JQI Seminar
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University of Maryland, USA

thp Institute for Theoretical Physics University of Cologne

Universal Quantum Physics in Driven Open Many-Body Systems

Sebastian Diehl

Institute for Theoretical Physics, University of Cologne

Collaboration:
J. Marino, M. Buchhold (Cologne)
M. Marcuzzi, I. Lesanovsky (Nottingham)

€ € € :

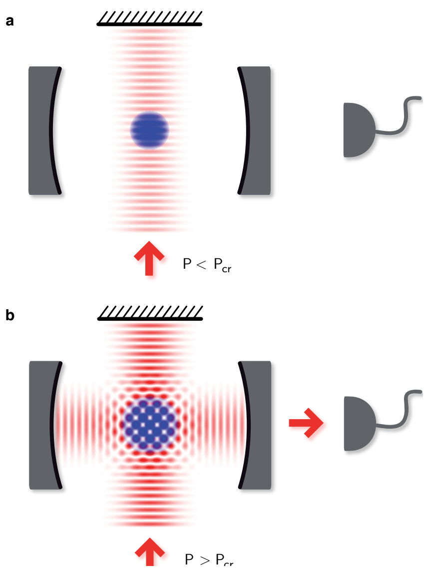
erc European Research Council

DFG Deutsche Forschungsgemeinschaft

Motivation: Driven open many-body dynamics

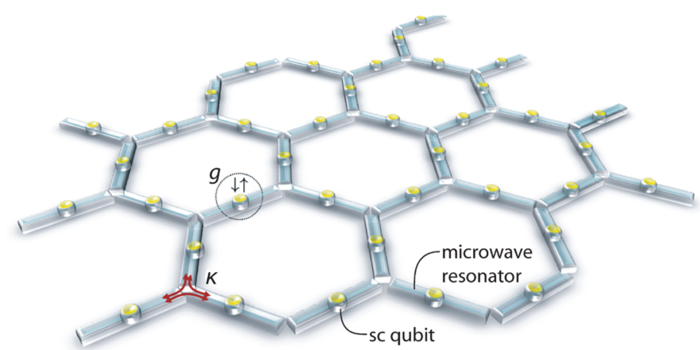
- experimental systems on the interface of quantum optics and many-body physics

- driven-open Dicke models



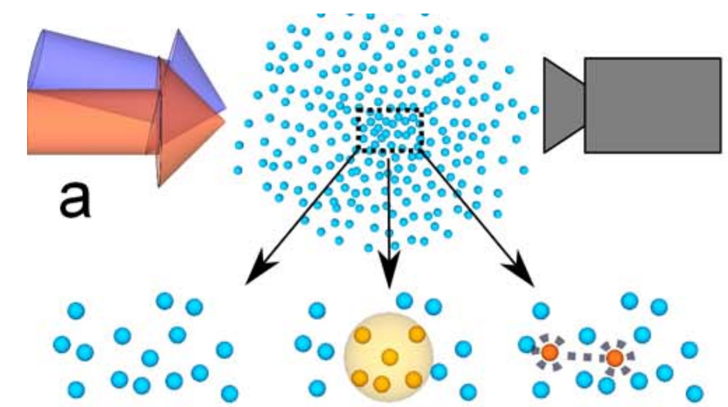
Baumann et al., Nature 2010
Ritsch et al., RMP 2013

- coupled microcavity arrays



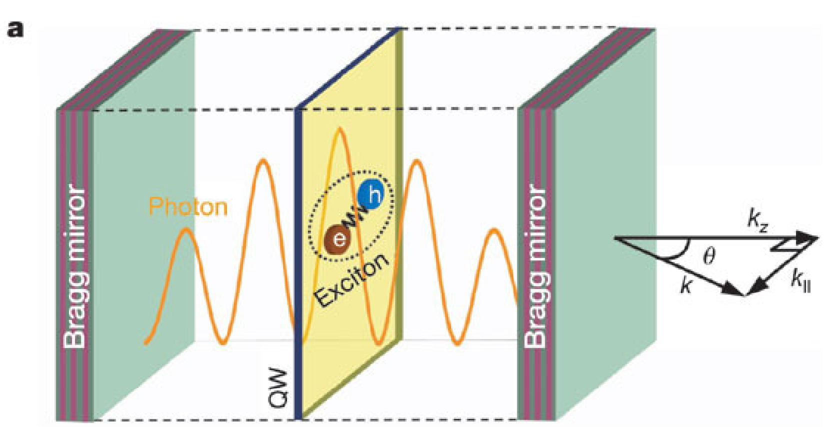
Koch et al., PRA 2010
Houck, Türeci, Koch, Nat. Phys. 2012

- driven-dissipative Rydberg systems



Carr et al. PRL 2013
Malossi et al. PRL 2014

- exciton-polariton systems in semiconductor quantum wells



Kasprzak et al., Nature 2006
Carusotto, Ciuti RMP 2013

- other platforms (light-matter):
  - ➡ polar molecules — Zhu et al. PRL 2013
  - ➡ photon BECs — Klaers et al. Nature 2010
  - ➡ trapped ions — Kim et al., Nature 2010; Islam et al., Nature 2011; Barreiro et al. Nature 2011; Britton et al. Nature 2012

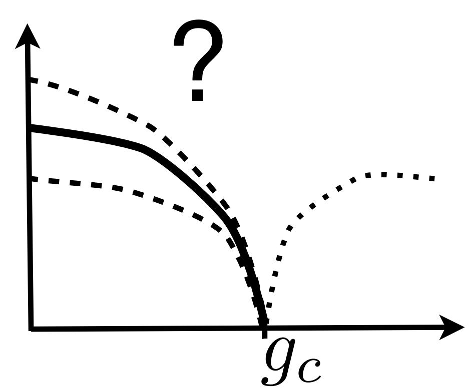
Non-Equilibrium Physics with Driven Open Quantum Systems (DOQS)

- Interdisciplinary research area: physics at various length scales

Quantum Optics	Many-body physics	Statistical mechanics
coherent and driven-dissipative dynamics on equal footing	continuum of spatial degrees of freedom	
Microscopic	"Thermodynamic"	Long wavelength

- Questions and Challenges:

Novel universal phenomena ?

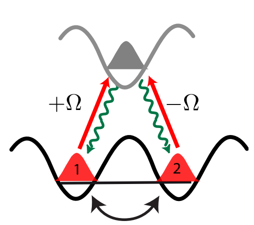


Efficient theoretical tools ?

$$Z[J] = \int \mathcal{D}\varphi \, e^{i(S[\varphi] + \int J\varphi)}$$

perform the transition form micro-to macrophysics:
quantum field theory out of equilibrium

Experimental platforms ?



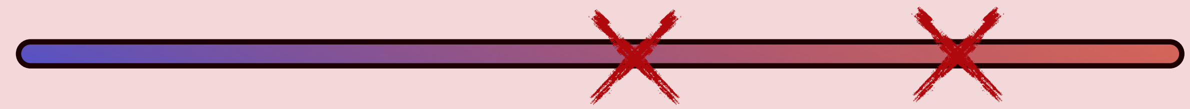
cold atoms, light-driven semiconductors, microcavity arrays, trapped ions ...

Overview: Non-equilibrium Phenomena in DOQS @ Cologne

Non-equilibrium dynamical criticality

- **Questions**:
- universality out of equilibrium?
- relation to equilibrium dynamical criticality?
- classical vs. quantum?

Phase structure of driven open quantum systems

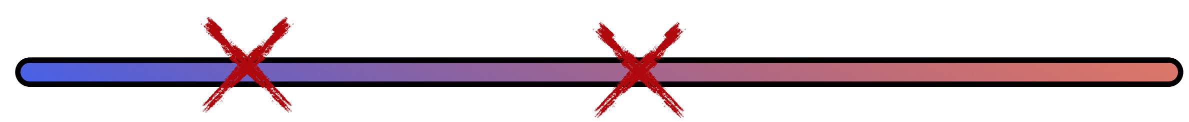


- **Questions**:
- fate of algebraic order & superfluidity?
- fate of Kosterlitz-Thouless transition?
- new phase transitions w/o equilibrium counterpart?

Interacting open system dynamics

- **Questions**:
- universal features in time evolution?
- identification of characteristic dynamical regimes?
- short vs. long time behavior?

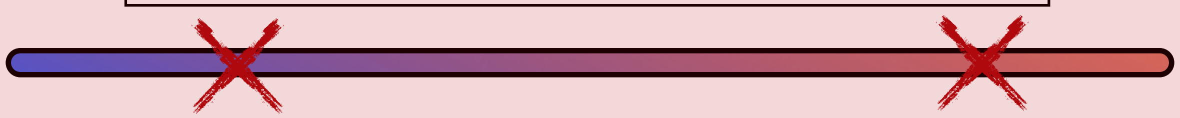
Order by Dissipation



- **Questions**:
- efficient cooling protocols?
- topological order out of equilibrium?
- potential for quantum information applications?

Overview: Non-equilibrium Phenomena in DOQS

Non-equilibrium dynamical criticality



- **Questions**:
- universality out of equilibrium?
- relation to equilibrium dynamical criticality?
- classical vs. quantum?

Driven Markovian Quantum Criticality
in Microcavity Arrays

J. Marino, SD, PRL (2016)

Phase structure of driven open quantum systems

- **Questions**:
- fate of algebraic order & superfluidity?
- fate of Kosterlitz-Thouless transition?
- new phase transitions w/o equilibrium counterpart?

New Absorbing State Phase Transition
in Driven Rydberg Ensembles

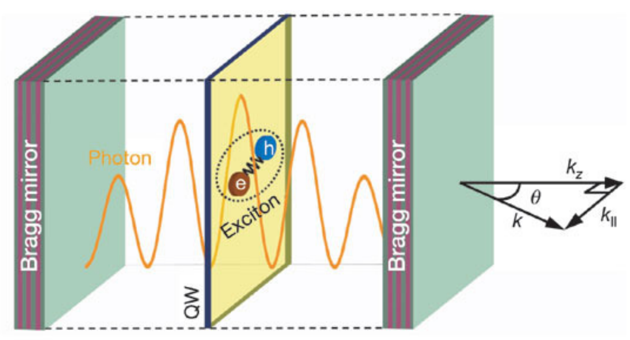
M. Marcuzzi, M. Buchhold, SD, I. Lesanovsky, arxiv (2016)

- guiding questions:

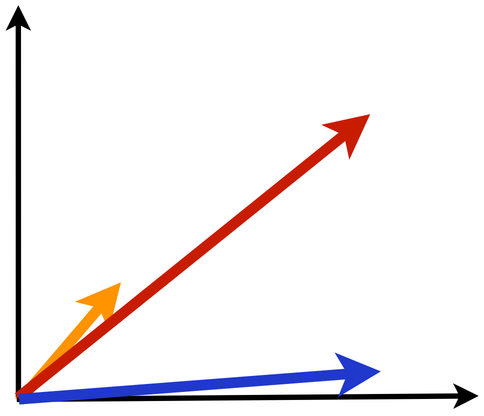
How much “non-equilibrium” remains at large distances?

How much “quantum” remains at large distances?

“What is Non-Equilibrium About It?”

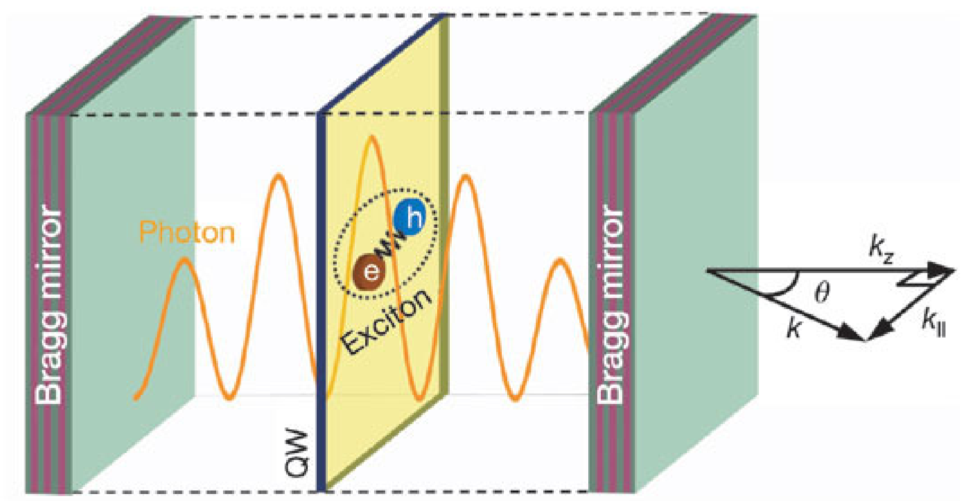


$$e^{i\Gamma[\Phi]} = \int \mathcal{D}\delta\Phi e^{iS_M[\Phi+\delta\Phi]}$$

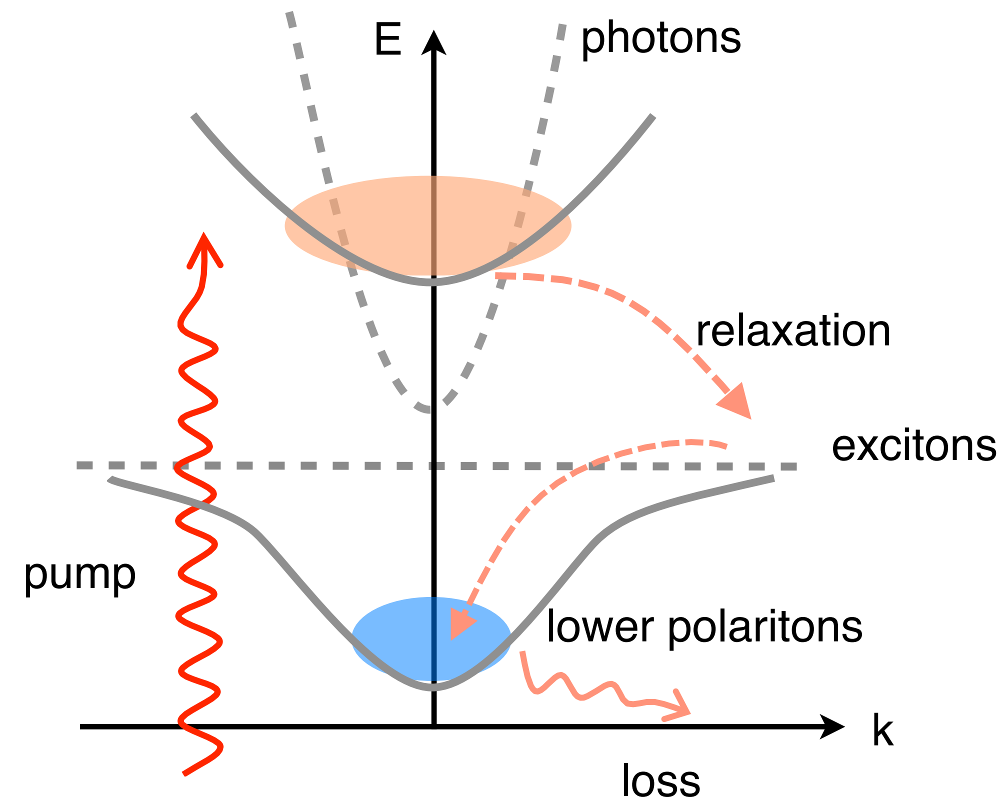


An Example: Exciton-Polariton Systems

Kasprzak et al., Nature 2006



Imamoglu et al., PRA 1996



- phenomenological description: stochastic driven-dissipative Gross-Pitaevskii-Eq

$$i\partial_t\phi = \left[-\frac{\nabla^2}{2m} - \mu + i(\gamma_p - \gamma_l) + (\lambda - i\kappa)\,|\phi|^2\right]\phi + \zeta$$

propagation

pump & loss rates

elastic collisions

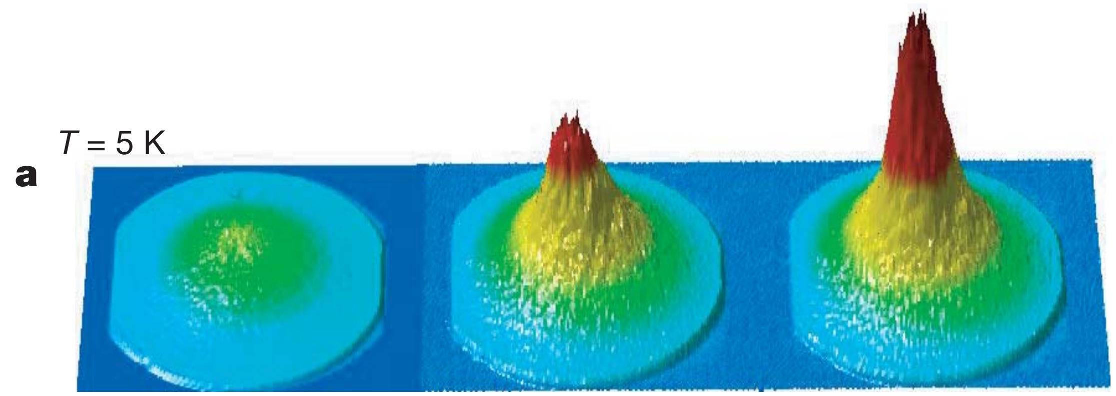
two-body loss

$$\langle\zeta^*(t,\mathbf{x})\zeta(t',\mathbf{x}')\rangle = \gamma\delta(t-t')\delta(\mathbf{x}-\mathbf{x}')$$

microscopic derivation and linear fluctuation analysis:
Szymanska, Keeling, Littlewood PRL (04, 06); PRB (07));
Wouters, Carusotto PRL (07,10)

An Example: Exciton-Polariton Systems

- Bose condensation seen despite non-equilibrium conditions



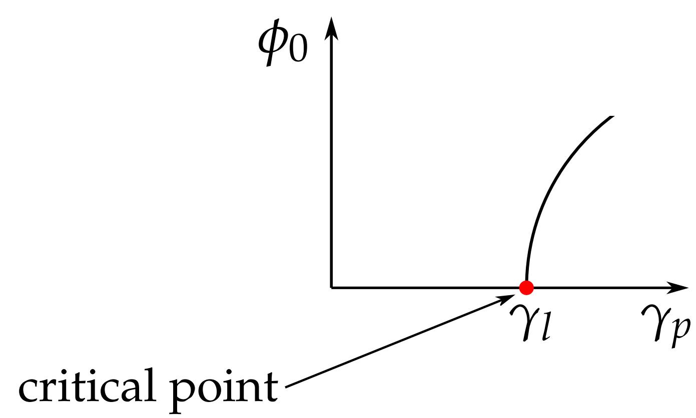
Kasprzak et al., Nature 2006

- stochastic driven-dissipative Gross-Pitaevskii-Eq

$$\cancel{i\partial_t}\phi = \left[-\cancel{\frac{\nabla^2}{2m}} - \mu + i(\gamma_p - \gamma_l) + (\lambda - i\kappa)\,|\phi|^2\right]\phi + \cancel{\zeta}$$

Szymanska, Keeling, Littlewood PRL (04, 06); PRB (07)); Wouters, Carusotto PRL (07,10)

- mean field
  - neglect noise
  - homogeneous solution $\phi(\mathbf{x},t) = \phi_0$



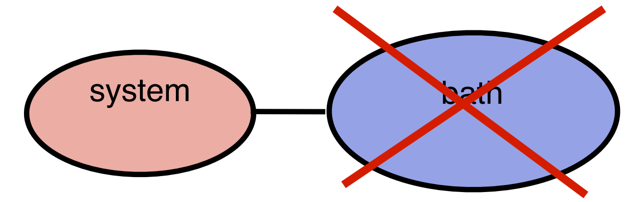
- naively, just as Bose condensation in equilibrium!
- Q: What is “non-equilibrium” about it?

