B^\top + A$$

Kraus, Schuch, Verstraete & Cirac, PRA (2010)

Example of a chiral PEPS: Chern insulator

$$A_\lambda = (-1+2\lambda)\begin{pmatrix} \omega & 0 \\ 0 & -\omega \end{pmatrix}$$

$$B_\lambda = \sqrt{\frac{\lambda-\lambda^2}{2}}\begin{pmatrix} I-\omega & I+\omega & -\sqrt{2}\,\omega & \sqrt{2}\,I \\ I-\omega & -I-\omega & \sqrt{2}\,I & -\sqrt{2}\,\omega \end{pmatrix}$$

$$D_\lambda = \begin{pmatrix} 0 & (-1+\lambda)\,I & -\frac{\lambda}{\sqrt{2}}\,I & \frac{\lambda}{\sqrt{2}}\,I \\ (1-\lambda)\,I & 0 & -\frac{\lambda}{\sqrt{2}}\,I & -\frac{\lambda}{\sqrt{2}}\,I \\ \frac{\lambda}{\sqrt{2}}\,I & \frac{\lambda}{\sqrt{2}}\,I & 0 & (-1+\lambda)\,I \\ -\frac{\lambda}{\sqrt{2}}\,I & \frac{\lambda}{\sqrt{2}}\,I & (1-\lambda)\,I & 0 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$0 < \lambda < 1$$

- Gapless Hamiltonian with short-range hoppings

$$H_1 = -i\sum_{\mathbf{k},s,t} \varepsilon(\mathbf{k})[\Gamma_{\text{out}}(\mathbf{k})]_{st} d_{\mathbf{k},s} d_{-\mathbf{k},t}$$

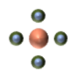
- Gapped Hamiltonian with powerlaw decaying hoppings ($1/r^3$), C = -1

$$H_2 = -i\sum_{\mathbf{k},s,t} [\Gamma_{\text{out}}(\mathbf{k})]_{st} d_{\mathbf{k},s} d_{-\mathbf{k},t}$$

Chirality of GFPEPS

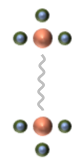
- Necessary (but not sufficient) condition:

$|P_1\rangle$



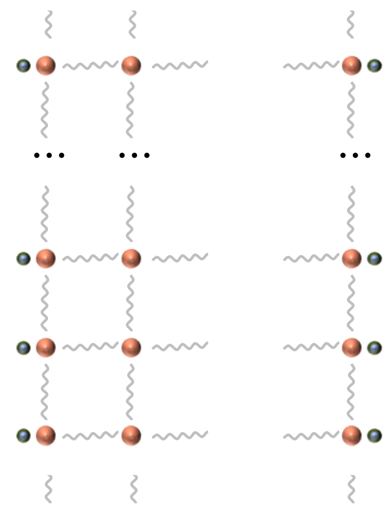
$$\sum x_\alpha c_\alpha |P_1\rangle = 0$$

$|P_2\rangle$



$$\sum x_{j,\alpha} c_{j,\alpha} |P_2\rangle = 0$$

$|P_{N_h}\rangle$



$$\sum x_{j,\alpha} c_{j,\alpha} |P_{N_h}\rangle = 0$$

- The existence of $\sum x_\alpha c_\alpha$ is related to the existence of chiral edge modes

- The GFPEPS is non-injective (otherwise adiabatically connected to a trivial state)

Approximating a Chern insulator

Do PEPS provide a good approximation to the ground/thermal state of a Chern insulator?

$$H = \sum_{\mathbf{k}} (a^\dagger_{\mathbf{k},\uparrow}, a^\dagger_{\mathbf{k},\downarrow})\,(\sigma\cdot\mathbf{d}(\mathbf{k}))\,(a_{\mathbf{k},\uparrow}, a_{\mathbf{k},\downarrow})^\top$$

$$\mathbf{d}(\mathbf{k}) = (\sin k_y, -\sin k_x, 2-\cos k_x - \cos k_y - e_S)$$

$$e_S = 1 \qquad C = -1$$

X.-L. Qi, Y.-S. Wu & S.-C. Zhang, PRB (2006)