Microscopic Description: Quantum Master Equation

- Quantum master equation:

Lindblad operators

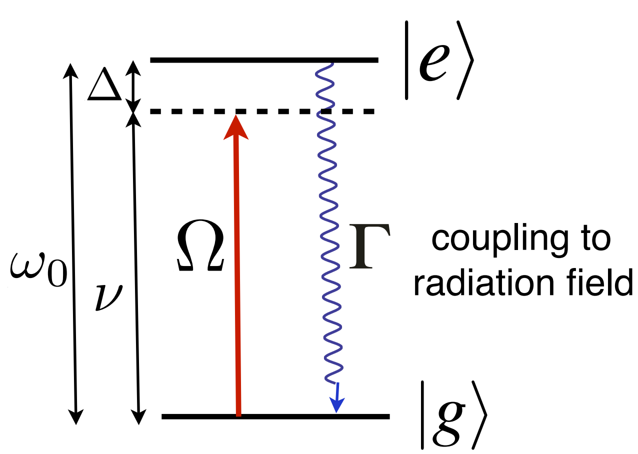
$$\partial_t \rho = \underbrace{-i[H,\rho]}_{\text{coherent evolution}} + \kappa \underbrace{\sum_i \left(L_i \rho L_i^\dagger - \tfrac{1}{2}\{L_i^\dagger L_i, \rho\}\right)}_{\text{dissipative evolution}}$$



$$\equiv \mathcal{L}[\rho]$$ -- Liouvillian operator

- Lindblad form: most general time-local evolution of density matrix
- validity: Born-Markov and rotating wave approximations -> driven systems

- single-body example: driven-dissipative two-level system



- simple fact: drive essential to access upper level

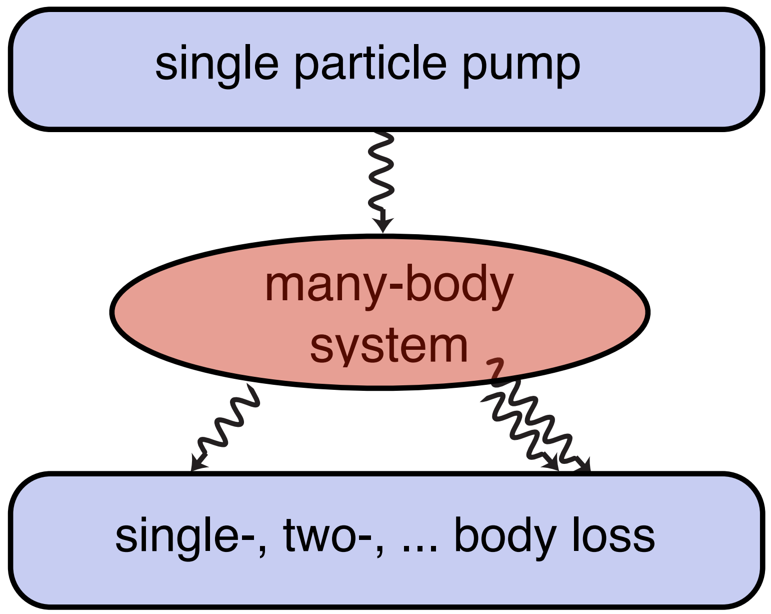
$$H = (|e\rangle, |g\rangle) \begin{pmatrix} \Delta & \Omega \\ \Omega & 0 \end{pmatrix} \begin{pmatrix} \langle e| \\ \langle g| \end{pmatrix}$$

$$L_i = |g\rangle\langle e| = \sigma^-$$

- Implications:
  - no guarantee for detailed balance
  - no obedience of the second law of thermodynamics (state purification)

Microscopic Description: Many-Body Systems

- generic microscopic many-body model:



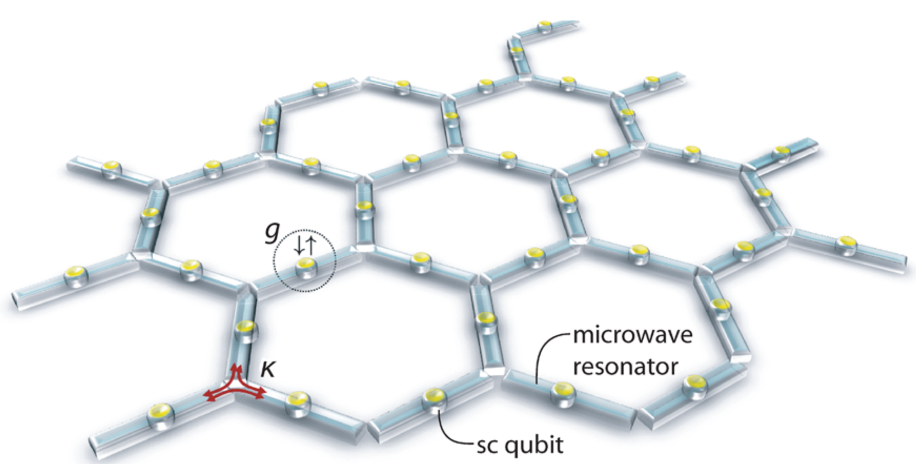
$$\partial_t \rho = -i[H, \rho] + \mathcal{D}[\rho] \equiv \mathcal{L}[\rho]$$

$$H = \int_{\mathbf{x}} \hat{\phi}^\dagger_{\mathbf{x}} \left(\tfrac{\triangle}{2M} - \mu\right) \hat{\phi}_{\mathbf{x}} + \tfrac{\lambda}{2} (\hat{\phi}^\dagger_{\mathbf{x}} \hat{\phi}_{\mathbf{x}})^2$$

$$\mathcal{D}[\rho] = \underbrace{\gamma_p \int_{\mathbf{x}} [\hat{\phi}^\dagger_{\mathbf{x}}\, \rho\, \hat{\phi}_{\mathbf{x}} - \tfrac{1}{2}\{\hat{\phi}_{\mathbf{x}} \hat{\phi}^\dagger_{\mathbf{x}}, \rho\}]}_{\text{single particle pump}} + \underbrace{\gamma_l \int_{\mathbf{x}} [\hat{\phi}_{\mathbf{x}}\, \rho\, \hat{\phi}^\dagger_{\mathbf{x}} - \tfrac{1}{2}\{\hat{\phi}^\dagger_{\mathbf{x}} \hat{\phi}_{\mathbf{x}}, \rho\}]}_{\text{single particle loss}} +$$

$$\underbrace{\kappa \int_{\mathbf{x}} [\hat{\phi}^2_{\mathbf{x}}\, \rho\, \hat{\phi}^{\dagger\,2}_{\mathbf{x}} - \tfrac{1}{2}\{\hat{\phi}^{\dagger\,2}_{\mathbf{x}} \hat{\phi}^2_{\mathbf{x}}, \rho\}]}_{\text{two particle loss}}$$

- quantum description of XP systems
- long wavelength limit of microcavity arrays: driven open Bose-Hubbard model (w/ incoherent pump)



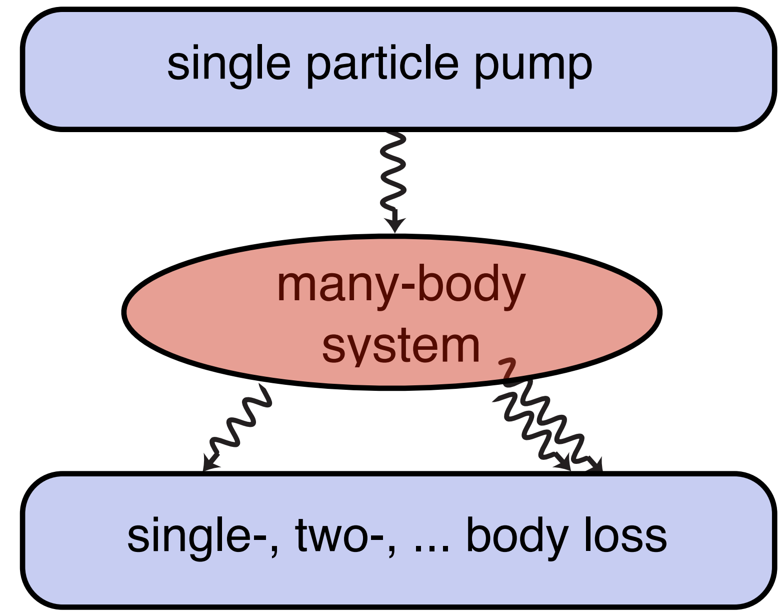
Hartmann et al.
Koch et al., PRA 2010
Houck, Türeci, Koch, Nat. Phys. 2012

➡ Methods to efficiently deal with these equations are scarce!

Theoretical Approach

- generic microscopic many-body model:

$$\partial_t \rho = -i[H,\rho] + \mathcal{D}[\rho] \equiv \mathcal{L}[\rho]$$



- Many-body problem: evaluation strategy

Review: L. Sieberer, M. Buchhold, SD, *Keldysh Field Theory for Driven Open Quantum Systems*, arxiv (2015)

Many-Body Master Equation → translation table → Microscopic Markovian Dissipative Action → Opens up toolbox of quantum field theory to DOQS

$$\partial_t \rho = -i[H,\rho] + \mathcal{D}[\rho] \quad \Longleftrightarrow \quad Z = \int \mathcal{D}(\Phi_+,\Phi_-) e^{\mathrm{i}S_M[\Phi_+,\Phi_-]}$$

many-body master equation

Keldysh real time functional integral

- symmetries
- control of IR fluctuations in many-body problem, eg. (F)RG
- flexible choice of degrees of freedom
- versatile: bosons, fermions, spins

M. Maghrebi, A. V. Gorshkov, PRB (2016)

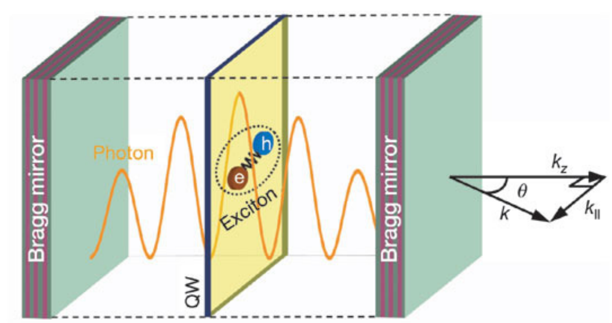
“What is non-equilibrium about it?”

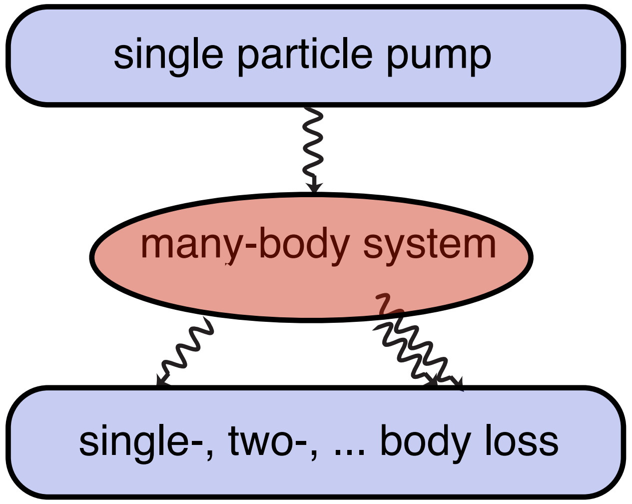
even in stationary state!

- typical differences to closed equilibrium systems:

  - absence of number conservation

    ➡ compatible with thermal equilibrium (Caldeira-Leggett Models)





  - absence of energy conservation

    ➡ driven system, incompatible with thermal equilibrium

“What is non-equilibrium about it?”: Absence of energy conservation

- Energy conservation: equilibrium dynamics generated by a time-independent Hamiltonian

➡ formally: symmetry of Keldysh action under

L. Sieberer, A. Chiochetta, U. Tauber, A. Gambassi, SD, PRB (2015)

previously: classical limit; H. K. Janssen (1976); C. Aron et al, J Stat. Mech (2011)

$$\mathcal{T}_\beta \Phi_\pm(t,\mathbf{x}) = \Phi^*_\pm(-t \pm \mathrm{i}\beta/2, \mathbf{x})$$

$$\Phi_\pm = \begin{pmatrix} \phi_\pm \\ \phi^*_\pm \end{pmatrix}$$

- compact functional formulation of KMS boundary condition

S. Jakobs, M. Pletyukhov, H. Schoeller, J. Phys. A Math. Theor. (2010).

- implies equilibrium conditions: quantum Fluctuation-Dissipation relations of all orders
- non-equilibrium detector: master equation action violates this symmetry explicitly
- offers a geometric interpretation

Geometric Interpretation

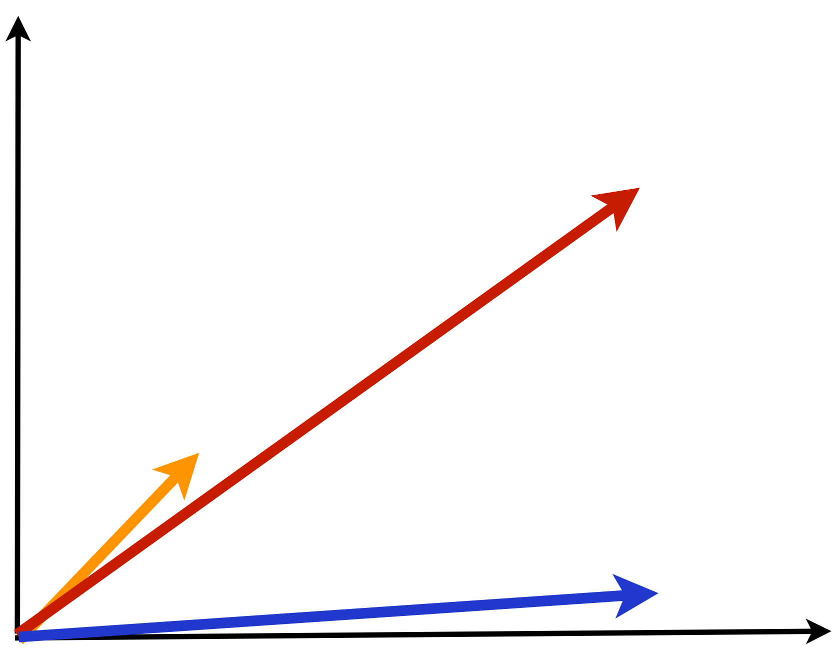
- couplings spanning the Keldysh action lie in the complex plane

$$\underbrace{\partial_t \rho = -i[H,\rho]}_{\Leftrightarrow\, S_H} + \underbrace{\mathcal{D}[\rho]}_{\Leftrightarrow\, S_{\mathcal{D}}} \quad \Longleftrightarrow \quad Z = \int \mathcal{D}(\Phi_+,\Phi_-) \mathrm{e}^{\mathrm{i}(S_H[\Phi_+,\Phi_-]+S_{\mathcal{D}}[\Phi_+,\Phi_-])}$$

Im

incoherent/ irrev. dynamics

$\Leftrightarrow S_{\mathcal{D}}$



Re

coherent/ reversible dynamics

$\Leftrightarrow S_H$

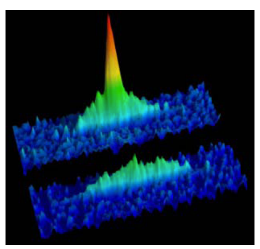
example: two-body processes λ

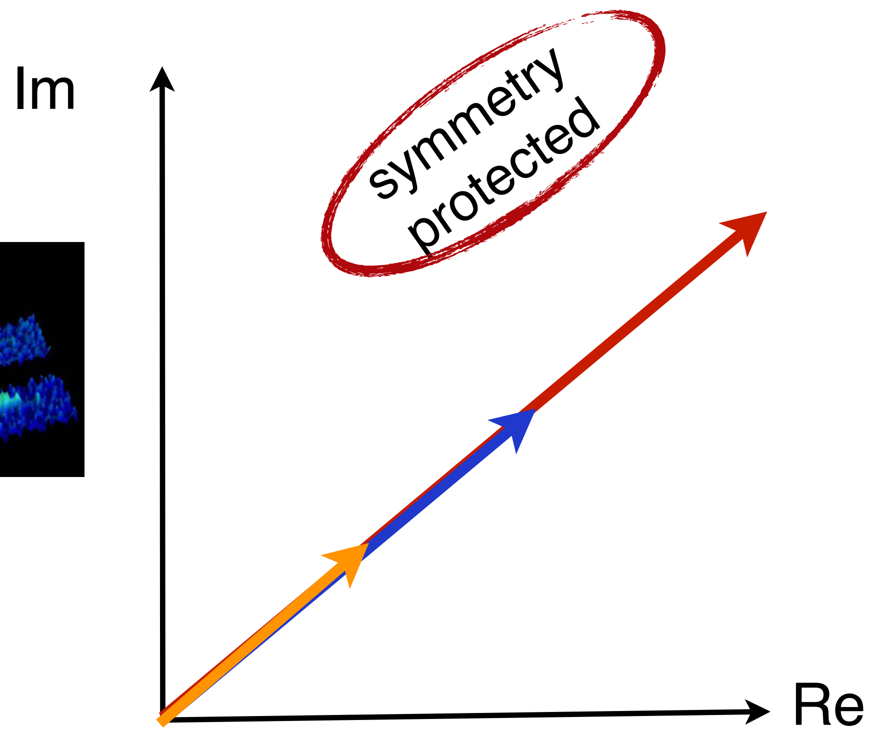
$\mathrm{Re}\lambda$	elastic two-body collisions
$\mathrm{Im}\lambda$	inelastic two-body losses

Review: L. Sieberer, M. Buchhold, SD, *Keldysh Field Theory for Driven Open Quantum Systems*, arxiv (2015)

Equilibrium vs. Non-Equilibrium Dynamics

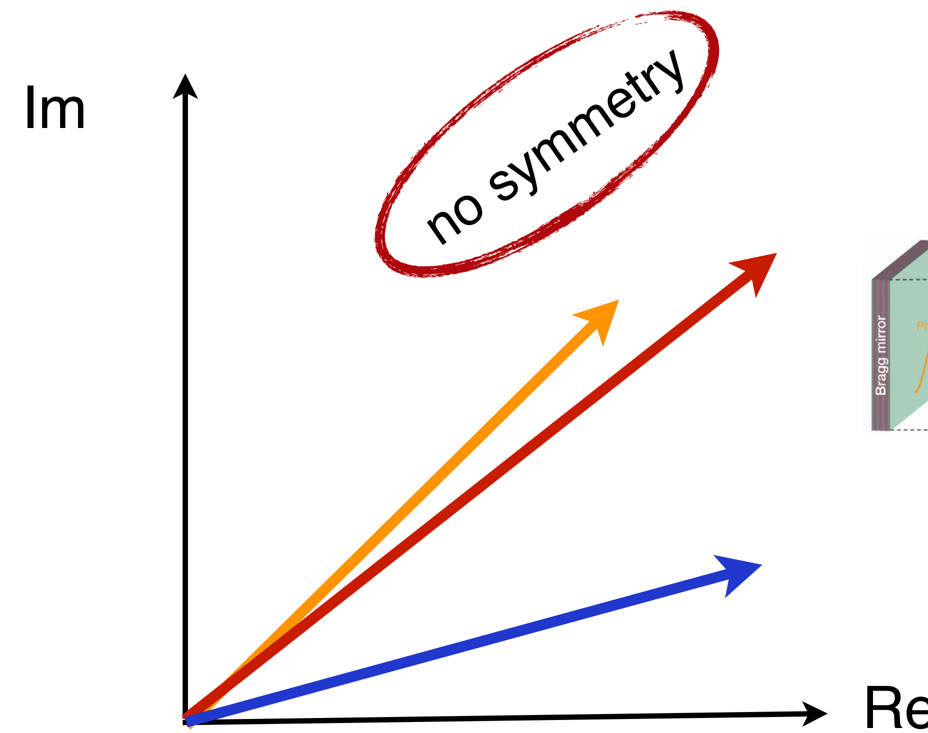
equilibrium dynamics

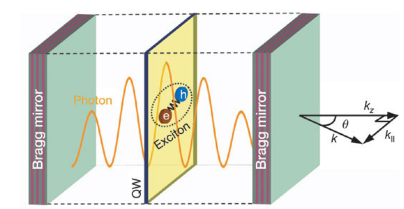




- coherent and dissipative dynamics may occur simultaneously
- but they are not independent

non-equilibrium dynamics



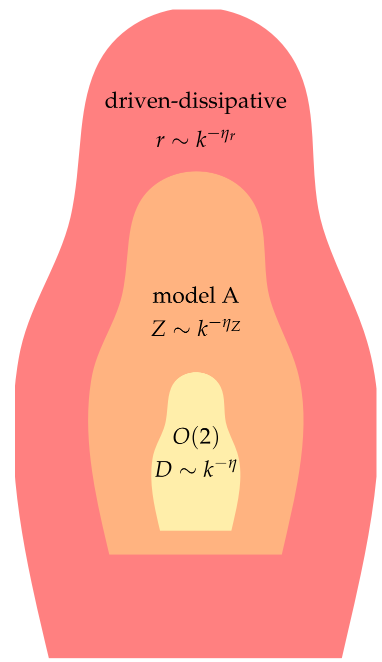


- coherent and dissipative dynamics do occur simultaneously
- they result from different dynamical resources

clever workaround: symmetry in decoupled subsystems, Hafezi, Adhikari, Taylor, PRB (2015)

➡ what are the physical consequences of the spread in the complex plane?

Dynamical Markovian Quantum Criticality



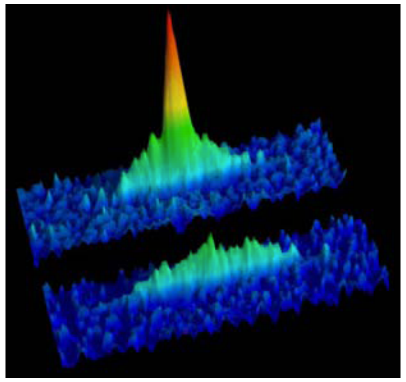
J. Marino, SD, PRL (2016)

Microscopic
Quantum Optics

“Thermodynamic”
Many-body physics

Long wavelength
Statistical mechanics

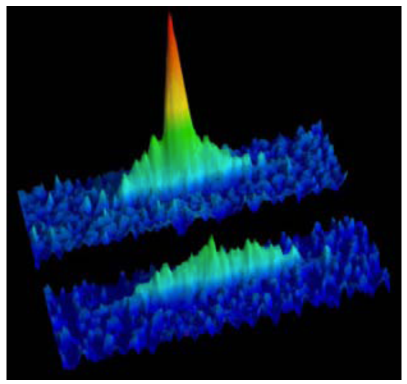
Critical Phenomena and Universality
(Equilibrium)



$\simeq$

Critical Phenomena and Universality (Equilibrium)

- Universality: The art of systematically forgetting about details



Bose-Einstein Condensate

$$\simeq$$

planar magnets

at the critical point

$$\tau = \frac{T - T_c}{T} \to 0$$

- The experimental witnesses: Critical exponents, e.g.

$$\langle \phi^*(r)\phi(0)\rangle \sim \frac{e^{-r/\xi}}{r^{d-2+\eta}}$$

correlation length

$$\xi \sim |\tau|^{-\nu} \to \infty$$

- The exponents:

ν "mass/gap exponent"

η "anomalous dimension"

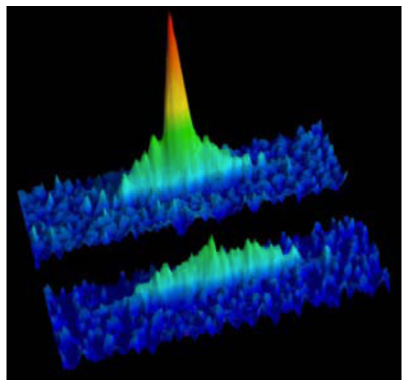
nontrivial statement:

no more independent exponents * than these!

* finite T equilibrium

Critical Phenomena and Universality (Equilibrium)

- Universality: The art of systematically forgetting about details

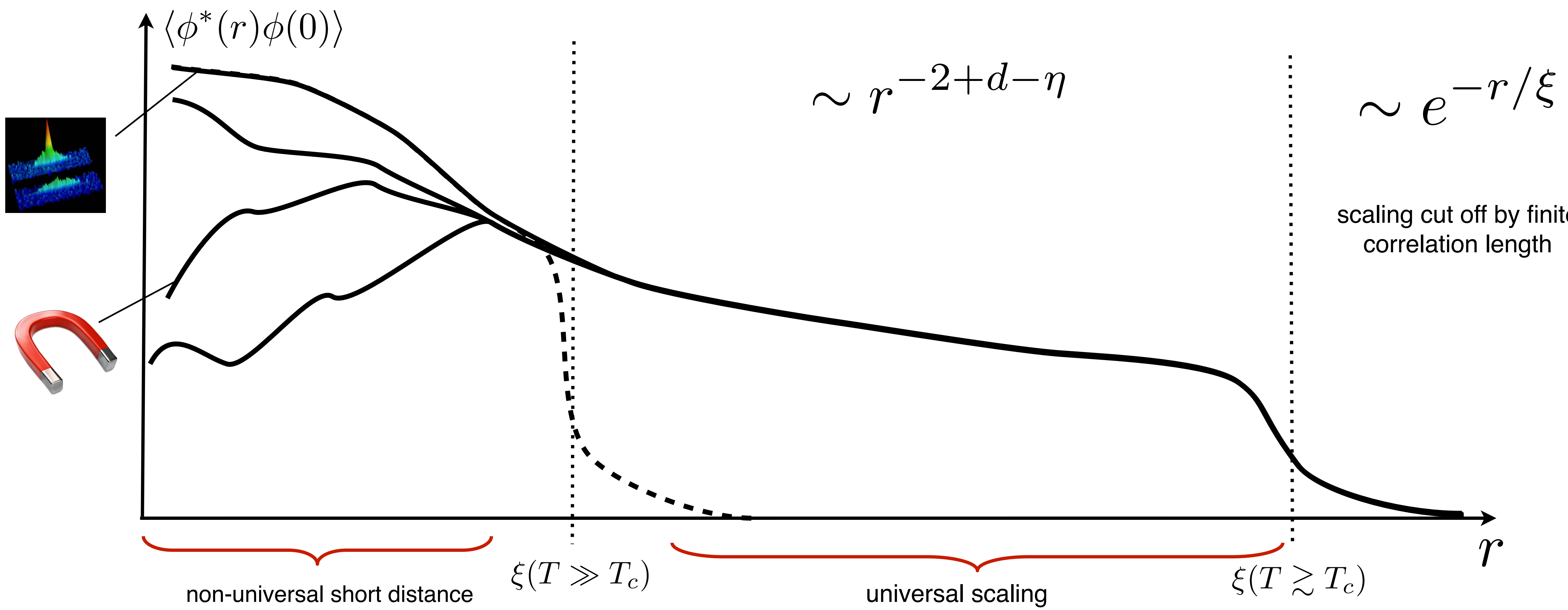


Bose-Einstein Condensate

$$\simeq$$

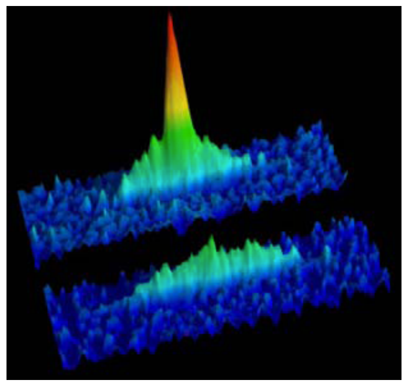
planar magnets

- The physical picture: universality induced by divergent correlation length



Critical Phenomena and Universality (Equilibrium)

- Universality: The art of systematically forgetting about details



Bose-Einstein Condensate

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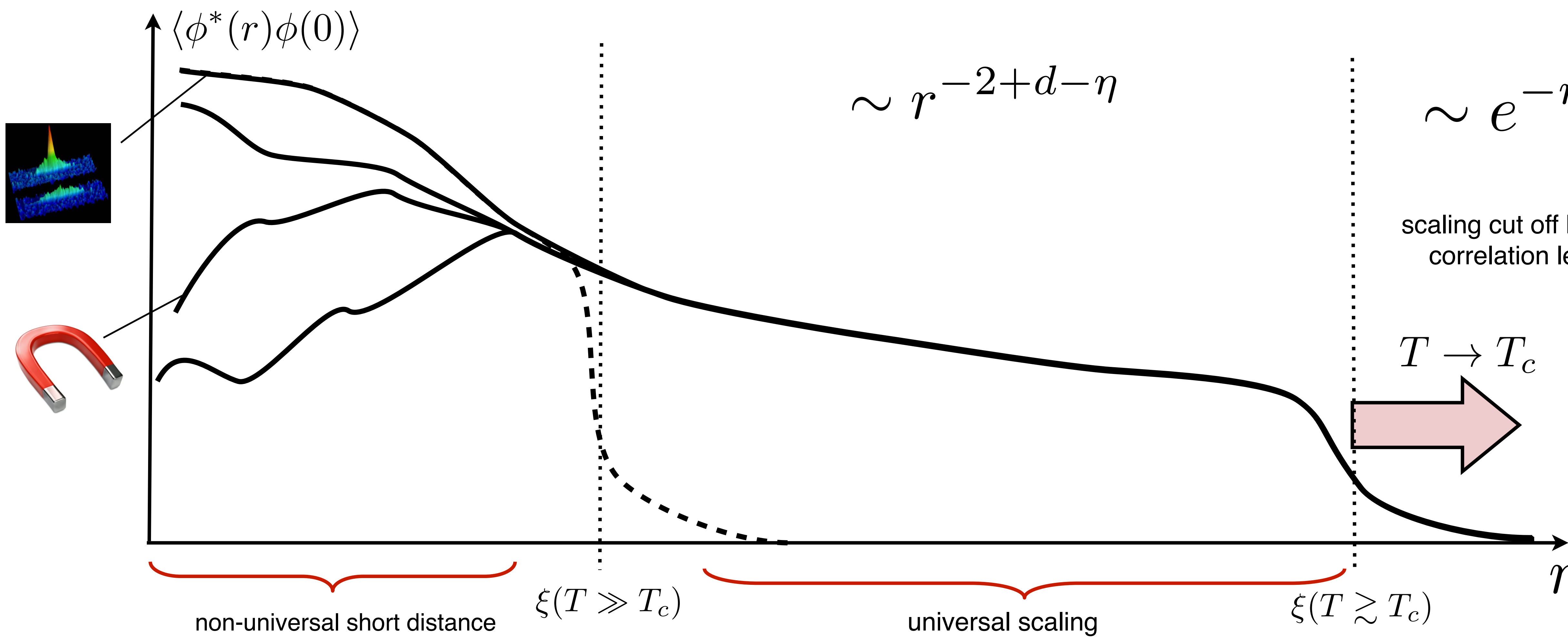
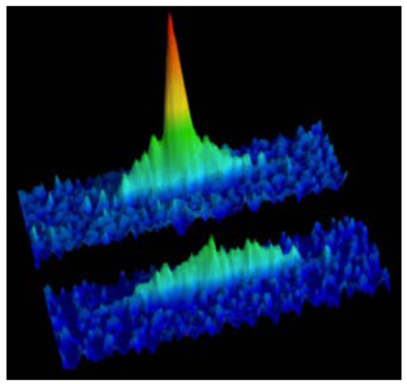
- The physical picture: universality induced by divergent correlation length



Critical Phenomena and Universality (Equilibrium)

- Universality: The art of systematically forgetting about details



Bose-Einstein Condensate

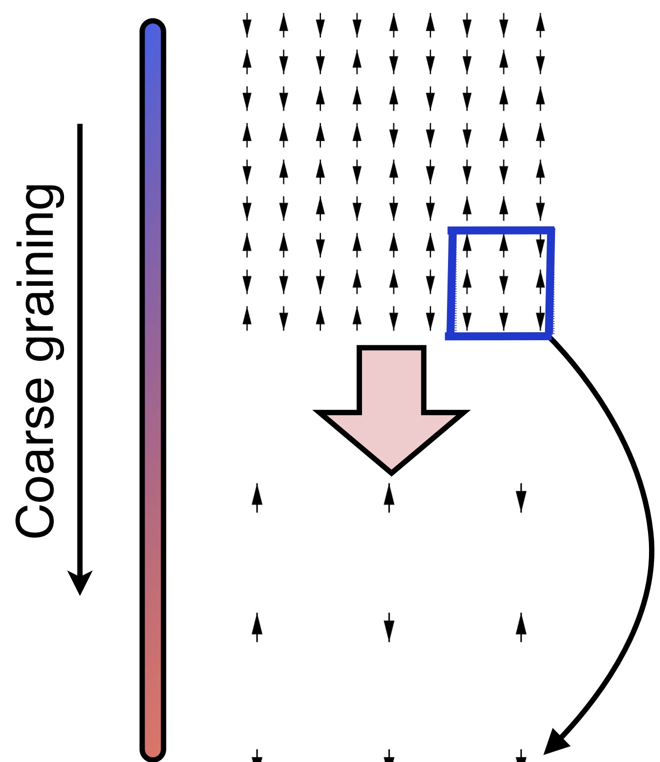
$\simeq$

planar magnets

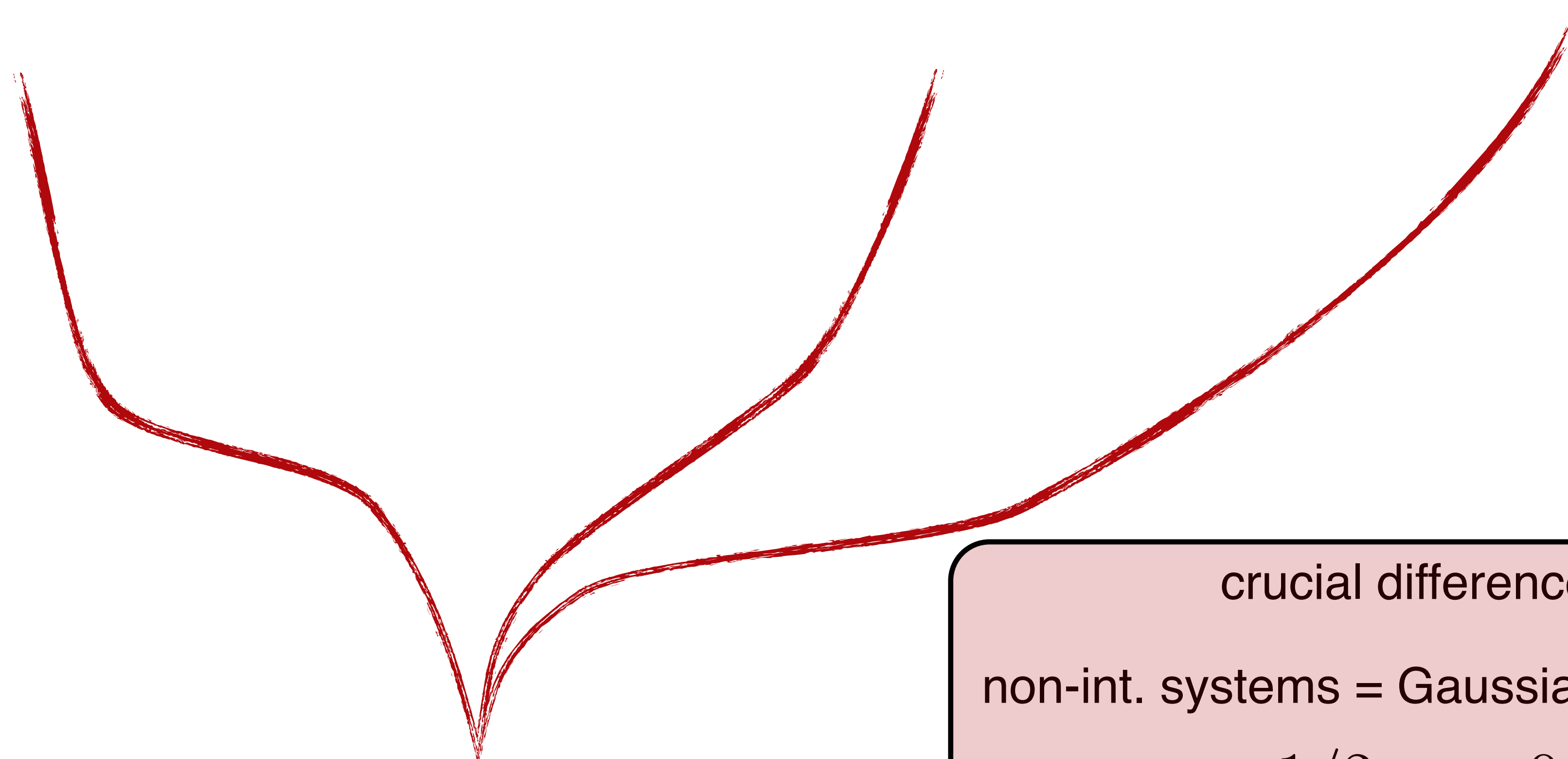
other systems...

- The description: Renormalization group

UV: microscopic physics



IR: long-wavelength physics



Wilson-Fisher fixed point

crucial difference:

non-int. systems = Gaussian fixed point

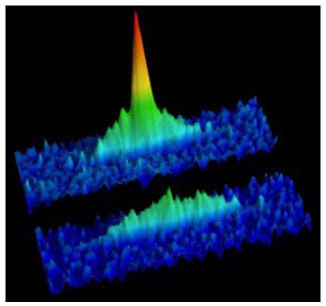
$$\nu = 1/2, \eta = 0$$

interacting systems = WF fixed point

ν, η non-rational

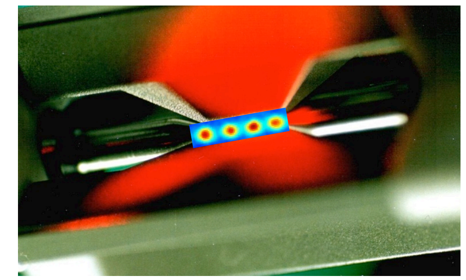
Universality Classes (Equilibrium)

- Universality classes: Memory of symmetries is kept



$\simeq$

$\neq$



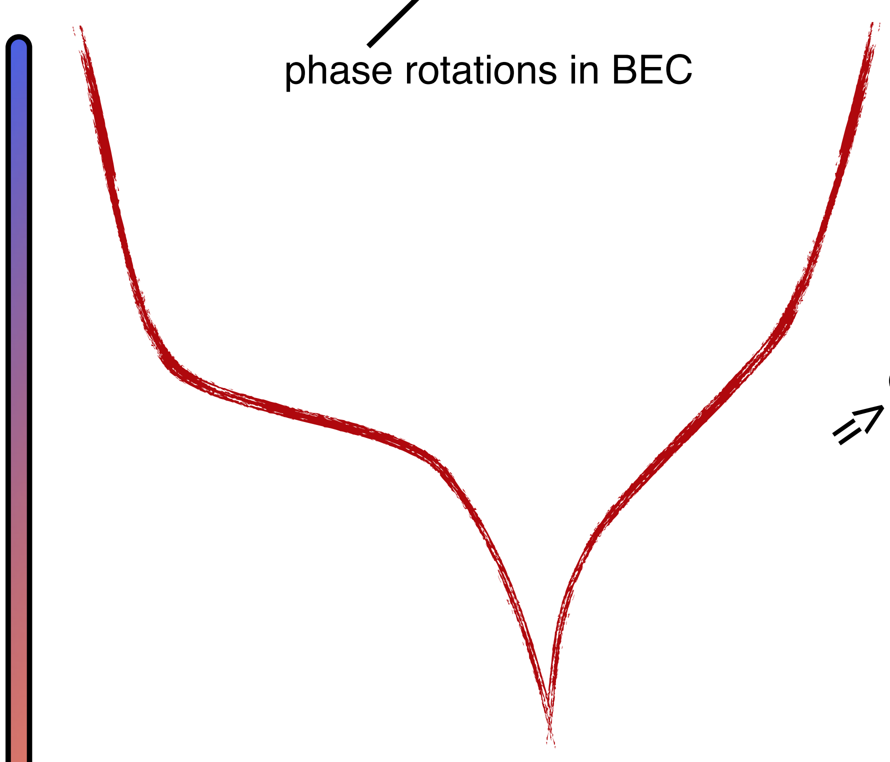
$\simeq$

Bose-Einstein Condensate | planar magnets | trapped ions | liquid-gas transition in carbon-dioxide