Approximating a Chern insulator

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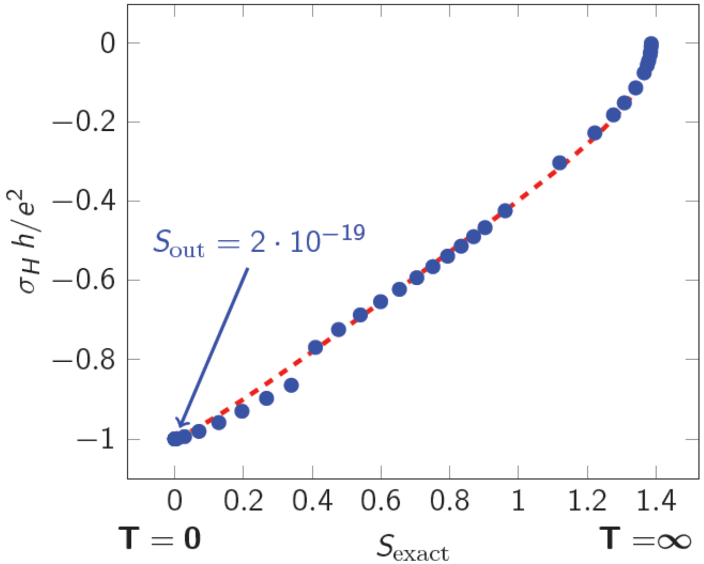
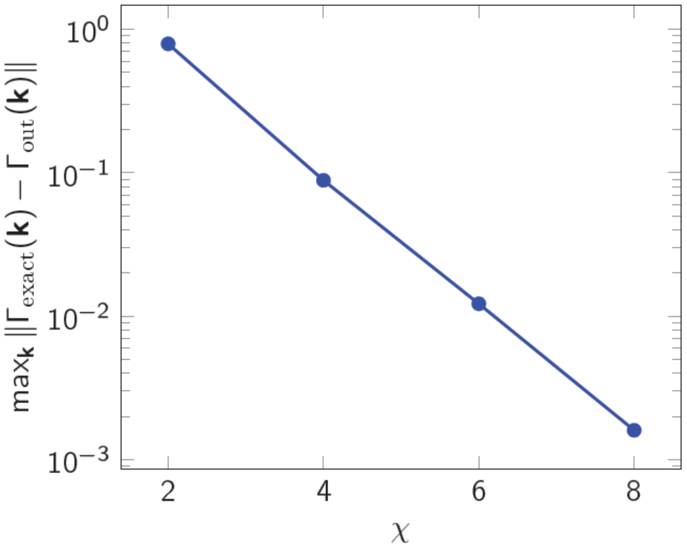




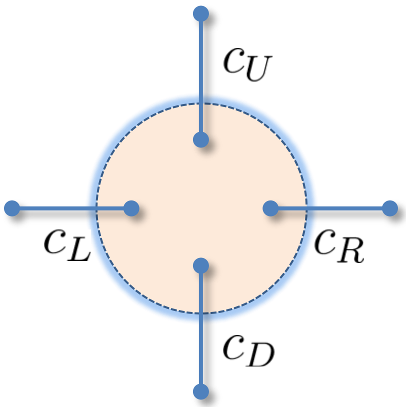
Bond dimension: $\chi = 2$

PEPS for interacting chiral topological states

In preparation...

Chiral PEPS example from projective construction

$$|\mathrm{GPEPS}\rangle = \left(\bigotimes_{\langle ij\rangle} \langle\omega|_{i,j}\right)\left(\bigotimes_{i=1}^{N} |P\rangle_i\right)$$

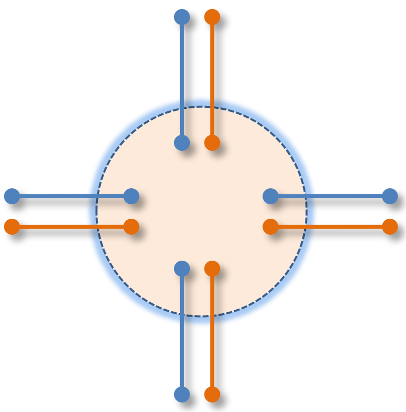


- topological superconductor with C = 1 (class D)

Projective construction:

$$|\Psi\rangle = P_{\mathrm{G}}|\mathrm{GPEPS}\rangle_1|\mathrm{GPEPS}\rangle_2$$

Gutzwiller projector -- only single occupancy allowed!