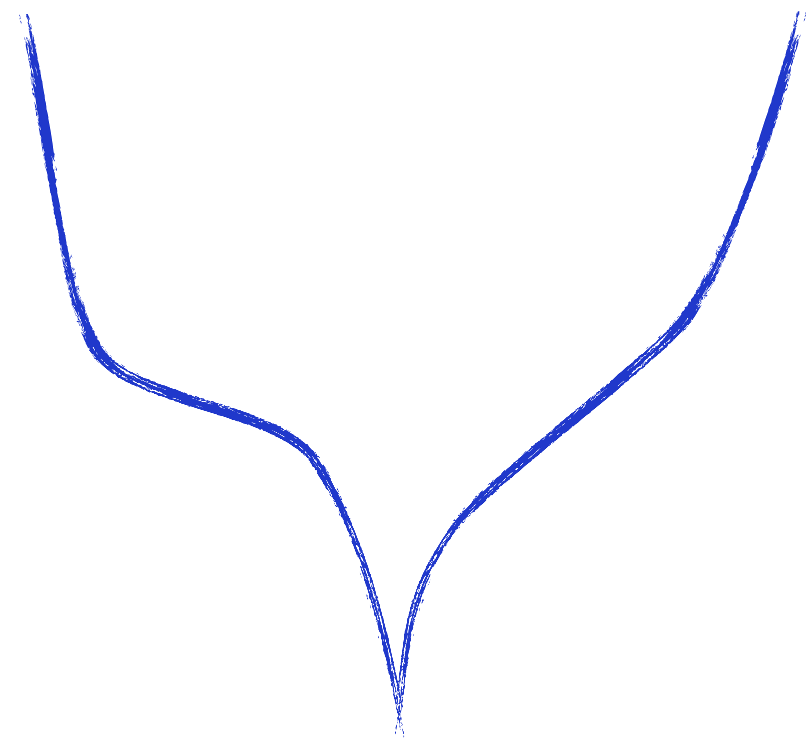
- Symmetries: $U(1) \simeq O(2)$ $\qquad$ Z_2

phase rotations in BEC

Coarse graining



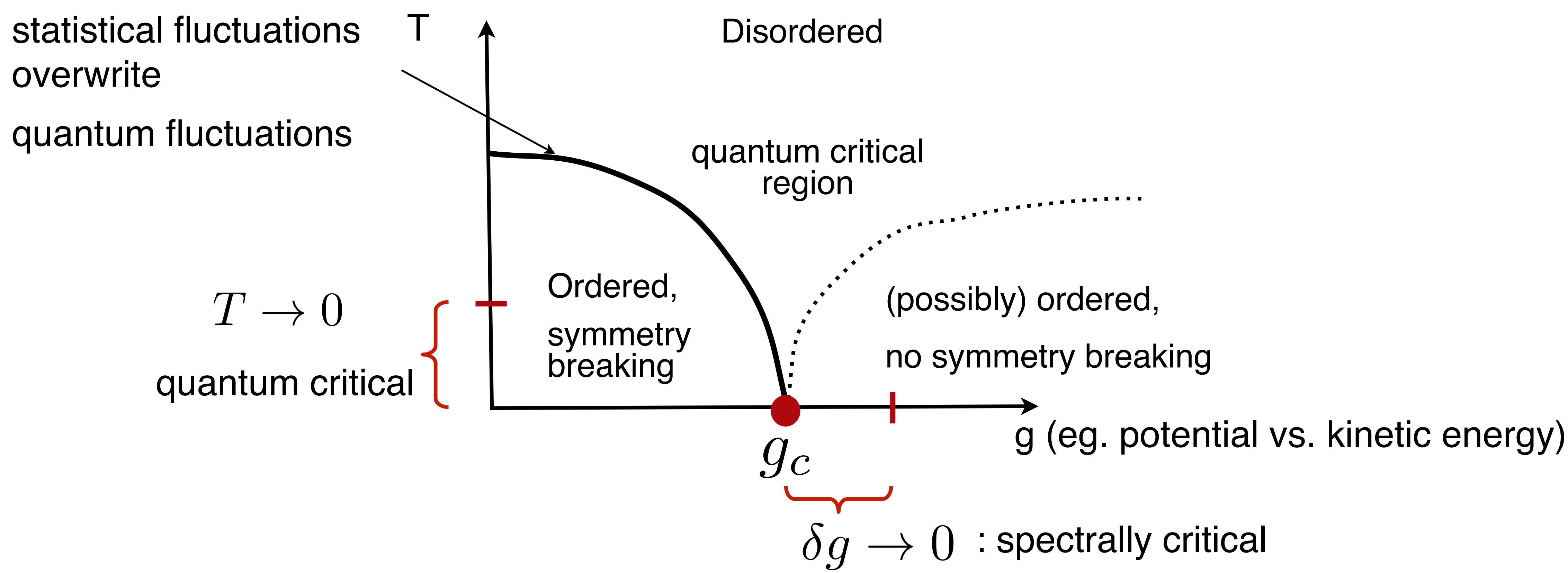
~ 80 stable elements
=> O(10^10) possible compounds
~ 10^23 particles
but only a handful universality classes



"O(2) universality class" $\qquad$ "Ising universality class"

Classical vs. Quantum Criticality

- generic quantum phase diagram



- double fine tuning, temperature is relevant perturbation to the quantum critical point

- quantum critical scaling for

$$T \ll \omega \ll \omega_G$$

quantum　　　　non-gaussian

From Micro- to Macrophysics: Functional RG

microphysics

macrophysics

Many-Body Master Equation → 1-1 mapping → Keldysh functional integral → 1-1 mapping → Keldysh Functional Renormalization Group

$$\partial_t \rho = -i[H, \rho] + \mathcal{D}[\rho]$$

operator representation

$$\mathrm{e}^{\mathrm{i}\Gamma[\Phi]} = \int \mathcal{D}\delta\Phi \mathrm{e}^{\mathrm{i}S_M[\Phi+\delta\Phi]}$$

functional integral representation

$$\partial_k \Gamma_k = \frac{\mathrm{i}}{2} \mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k\right.$$

functional differential equation rep.

Wetterich, 93

closed system Keldysh:
Gasenzer, Pawlowski,PLB 08;
Berges, Hoffmeister, Nucl. Phys. B, 09

open system Keldysh review
Sieberer, Buchhold, SD, arxiv (2015)

From Micro- to Macrophysics: Functional RG

microphysics

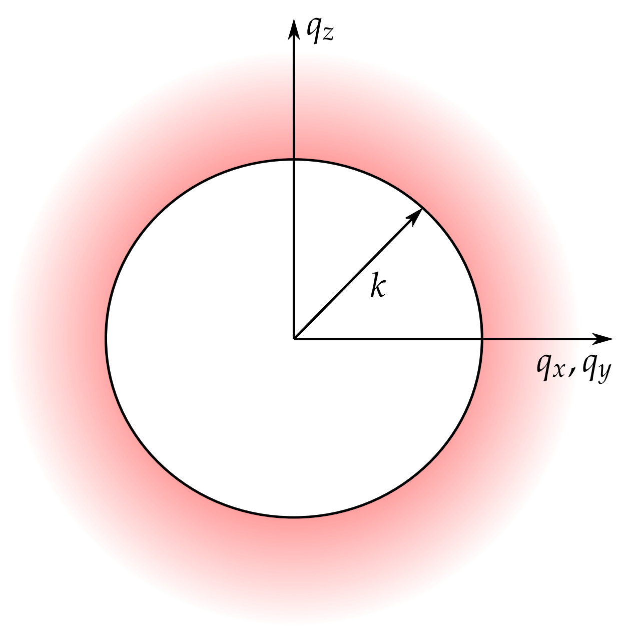
macrophysics

- how does it work?
  Smooth interpolation

$$\partial_k \Gamma_k = \frac{\mathrm{i}}{2} \mathrm{Tr} \left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_k R_k \right]$$

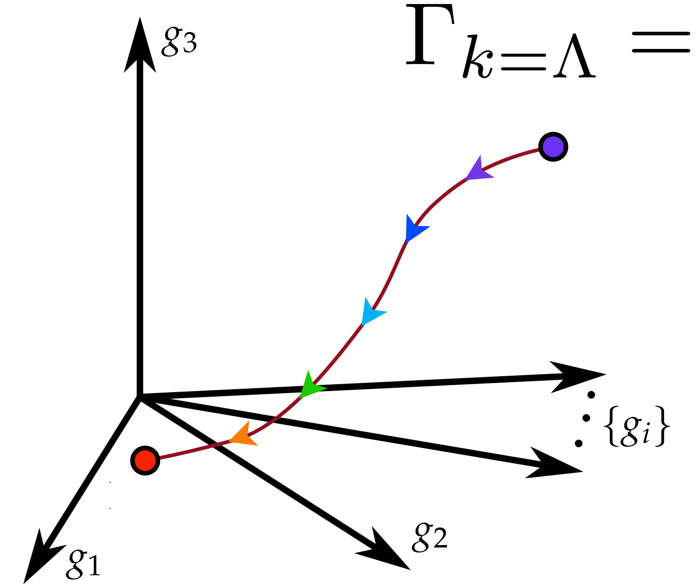
second field variation

infrared regulator



coarse graining in real space = integrating out high modes in momentum space

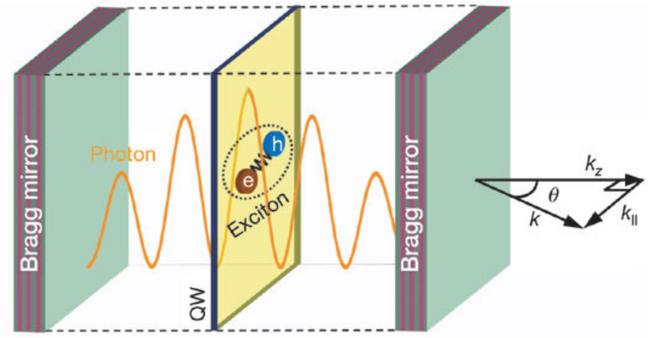
$\Gamma_{k=\Lambda} = S$ Markovian dissipative action



$\Gamma_{k=0} = \Gamma$ full effective action

mode elimination induces RG flow of coupling of effective action

Driven Classical Criticality

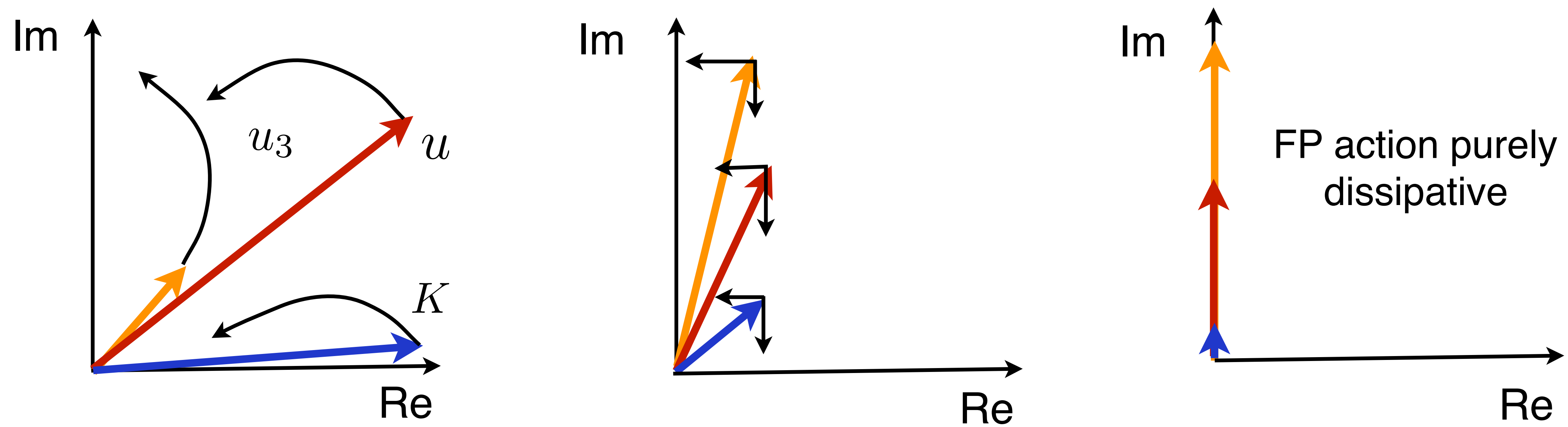


L. Sieberer, S. Huber, E. Altman, SD,
PRL 110, 195301 (2013) and PRB 89, 134310 (2014);
U. C. Tauber, SD, PRX 4, 021010 (2014)

Classical driven criticality: Schematic RG flow

L. Sieberer, S. Huber, E. Altman, SD, PRL (2013)

- Flow in the complex plane of couplings



non-linear initial flow — linearized IR flow — fixed point

- initial values: $\Gamma_{k\approx\Lambda_0} \approx S$

- universal domain encoding universality class
- scaling of running couplings

$$g = ak^{\eta_a} + ibk^{\eta_b}$$

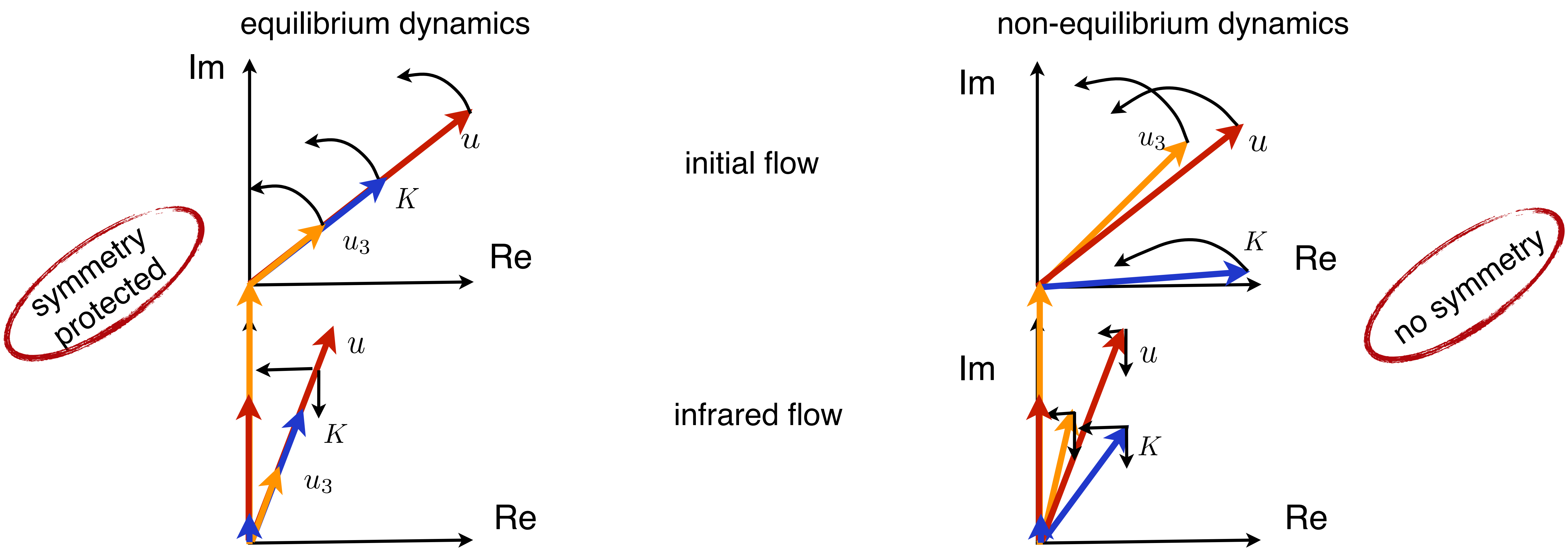
crit. exponent

- key results (classical):

➡ universal decoherence (new independent critical exponent)
➡ asymptotic thermalization
➡ reveals equilibrium vs. non-equilibrium fine structure

Universal decoherence, fine structure, and thermalization

- decoherence <=> purely imaginary fixed point action
- global thermal equilibrium is ensured by symmetry:



- eigenvalue of flow speed

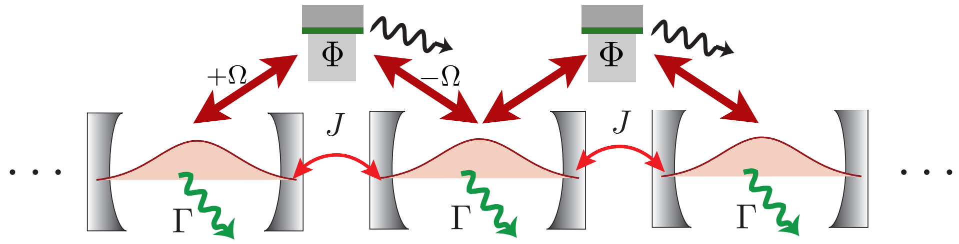
$$\eta_R \approx -0.143$$

- lowest eigenvalue

$$\eta_r \approx -0.101$$

➡ equilibrium and driven systems are in different universality classes
➡ physical reason: independence of coherent and dissipative dynamics
➡ asymptotic thermalization: all couplings aligned on Im axis

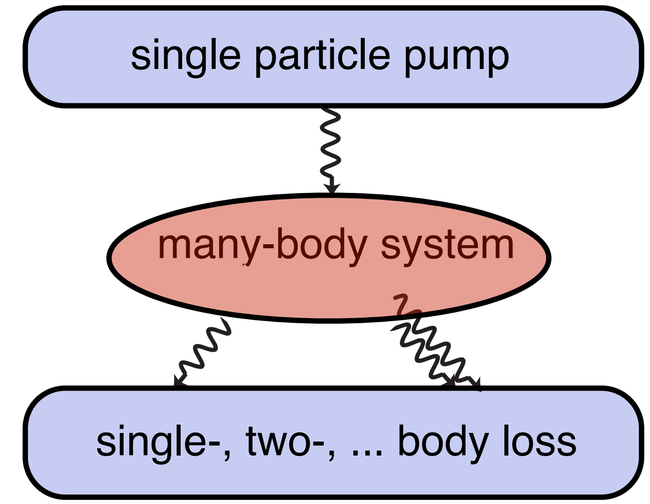
Driven Quantum Criticality



J. Marino, SD, PRL (2016)

Non-equilibrium analogue of quantum criticality (1D)

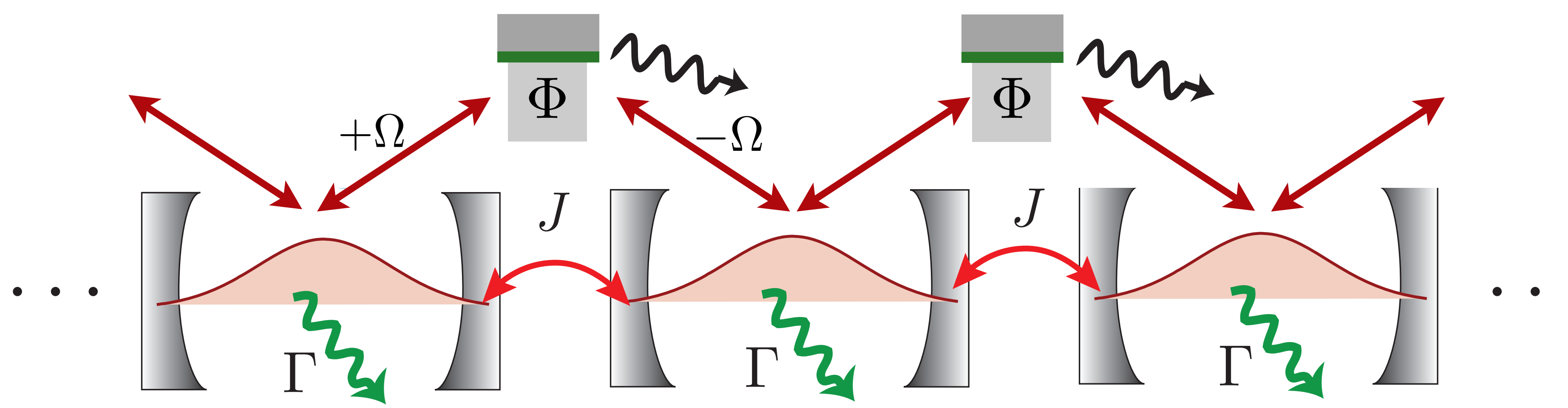
- Lindblad Master equation with additional strong quantum diffusion (1D)



$$\gamma_d \int_{\mathbf{x}} [\nabla a(x)\, \rho\, \nabla a^\dagger(x) - \tfrac{1}{2}\{\nabla a^\dagger(x) \nabla a(x), \rho\}]$$

- possible realization: microcavity arrays

cf. D. Marcos et al., NJP (2012)



$$H_c = \Omega \sum_i \sigma_i^+ (a_i - a_{i+1}) + h.c.$$

$$\Omega \ll \gamma_q$$

$$\mathcal{D}[\rho] = \gamma_q \sum_i [\sigma_i^- \,\rho\, \sigma_i^+ - \tfrac{1}{2}\{\sigma_i^+ \sigma_i^-, \rho\}]$$

Non-equilibrium analogue of quantum criticality (1D)

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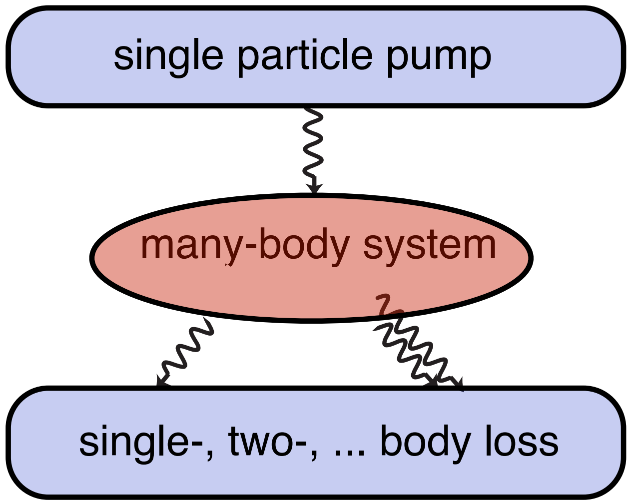
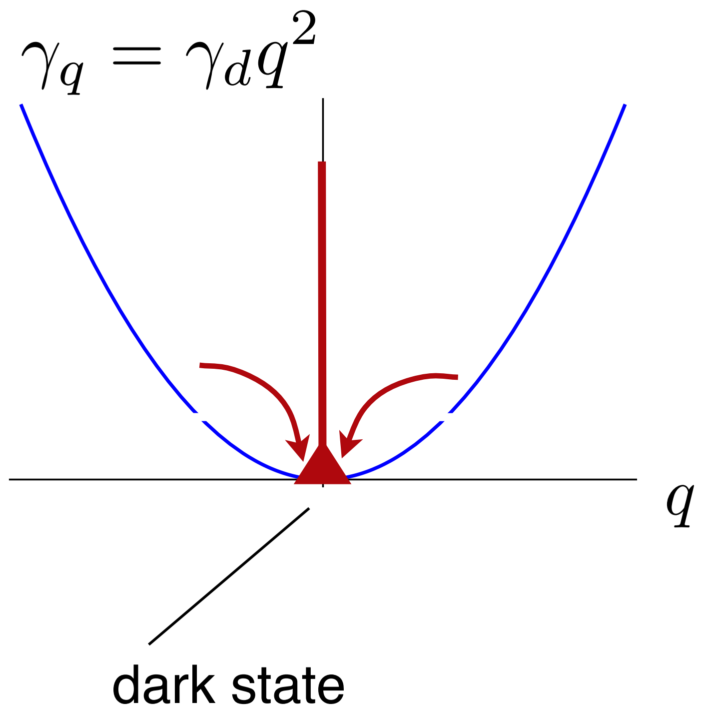


$$\gamma_d \int_{\mathbf{x}} [\nabla a(x)\, \rho\, \nabla a^\dagger(x) - \tfrac{1}{2}\{\nabla a^\dagger(x) \nabla a(x), \rho\}]$$

- physical interpretation: Dark state   number conserving variant: SD et al., Nature Phys. (2008)
  - in Fourier space

$$\int_q \gamma_q [a_q\, \rho\, a_q^\dagger - \tfrac{1}{2}\{a_q^\dagger a_q, \rho\}]$$



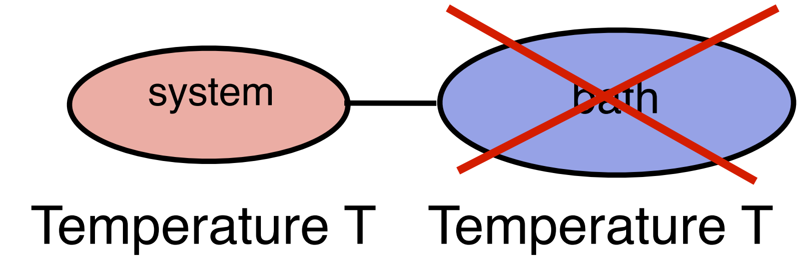
➡ noiseless "dark" state at q=0

➡ favors accumulation of bosons at q=0 ("BEC")

➡ competition w/ interactions yields phase transition

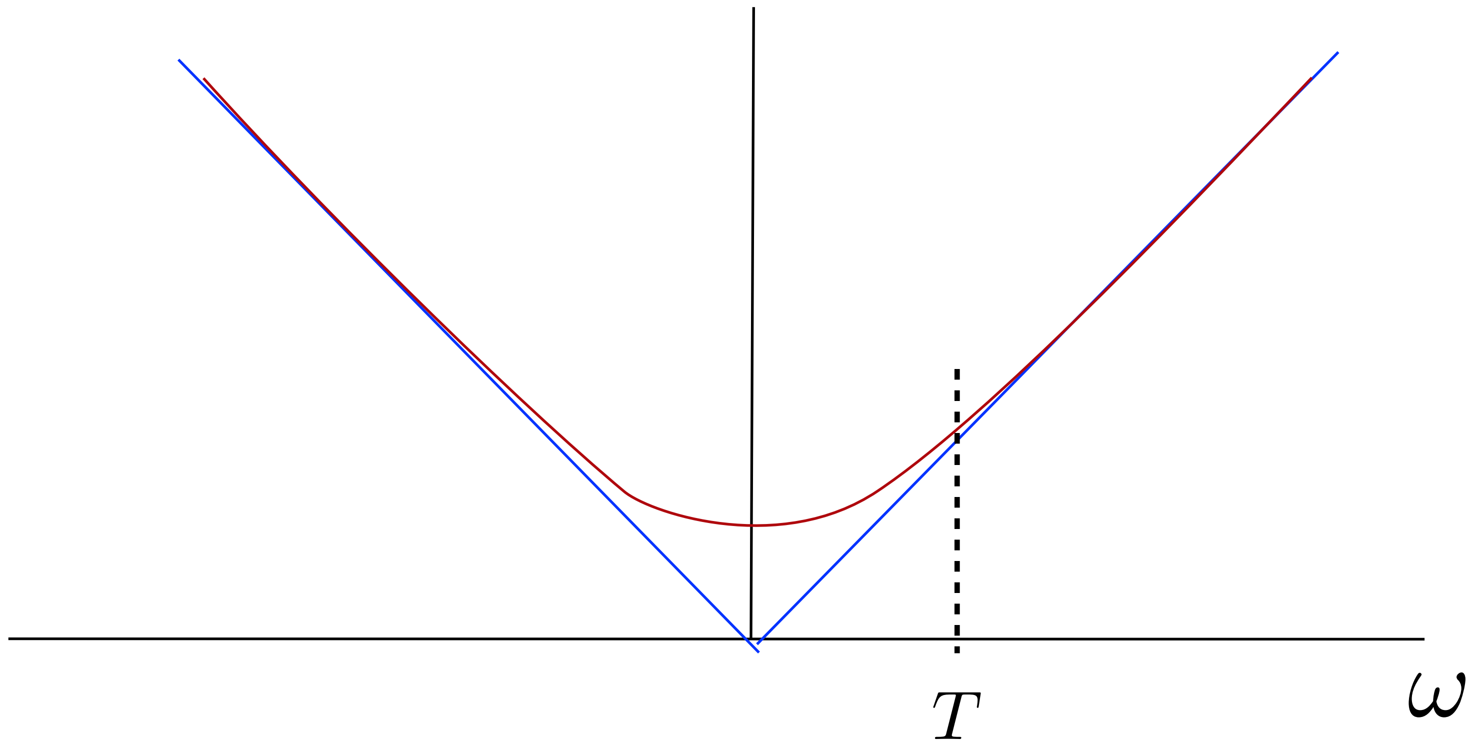
“What is quantum about it?”

- analogy to an equilibrium system: noise level



global equilibrium

$$P^K(\omega) \sim \omega \coth \tfrac{\omega}{2T}$$



- two regimes

$$\omega/2T \ll 1 : \quad P^K(\omega) \approx 2T, \quad P^K(t-t') \sim \delta(t-t')$$

classical/markovian

$$\omega/2T \gg 1 : \quad P^K(\omega) \approx |\omega|, \quad P^K(t-t') \sim (t-t')^{-2}$$

quantum/non-markovian

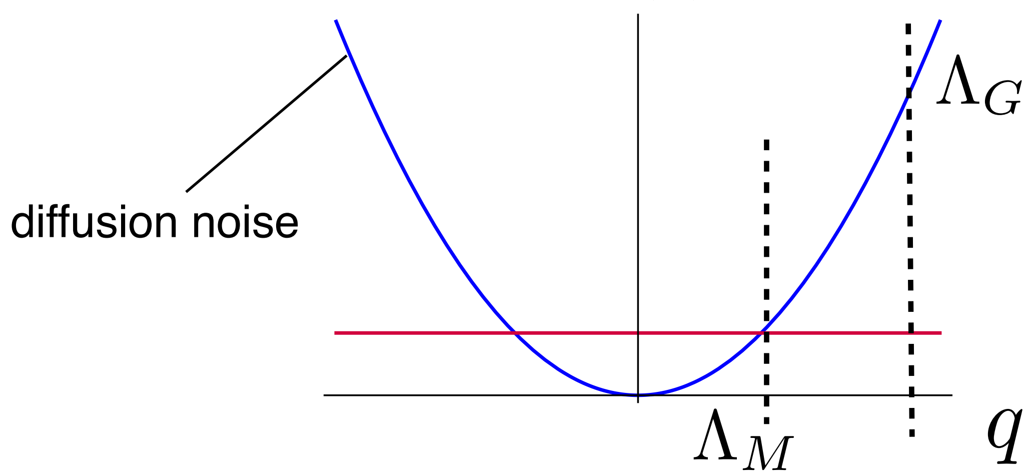
➡ scaling of the noise level
➡ existence of one noiseless mode

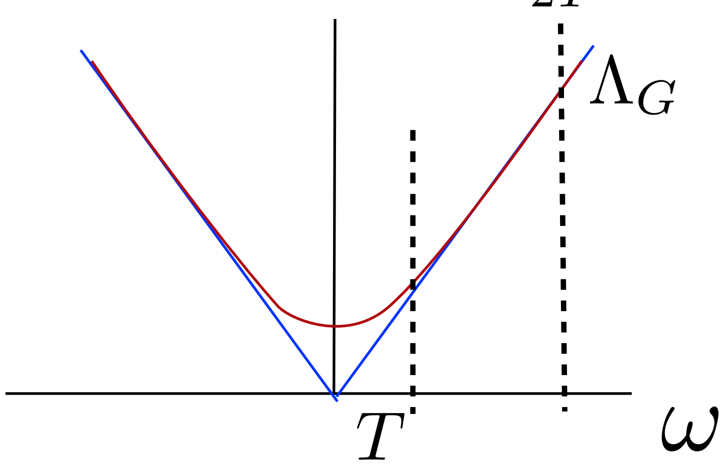
Non-equilibrium analogue of quantum criticality

- strongly momentum dependent noise level

markovian non-equilibrium:
weak noise at long wavelength

equilibrium:
weak noise at long timescales





non-eq variant: cf. Dalla Torre et al., Nat Phys. (2010)

- identical canonical scaling to quantum problem for $z = 2 \quad (\omega \sim q^2)$
- but spatial vs. temporal noise
- anomalous scaling regime: two scales

Ginzburg scale $\Lambda_G \simeq \frac{\kappa}{\gamma_d}$ two-body loss

one-loop perturbative

Markov scale $\Lambda_M \simeq \Lambda_G \left(\frac{\tilde{\gamma}_* + \frac{b_*}{2+a_*}}{2 + \frac{b_*}{2+a_*}} \right)^{\frac{1}{2+a_*}}$

rescaled Markov noise at FP

integration of one-loop flow
cf. Chiochetta, Mitra, Gambassi, arxiv (2014)

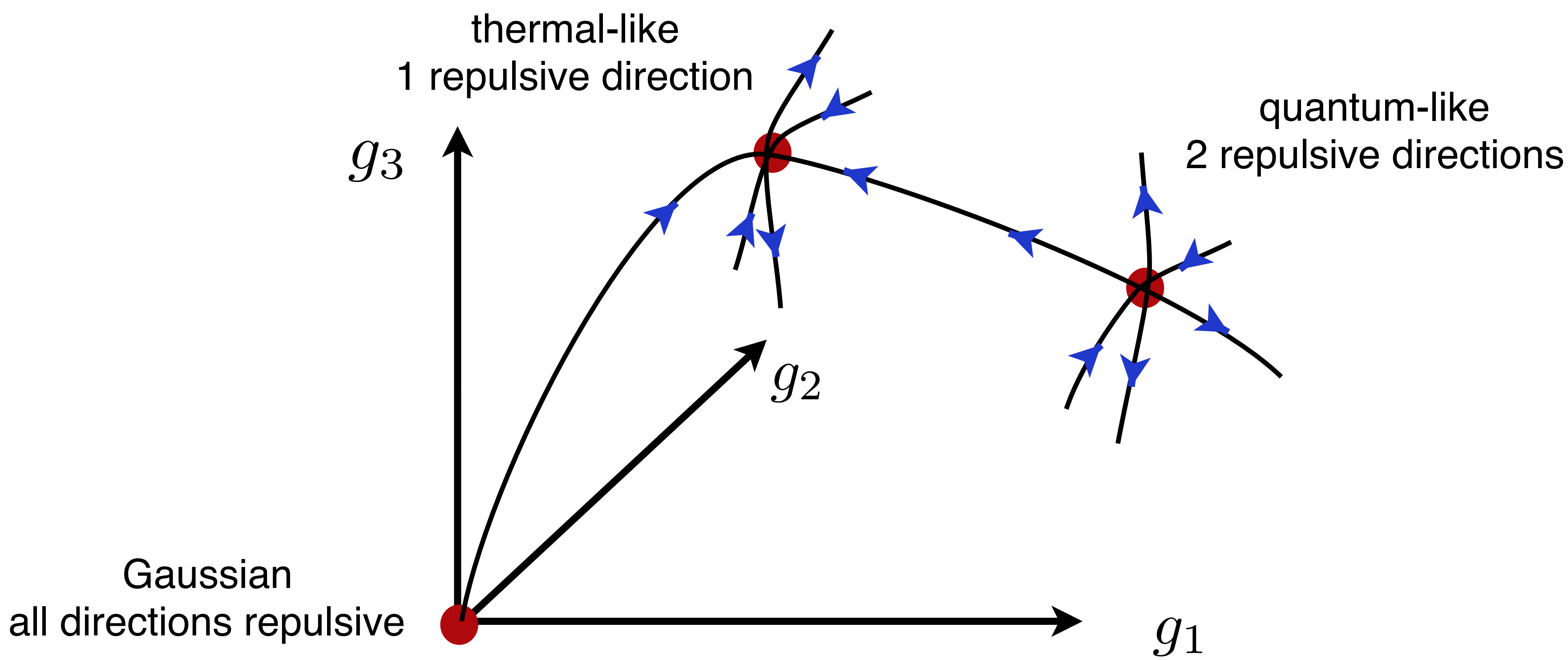
$a_* \approx 0.3$

$b_* \approx 0.2$

- non-gaussian critical scaling for $\Lambda_M \ll \Lambda_G$

(1) No quantum-classical correspondence

- new fixed point with more repulsive directions (fine tuning of loss rate)



- results for critical exponents

		static			dynamic		noise	
	Crit. Exps.	ν	η_{K_R}	η_{K_I}	η_{ZR}	η_{ZI}	η_{γ_d}	η_γ
1+2 dimensions	DD Quantum	0.405	-0.025	-0.025	0.08	0.04	-0.26	$\times$
3 dimensions	DD SC	0.72	-0.22	-0.12	0.16	0	$\times$	-0.16

different degree of divergence of correlations length

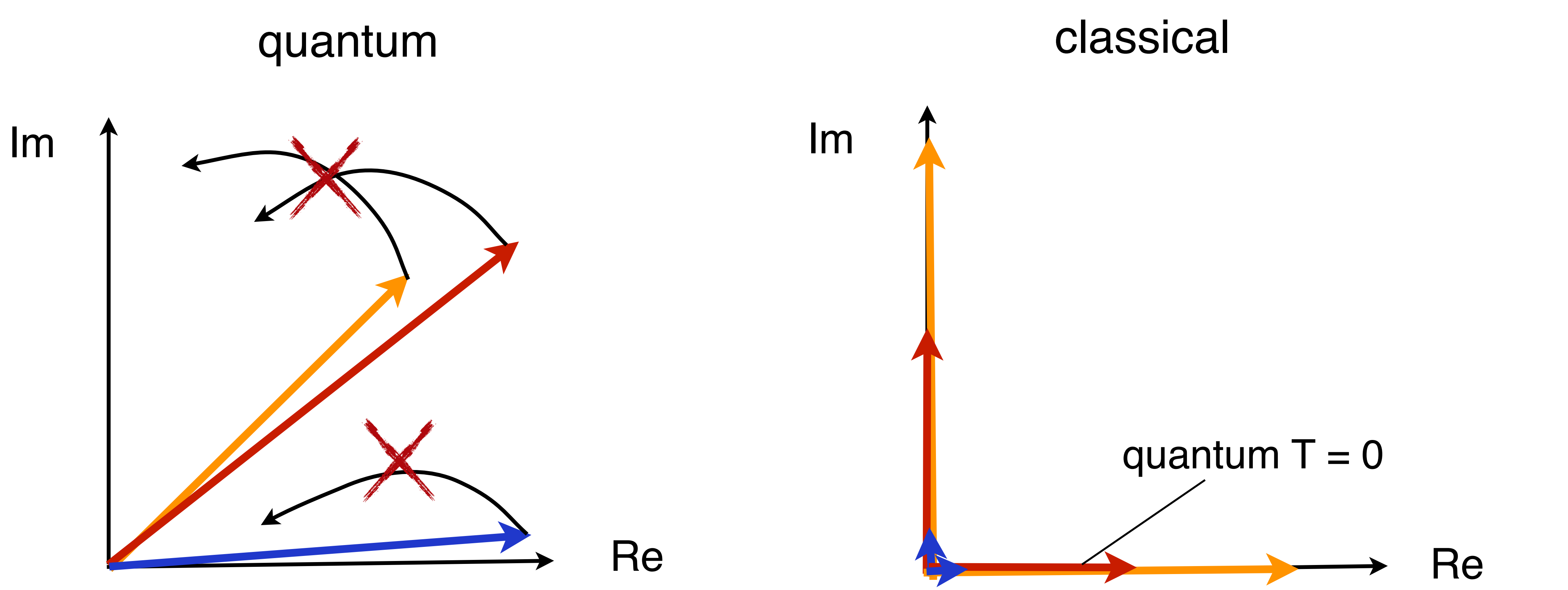
$$\xi \sim (t - t_c)^{-\nu}$$

➡ new non-equilibrium universality class

(2) Absence of Asymptotic Decoherence

- coherent dynamics does not fade out:
- exponent degeneracy: $\eta_A = \eta_D = -0.03$ $\quad A \sim k^{\eta_A}, \quad D \sim k^{\eta_D}$

"effective mass" diffusion



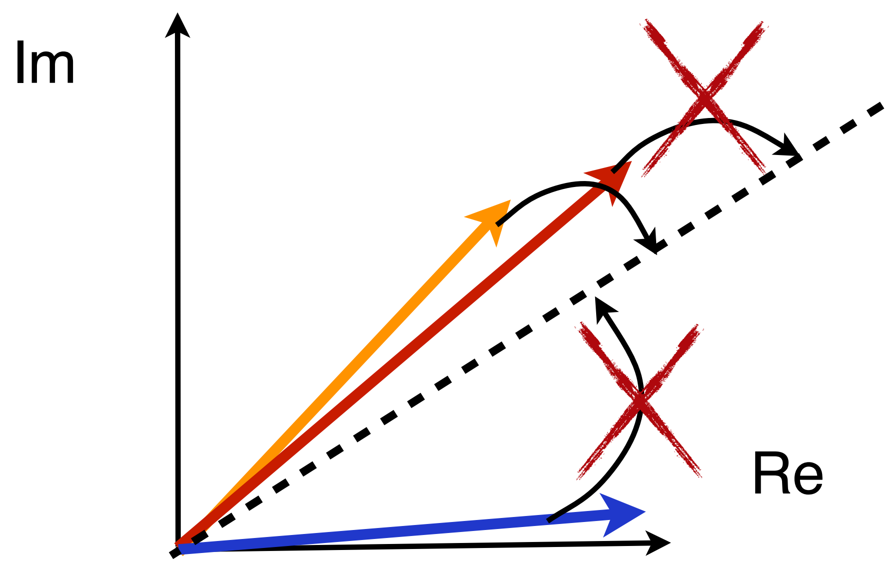
➡ mixed fixed point with finite dissipative and coherent couplings