Projective construction of $SO(n)_1$ state

$$|\Psi\rangle = P_{\mathrm{G}} \prod_{\alpha=1}^{n} \exp\left(\sum_{i<j} \frac{1}{z_i - z_j} a_{i,\alpha}^{\dagger} a_{j,\alpha}^{\dagger} \right) |0\rangle$$

$$\Psi(\alpha_1, \ldots, \alpha_N) = \langle \chi^{\alpha_1}(z_1) \chi^{\alpha_2}(z_2) \cdots \chi^{\alpha_N}(z_N) \rangle$$

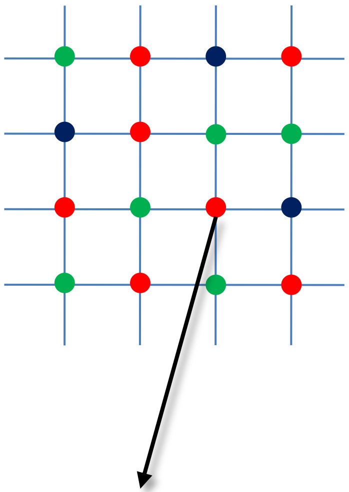
Bulk-edge correspondence (Moore-Read):

- Edge CFT: $SO(n)_1$ with central charge c = n/2
- Anyonic quasiparticles

$I, s, \bar{s}, v$ n even

I, s, v n odd

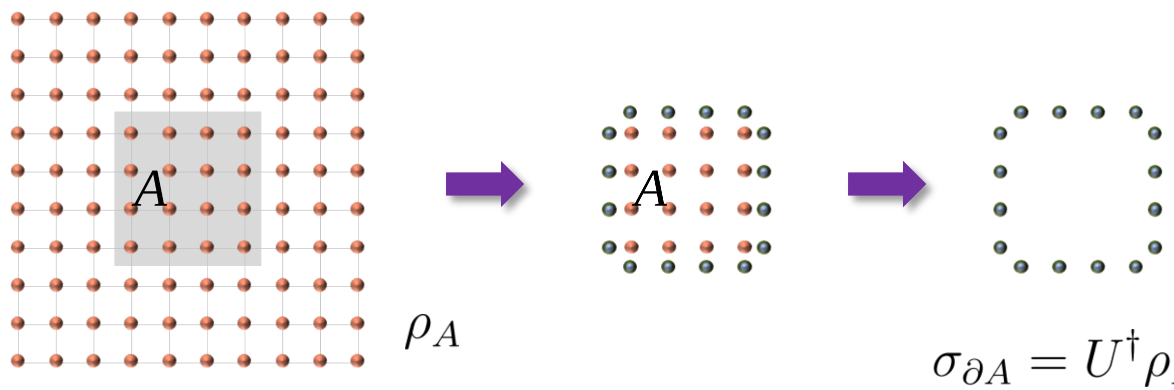
$$h_I = 0 \quad h_s = h_{\bar{s}} = \frac{n}{16} \quad h_v = \frac{1}{2}$$



$z = x + iy$

HHT, Phys. Rev. B 87, 041103 (2013)

Boundary theory of PEPS



$$\sigma_{\partial A} = U^\dagger \rho_A U$$

isometry

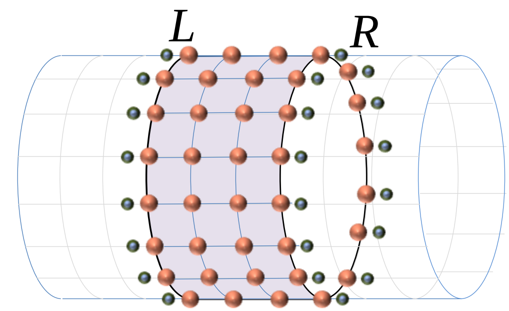
Boundary Hamiltonian:

$$\sigma_{\partial A} = e^{-H_{\partial A}}$$

- ... gives entanglement spectrum
- ... can be easily determined (exactly or approximiately)

Cirac, Poilblanc, Schuch & Verstraete, PRB (2011)

Boundary theory of chiral PEPS



Entanglement spectrum for chiral states $|\Psi_a\rangle$:

$$\sigma_{L,a} = e^{-H_L}\big|_a$$

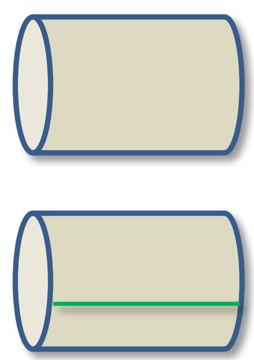
chiral CFT   topological sector a

- $|\Psi_a\rangle$ are "minimally-entangled" states!

PEPS

No flux

With flux



Each contains two sectors!

Li & Haldane, PRL (2008); Qi, Katsura & Ludwig, PRL (2012);
Zhang, Grover, Turner, Oshikawa & Vishwanath, PRB (2012)

Outlook

- Chiral PEPS with exponentially decaying correlations and gapped short-range parent Hamiltonian?

    - Approach different from projective construction and discretization of conformal blocks?

- Gauge symmetry of PEPS local tensor as a unified description of both chiral and non-chiral topological states?

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Thank you for your attention!