(3) Absence of Asymptotic Thermalization

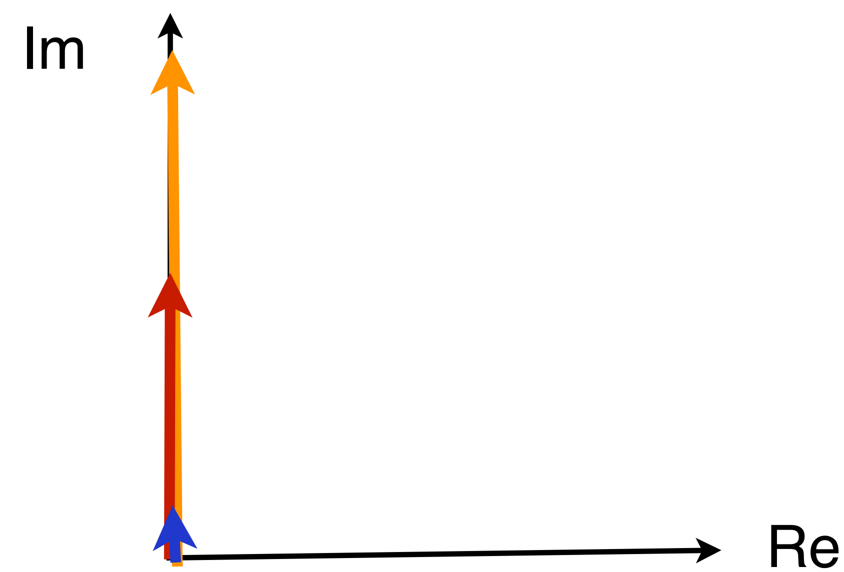
L. Sieberer, A. Chiochetta, U. Tauber, A. Gambassi, SD, PRB (2015)

- symmetry as straightforward diagnostic tool for Schwinger-Keldysh actions
- symmetry explicitly violated microscopically by markovian quantum dynamics
- not emergent:

quantum



classical



- formally:

$$\boxed{Z \sim k^{\eta_Z} \quad , \quad \gamma_d \sim k^{\eta_{\gamma_d}}}$$

$$Z \sim k^{\eta_Z}, \quad \gamma \sim k^{\eta_\gamma}$$

quasiparticle residue — noise level

$$\eta_Z = 0.08, \quad , \quad \eta_{\gamma_d} = -0.26$$

$$\eta_Z = \eta_\gamma = 0.16$$

➡ microscopic and universal asymptotic violation of quantum FDR

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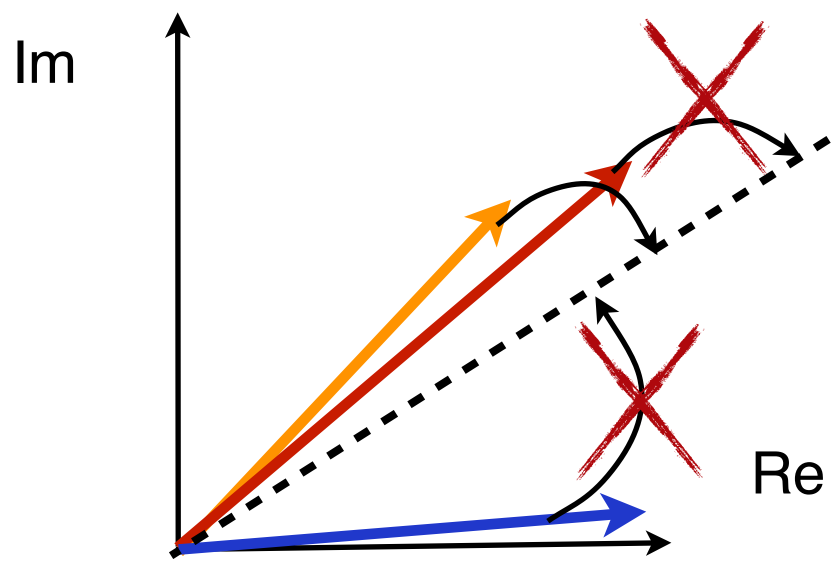
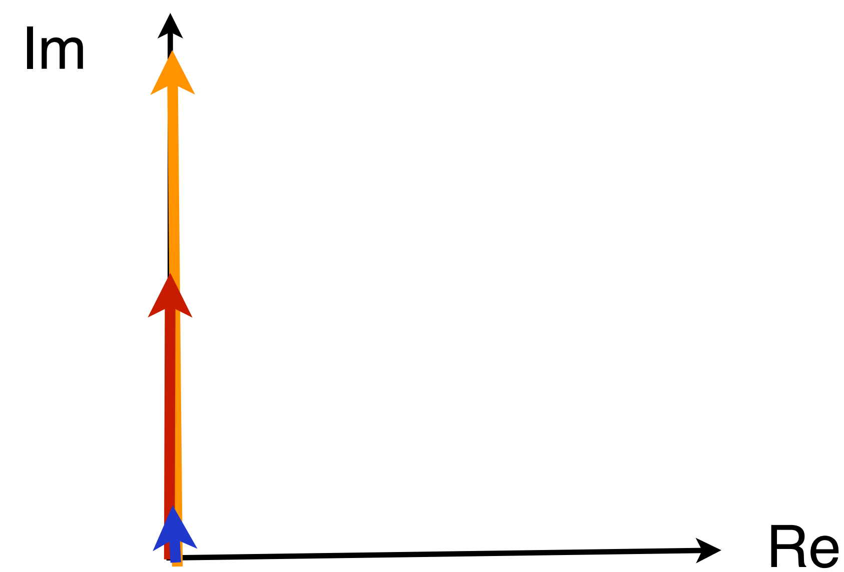


classical



- formally:

$$\boxed{Z \sim k^{\eta_Z} e^{\mathrm{i}\eta'_Z \log k/\Lambda}, \qquad \gamma_d \sim k^{\eta_{\gamma_d}}}$$

quasiparticle residue — noise level

$$Z \sim k^{\eta_Z}, \qquad \gamma \sim k^{\eta_\gamma}$$

$$\eta_Z = 0.08, \quad \eta'_Z = 0.03, \quad \eta_{\gamma_d} = -0.26$$

$$\eta_Z = \eta_\gamma = 0.16$$

➡ limit-cycle like oscillations with (huge!) period
(observable: spectral density)

$$\frac{k_{n+1}}{k_n} = e^{\frac{2\pi}{\eta'_Z}}$$

Observable consequences of driven criticality

- static exponents: first order spatial coherence function

$$\langle \phi^*(r)\phi(0)\rangle \sim \frac{e^{-r/\xi}}{r^{1+\eta_D}} \qquad \xi \sim |\Delta|^{-\nu}$$

distance from phase transition

- dynamical exponents: experiments probing the dynamical single-particle renormalized response (RF spectroscopy for ultracold atoms, homodyne detection)

$$\chi(\omega,\mathbf{q}) \equiv G^R(\omega,\mathbf{q}) = \frac{Z^{-1}}{\omega - \omega_{\mathbf{q}}} \qquad \omega_{\mathbf{q}} \approx A\mathbf{q}^2 - iD\mathbf{q}^2$$

complex dispersion at criticality

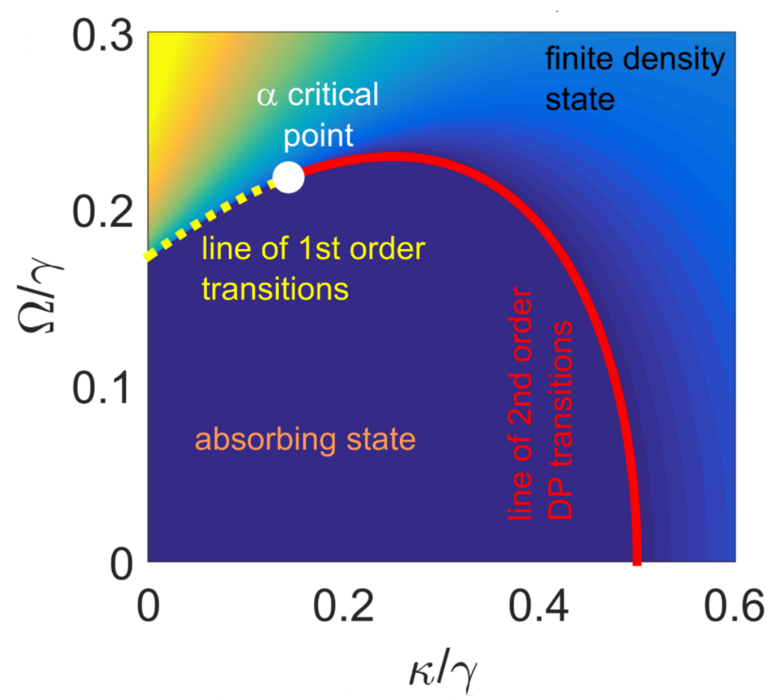
- with anomalous behavior

$$Z \sim |\mathbf{q}|^{\eta_Z} e^{i\eta_Z' \log|\mathbf{q}|/\Lambda}$$

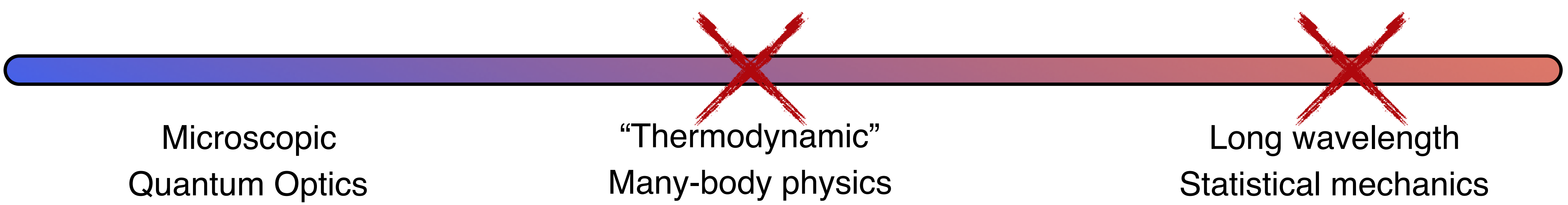
$$A \sim |\mathbf{q}|^{\eta_A}, \quad D \sim |\mathbf{q}|^{\eta_D} \qquad \eta_A = \eta_D$$

(absence of decoherence)

New Absorbing State Transitions in Rydberg Ensembles

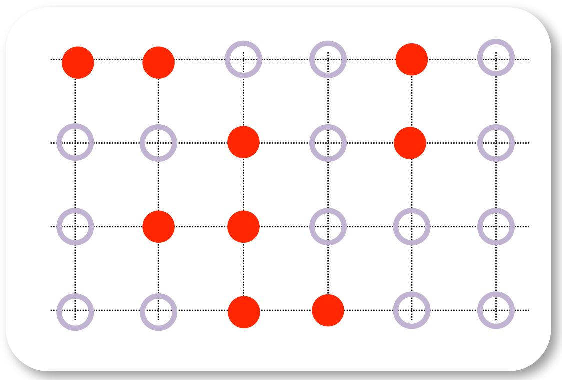


M. Marcuzzi, M. Buchhold, SD, I. Lesanovsky,
arxiv:1601.07305 (2016)



A Classical Non-Equilibrium Phase Transition

- the contact process

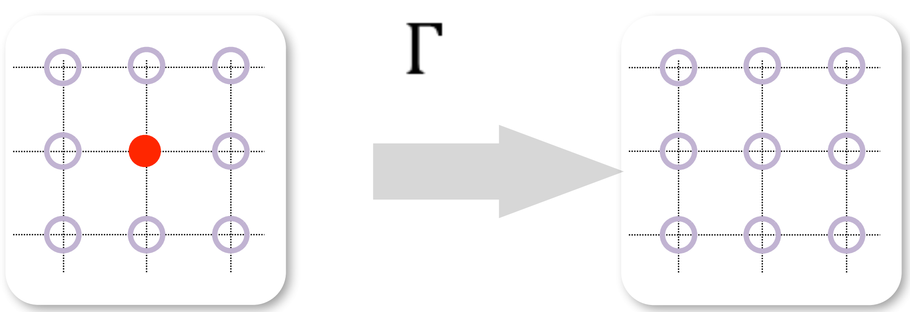


single-component order parameter:

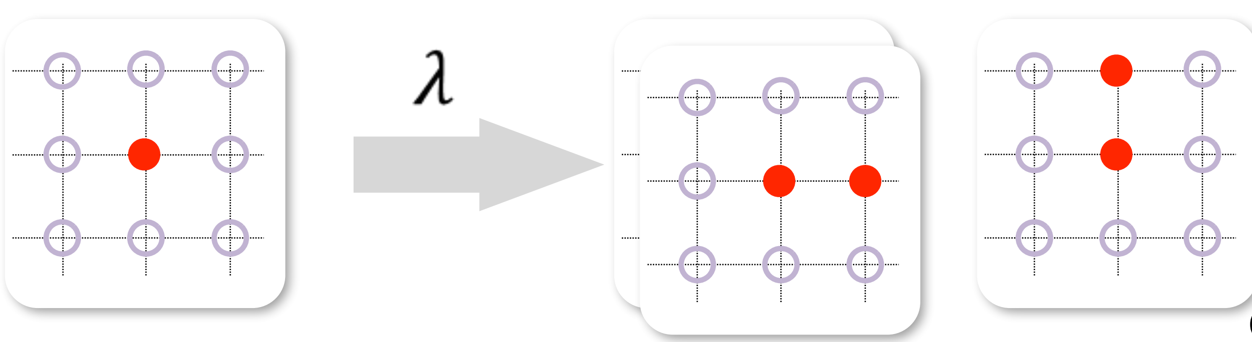
density of active sites

"The Ising model of non-equilibrium physics"

- local dynamical rules:



decay



branching

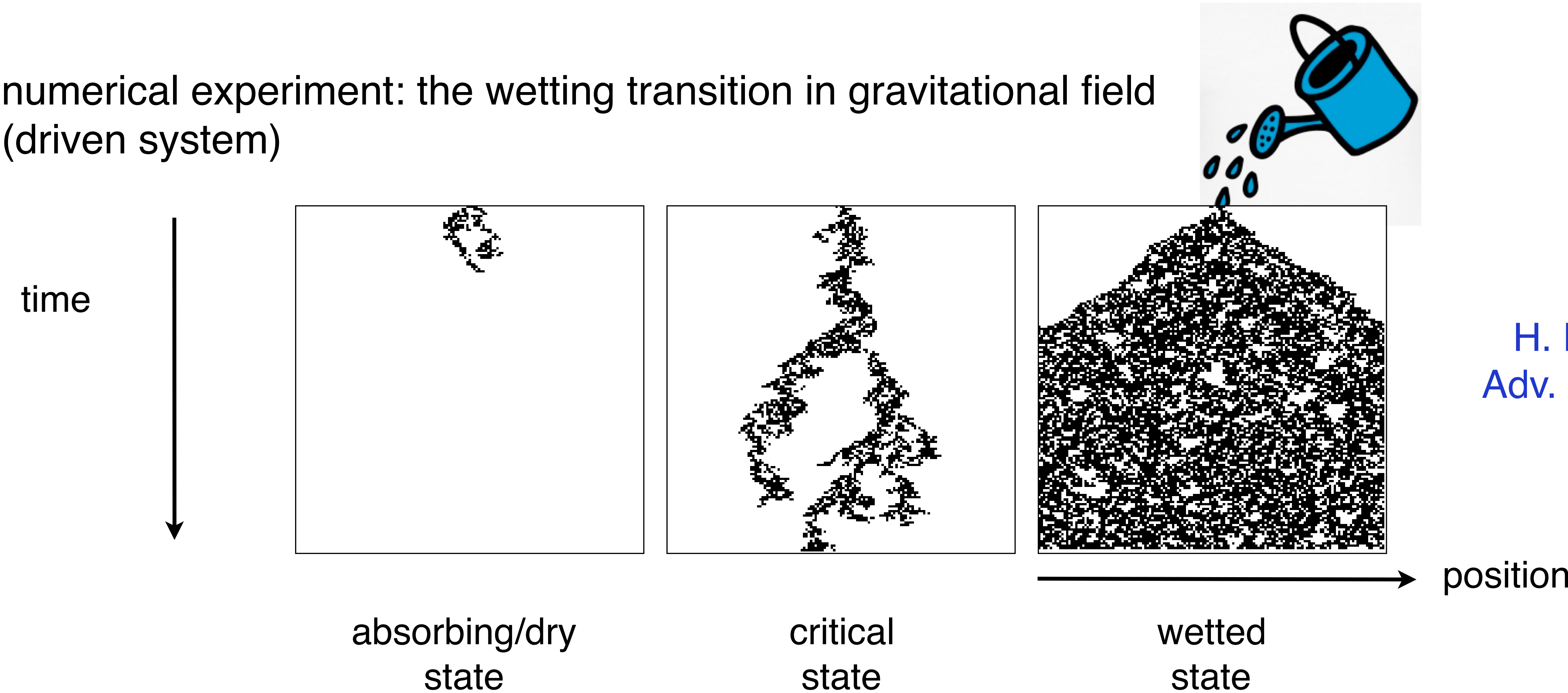
etc.

no offspring from vacuum

- implications:
  - violation of detailed balance
  - unique absorbing state (no active sites)

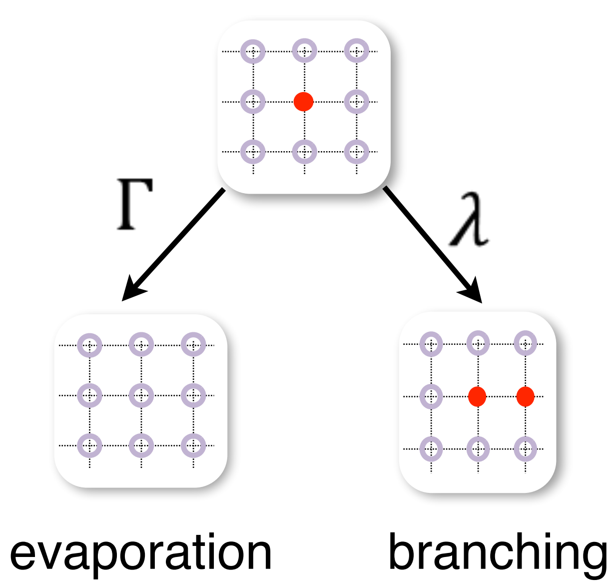
A Classical Non-Equilibrium Phase Transition

- numerical experiment: the wetting transition in gravitational field (driven system)

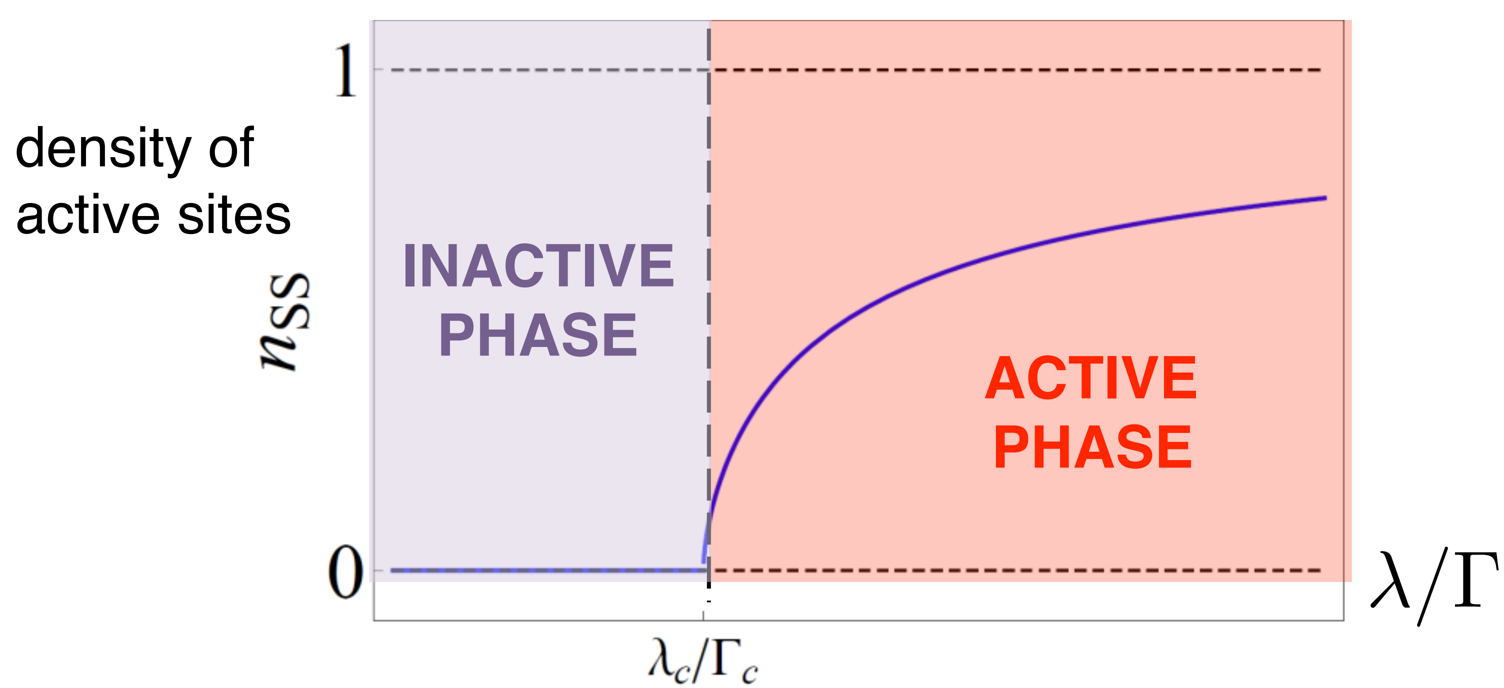


H. Hinrichsen,
Adv. Phys. (2000)

- physical processes



- phase transition:

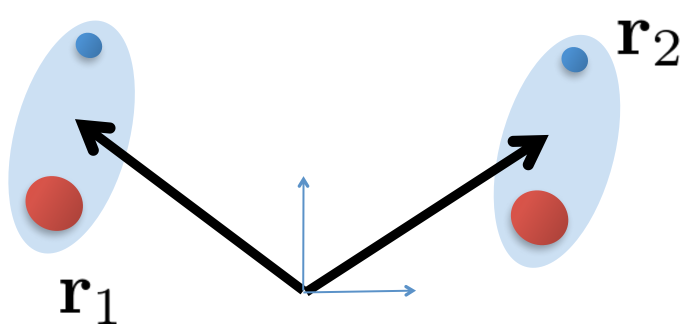


Directed Percolation with Rydberg Atoms

- phase transition in Directed Percolation universality class: “Ising model of non-equilibrium physics”
- currently no experimental realization except in 2D Takeuchi et al. PRL (2007)

- implementation proposal with Rydberg atoms M. Marcuzzi et al., NJP (2015)
  - strong, rapidly decaying van-der-Waals interaction in Rydberg state

$$V_{km} = \frac{C_6}{|\mathbf{r}_k - \mathbf{r}_m|^6}$$



  - no spatial dynamics on exp. timescales
  - versatile platform:

driven dissipative Rydberg gases Hoening et al. PRA (2014); Marcuzzi el al. PRL (2014); Weimer, PRL (2015)

Rydberg dressing and long range interactions (Pohl & Gorshkov groups)

Rydberg polaritons, light propagation in non-linear media Firstenberg et al., Nature (2012), Magrebi et al. PRL (2015)

Rydberg crystals Schauss et al. Nature (2012) Schauss et al. Science (2015)

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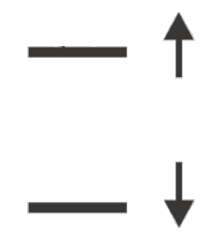
  - many-body excitation dynamics in two-level approximation



Rydberg state

ground state

$$\partial_t \rho = -i[H,\rho] + \sum_i L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\}$$

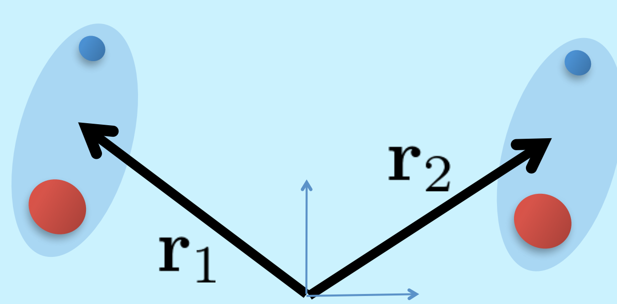
Hamiltonian (atoms at positions r_k)

$$H = \Omega \sum_k \sigma_x^k + \Delta \sum_k n_k + \frac{1}{2}\sum_{km} V_{km} n_k n_m$$

Rydberg occupation

strong, rapidly decaying van-der-Waals interaction

$$V_{km} = \frac{C_6}{|\mathbf{r}_k - \mathbf{r}_m|^6}$$



$\mathbf{r}_1$ $\mathbf{r}_2$

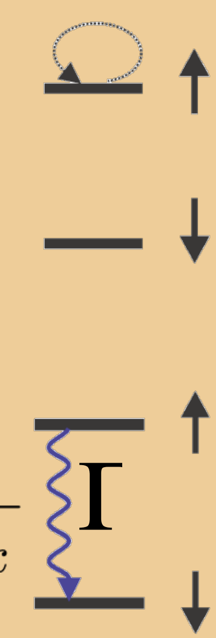
Jump operators

dephasing

$$L_k^{(1)} = \sqrt{\gamma} n_k$$

decay

$$L_k^{(2)} = \sqrt{\Gamma} \sigma_k^-$$



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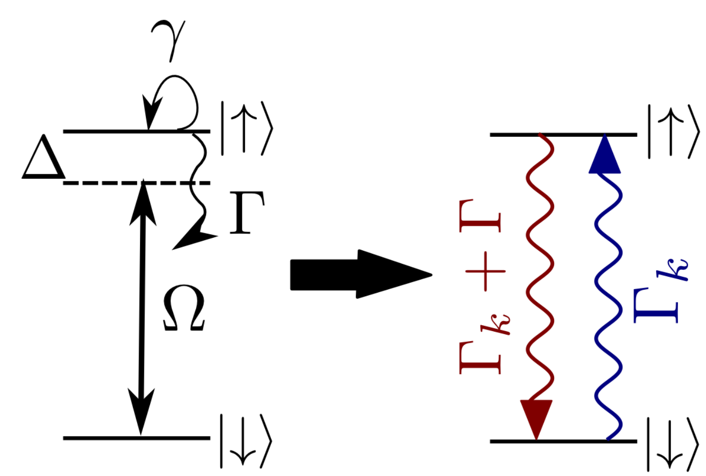
- implementation proposal with Rydberg atoms M. Marcuzzi et al., NJP (2015)
  - Effective classical dynamics of occupation probability for strong dephasing:

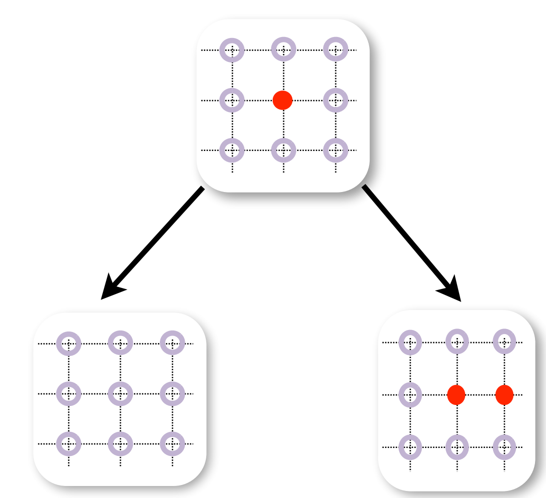
$$\dot{\mathbf{v}} = \sum_k \Gamma_k \left[\sigma_k^+ - p_k\right] \mathbf{v} + \sum_k (\Gamma + \Gamma_k) \left[\sigma_k^- - n_k\right] \mathbf{v}$$



  - occupation dependent rate:

$$\Gamma_k = \frac{\Omega^2 \gamma}{\left(\frac{\gamma}{2}\right)^2 + \left(\Delta + \sum_{q \neq k} V_{kq} n_q\right)^2}$$

  - directed percolation limit: $\Delta \gg \gamma \quad \& \quad \Delta = -V$ nn interaction



$$\dot{\mathbf{v}} \approx \frac{4\Omega^2}{\gamma} \sum_{k,l} n_l [\sigma_k^+ - p_k] \mathbf{v} + \sum_{k,l} (\Gamma / z + \frac{4\Omega^2}{\gamma} n_l) [\sigma_k^- - n_k] \mathbf{v}$$

conditional excitation: branching

decay

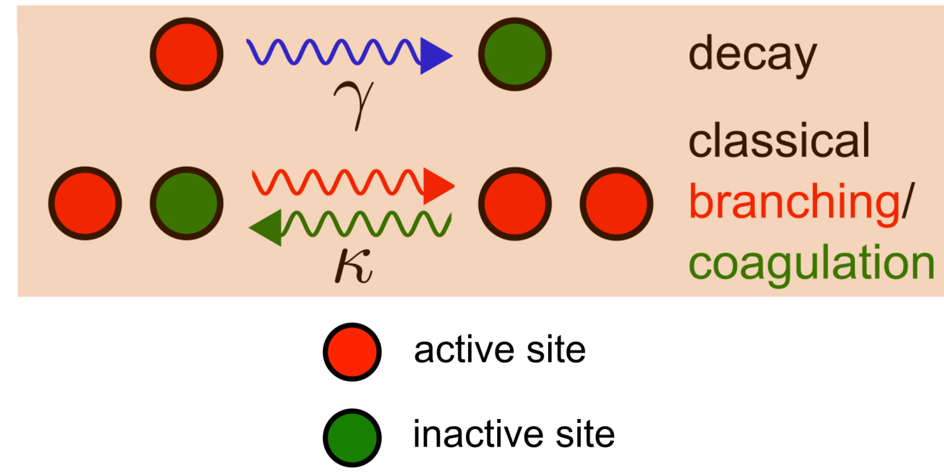
M. Marcuzzi, M. Buchhold, SD, I. Lesanovsky, arxiv (2016)

Quantum Variant of Directed Percolation Dynamics

- Can we formulate a quantum analog of the DP processes?
  - additional coherent dynamics

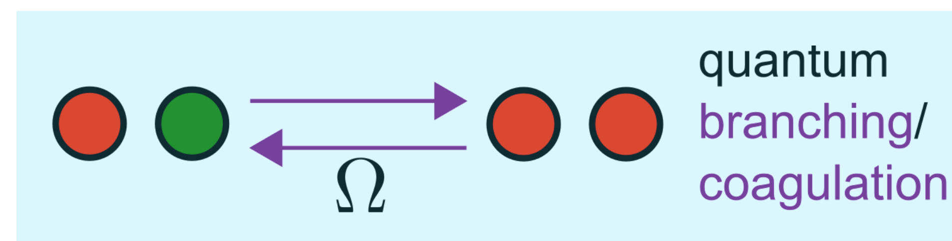
$$\partial_t \rho = -i[H, \rho] + \mathcal{L}(\rho)$$

incoherent contact process



$$H = \Omega \sum_{\langle i,j \rangle} n_i \left(\sigma_j^+ + \sigma_j^- \right)$$

coherent contact process



- new "quantum" scale
- implementation:
  - Quantum branching/coagulation: energetic constraint
    coherent pump laser, detuning $\Delta = -V$
  - Classical branching/coagulation:
    now: weak spontaneous emission
    but: pump laser with strong phase noise $\gamma \gg \Omega$

Walls, Milburn, PRA (1985)

M. Marcuzzi, M. Buchhold, SD, I. Lesanovsky, arxiv (2016)

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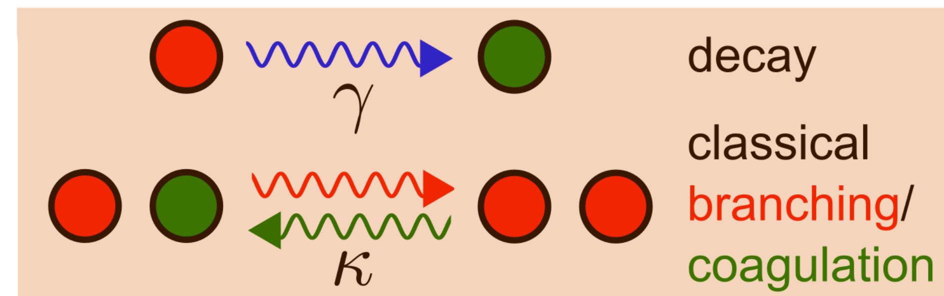
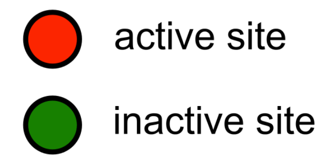
$$\partial_t \rho = -i[H,\rho] + \mathcal{L}(\rho)$$

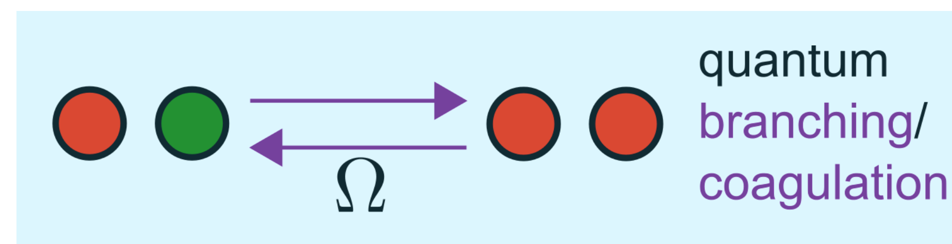
incoherent contact process





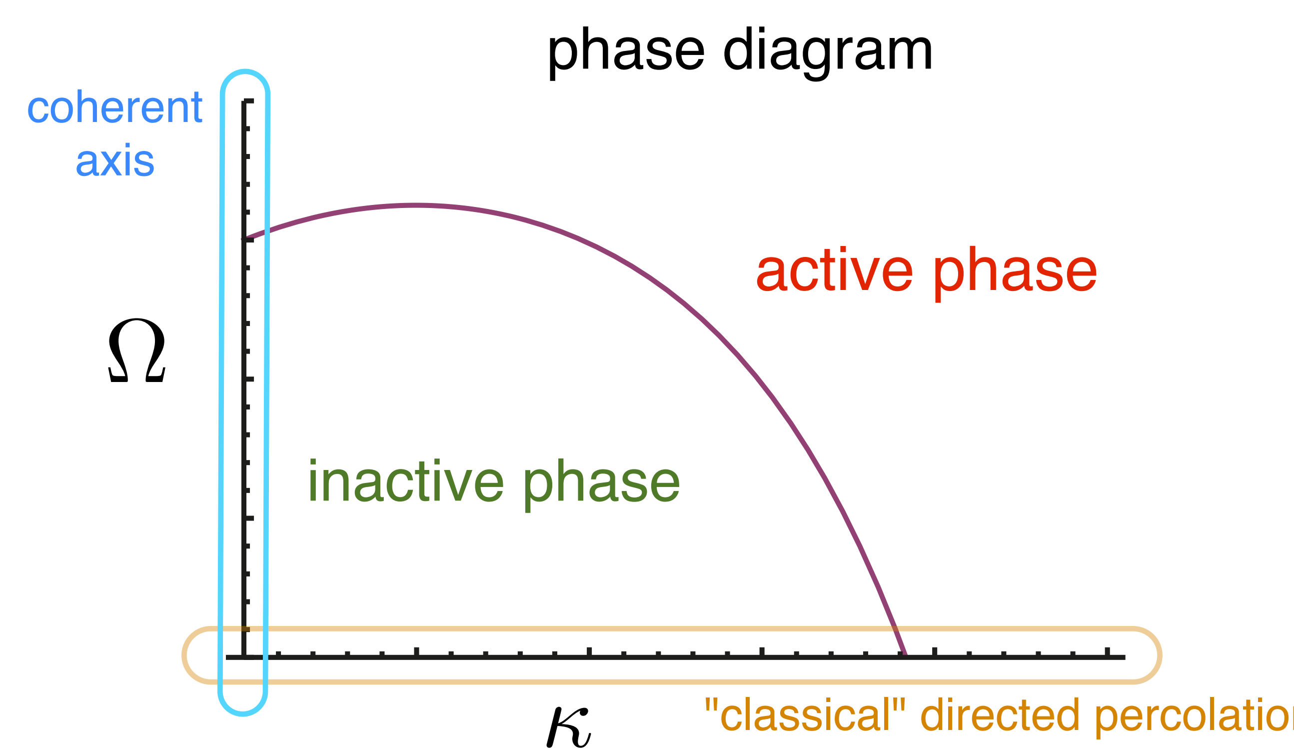
$$H = \Omega \sum_{\langle i,j \rangle} n_i \left(\sigma_j^+ + \sigma_j^-\right)$$

coherent contact process



- new "quantum" scale

- implications:
  - additional axis in phase diagram
  - relevant in d < 2: modified universality class
  - non-universal modifications in arbitrary dimensions



Field Theoretical Approach

- Heisenberg-Langevin introduces coupling of occupations to coherences

$$\partial_t n_X = \underbrace{D\nabla^2 n_X + \Delta n_X - 2\kappa n_X^2}_{\text{classical processes}} + \underbrace{\Omega \sigma^y_X n_X}_{\text{coupling to coherences}} + \underbrace{\xi_X}_{\text{quantum noise}}$$

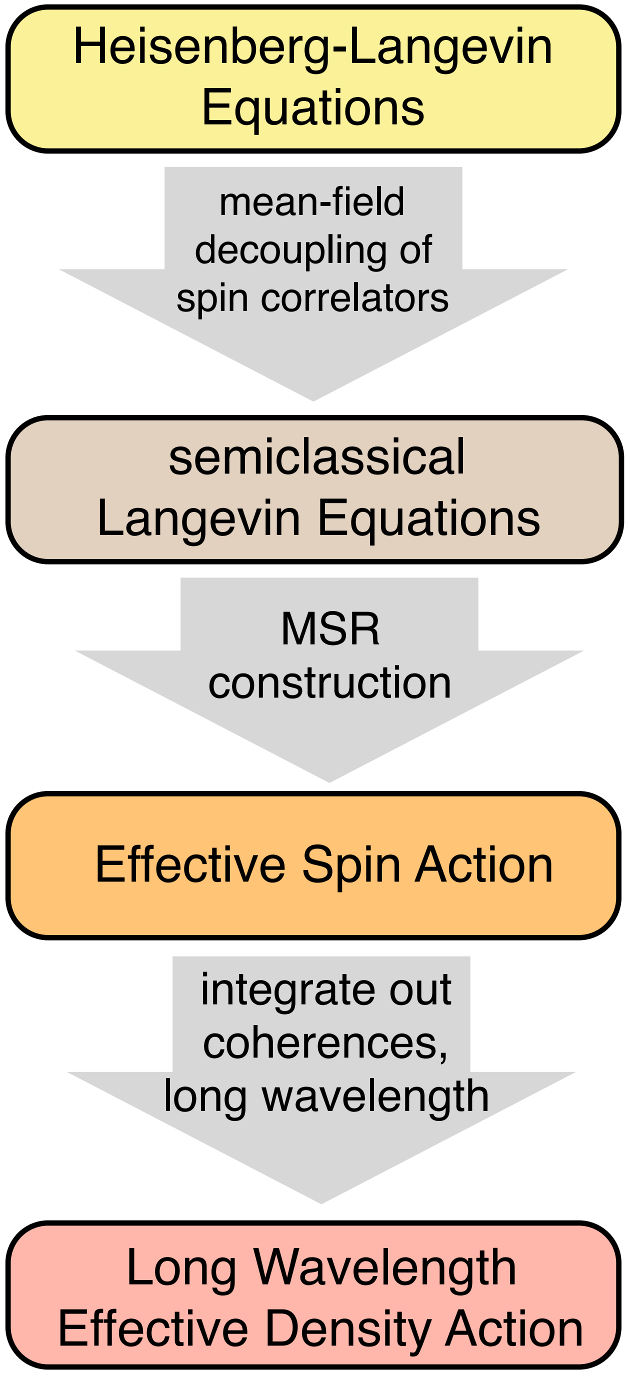
- Long wavelength occupation density action:

$$S = \int_X \tilde{n}_X \left[\left(\partial_t - D\nabla^2 - \Delta \right) n_X + u_3 n_X^2 + u_4 n_X^3 \right]$$

$$- \int_X \tilde{n}_X^2 n_X + \mu_4 \tilde{n}_X^2 n_X^2$$

conventional field theory of directed percolation

new elements due to the quantum scale



Field Theoretical Approach

- Long wavelength occupation density action:

$$S = \int_X \tilde{n}_X \left[\left(\partial_t - D\nabla^2 - \Delta\right) n_X + u_3 n_X^2 + u_4 n_X^3\right]$$
$$- \int_X \tilde{n}_X^2 n_X + \mu_4 \tilde{n}_X^2 n_X^2$$

- qualitatively new features due to quantum dynamics:

  - potentially negative cubic non-linearity

similar findings:

M. Maghrebi, A. V. Gorshkov, PRB (2016)

$$u_3 \sim \kappa - \frac{\Omega^2}{2d+1}$$

  - quartic terms u_4, μ_4 break rapidity inversion symmetry of DP

$$n(t, \mathbf{x}) \leftrightarrow -\tilde{n}(-t, \mathbf{x})$$

➡ physical consequences?

Mean-field phase diagram

- Neglect fluctuations in the action

$$S = \int_X \tilde{n}_X \frac{\delta\Gamma(n_X)}{\delta n_X}$$

effective potential

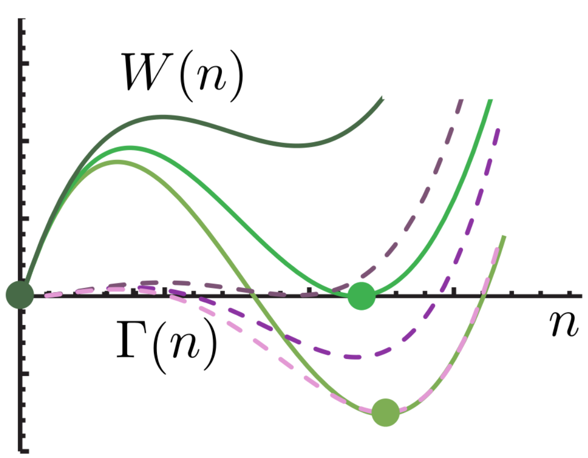
- Minima of effective potential determine stationary phase

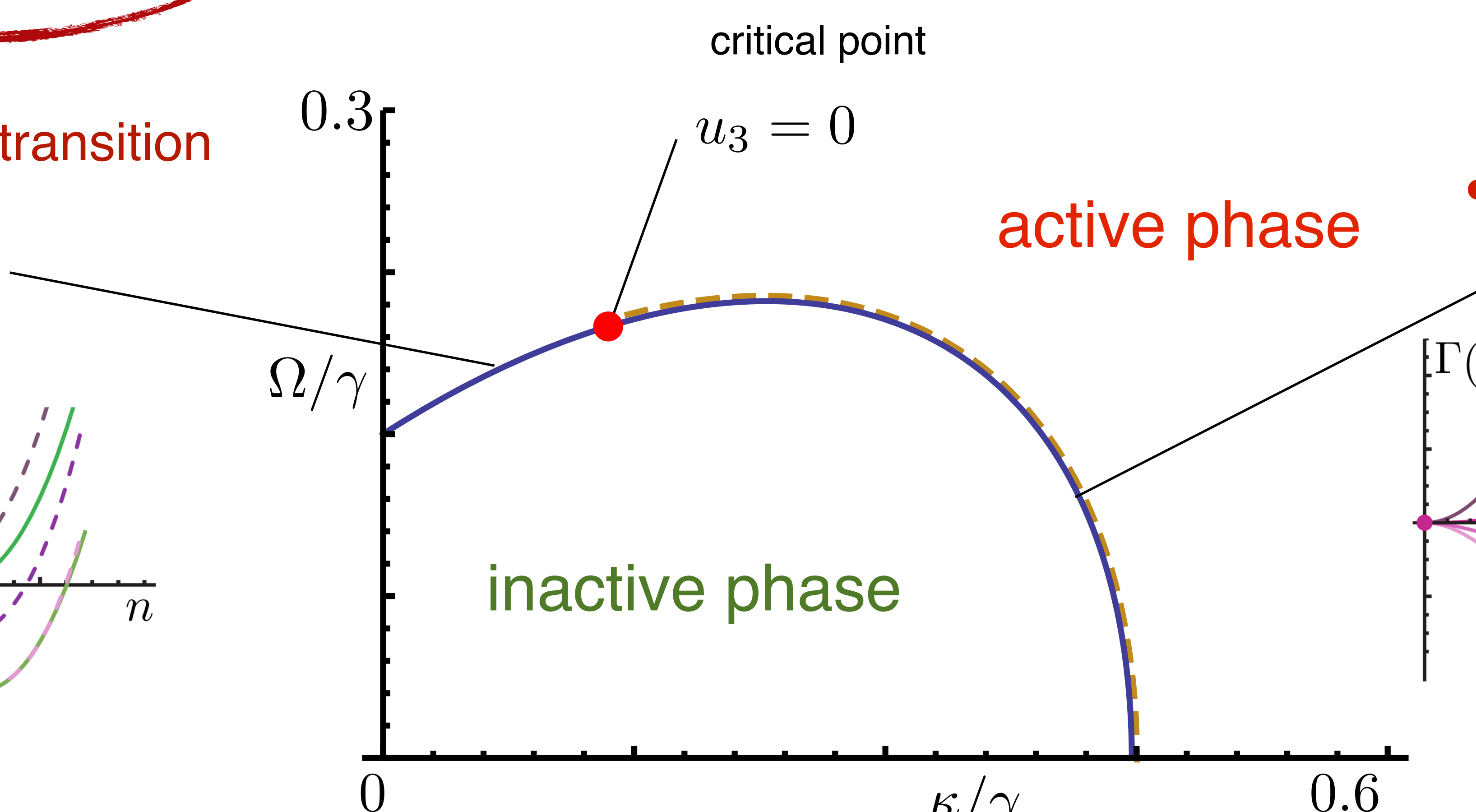
$$\begin{cases} n > 0 & \text{active phase} \\ n = 0 & \text{inactive phase} \end{cases}$$

$$\Gamma(n) = \frac{\Delta}{2} n^2 + \frac{u_3}{3} n^3 + \frac{u_4}{4} n^4$$

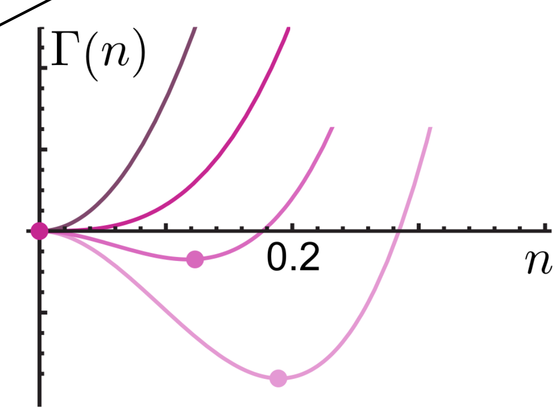
$$u_3 \sim \kappa - \frac{\Omega^2}{2d+1}$$

- 1st order transition





- 2nd order transition



Mean-field phase diagram

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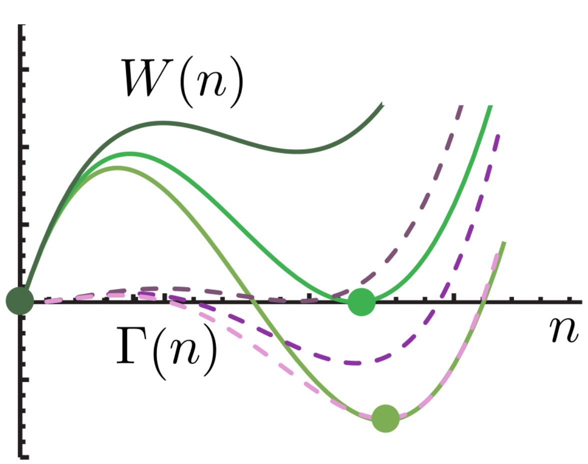
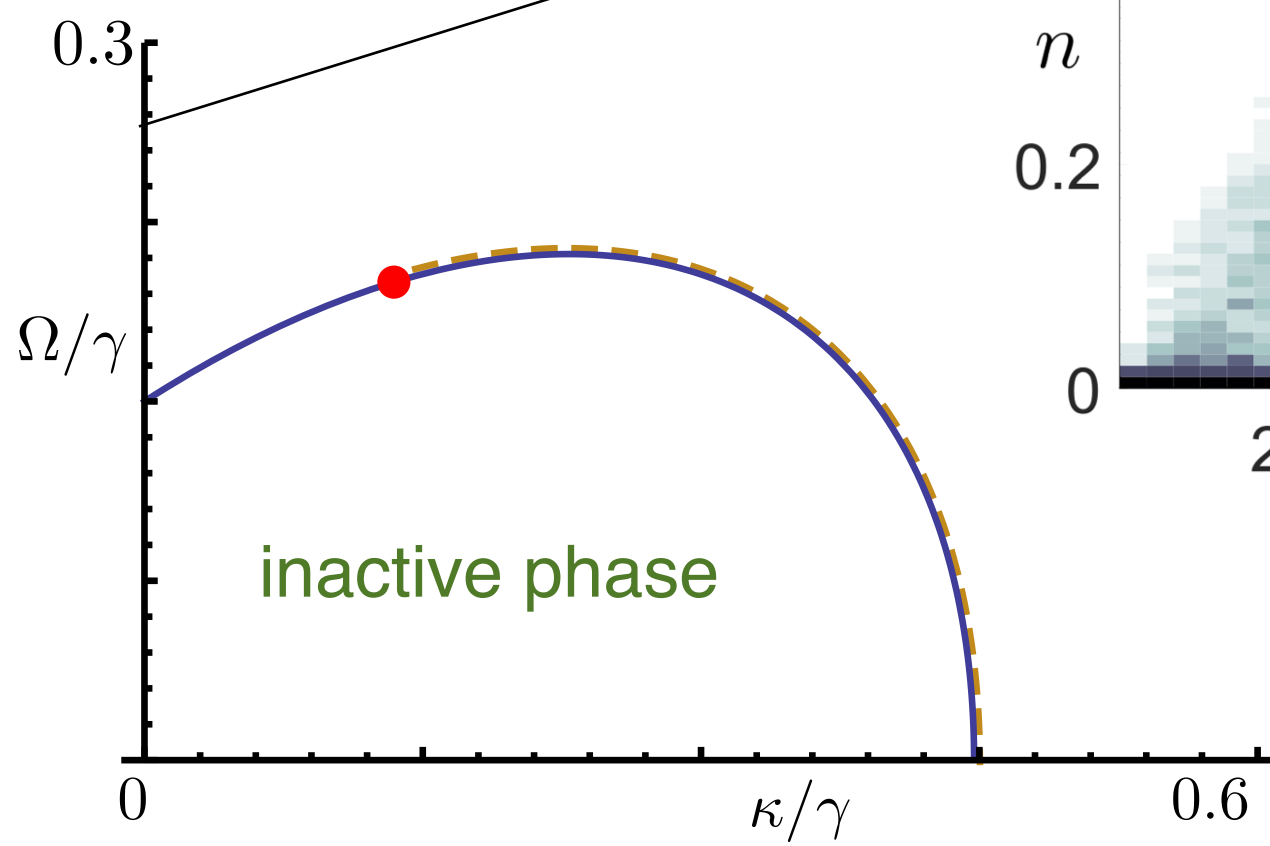
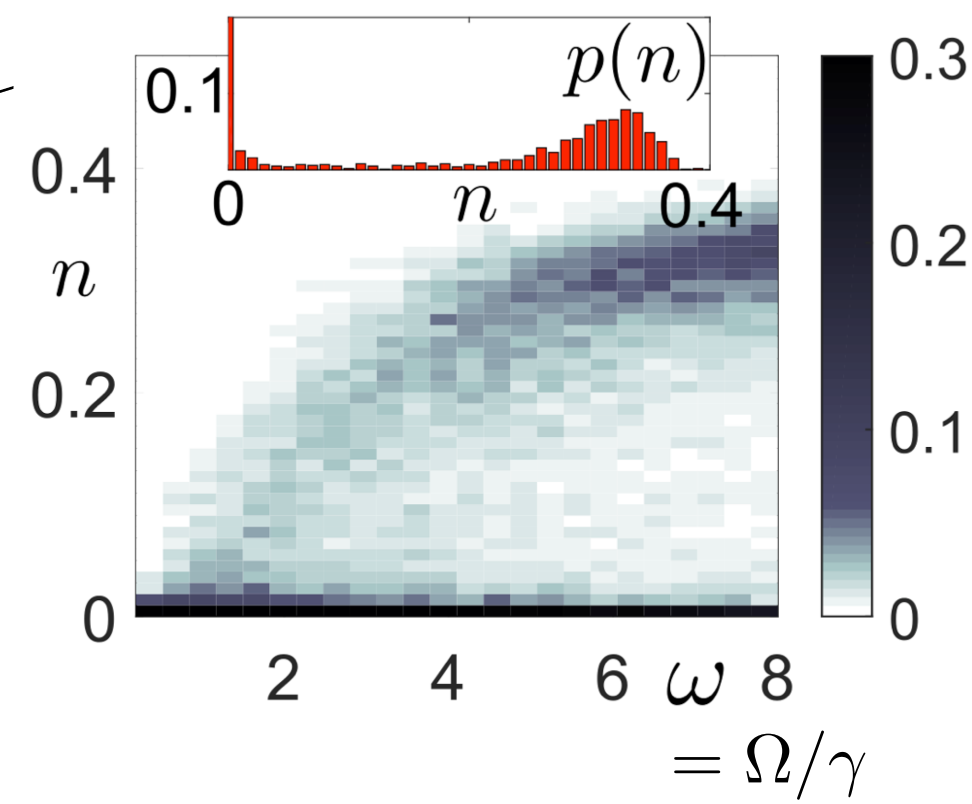
- 1st order transition

$u_3 < 0$



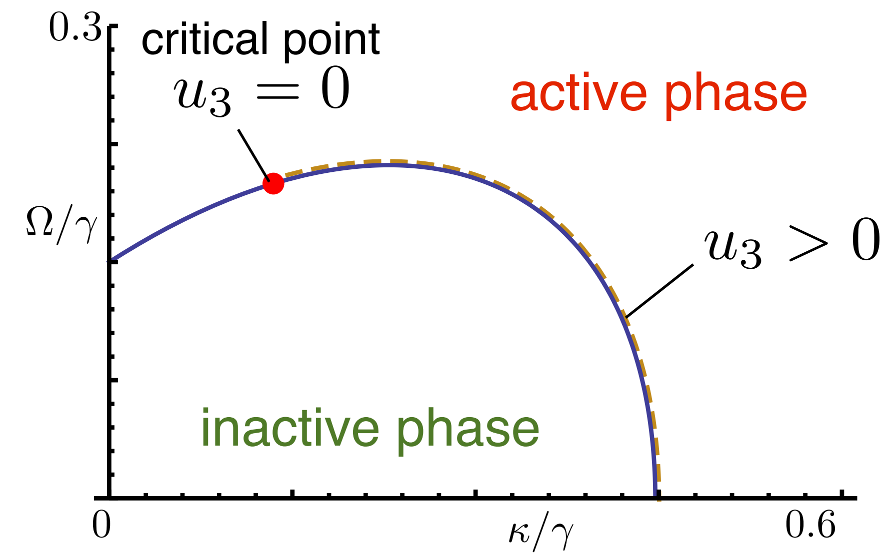


numerics: bimodal structure



A New Universality Class?

- Fluctuations: Universal scaling behavior at 2nd order transition



gap vanishes

$$S = \int_X \tilde{n}_X \left[\left(\partial_t - D\nabla^2 - \Delta\right) n_X + u_3 n_X^2 + u_4 n_X^3 \right]$$

relevant in d < 2

$$- \int_X \tilde{n}_X^2 n_X + \mu_4 \tilde{n}_X^2 n_X^2$$

relevant in d < 4

$u_3 > 0$	$u_3 = 0$ bicritical point
• d ≥2: quantum terms vanish under RG symmetry restored $n \leftrightarrow -\tilde{n}$ classical directed percolation $\omega \sim q^{2-\eta}$ $\eta_{\rm DP} \approx \frac{d-4}{12}$ 1st order ϵ-expansion	• d ≥2: only trivial RG flow ⟶ no effect of long wavelength fluctuations mean-field critical exponents $\eta = 0$
• d < 2: quantum terms relevant under RG ⟶ symmetry explicitly broken absorbing state transition w/o obvious symmetries $\eta \neq \eta_{\rm DP}$	• d < 2: all couplings relevant no symmetries + $u_3 = 0$ ⟶ further new universality class ?

equivalence?

Summary & Outlook

Review: L. Sieberer, M. Buchhold, SD, *Keldysh Field Theory for Driven Open Quantum Systems*, arxiv (2015)

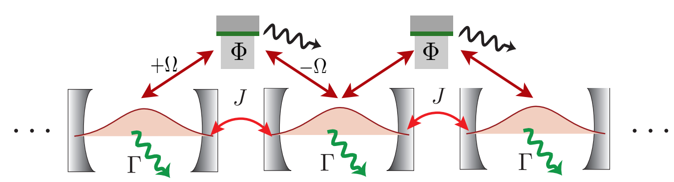
Driven open many-body systems provide an arena for non-equilibrium quantum statistical mechanics

the tool: Keldysh field theory

Quantum non-equilibrium criticality (1D)
New non-equilibrium universality class

Constrained microscopic quantum dynamics
New absorbing state transition

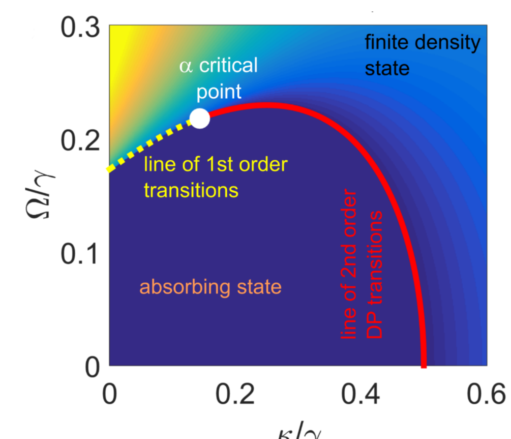
- **Platform**: driven microcavity arrays
- non-equilibrium persists:
  - no emergent equilibrium symmetry for fixed point action
  - no quantum-classical mapping



- quantum persists: no decoherence
- universal limit cycle for quasiparticle residue

J. Marino, SD, PRL (2016) and in prep.

- **Platform**: driven Rydberg gases
- coherent dynamics gives new axis in phase diagram



- first order transition associated to quantum scale
- new universality class without DP symmetries?

M. Marcuzzi, M. Buchhold, SD, I. Lesanovsky, arxiv (2016)

- Perspectives:
  - Open system quantum Hall effect — Hafezi group, PRL (2014), arxiv (2105)
  - Driven open fermion ensembles — Imamoğlu group, Science (2014)
  - Keldysh symmetries and dynamical slow modes

1st order absorbing transition

- 1st order: gap remains finite $\Delta \neq 0$

  $\longrightarrow$ spatial fluctuations irrelevant $\cancel{D\nabla^2 n_X}$

- Effective description: uniform density

$$S = V \int_t \tilde{n}_t \left(\dot{n}_t + \Gamma'\right) - \Xi \tilde{n}_t^2 \qquad \Xi = \frac{1}{2} n_t + \mu_4 n_t^2$$

spatial volume

effective potential

fluctuation strength

- Fluctuations increase with density

  optimal path: $\tilde{n}_t = \Gamma'/\Xi$

$$P(n) = \frac{1}{Z} \exp(-V \int_0^n \Gamma'/\Xi)$$

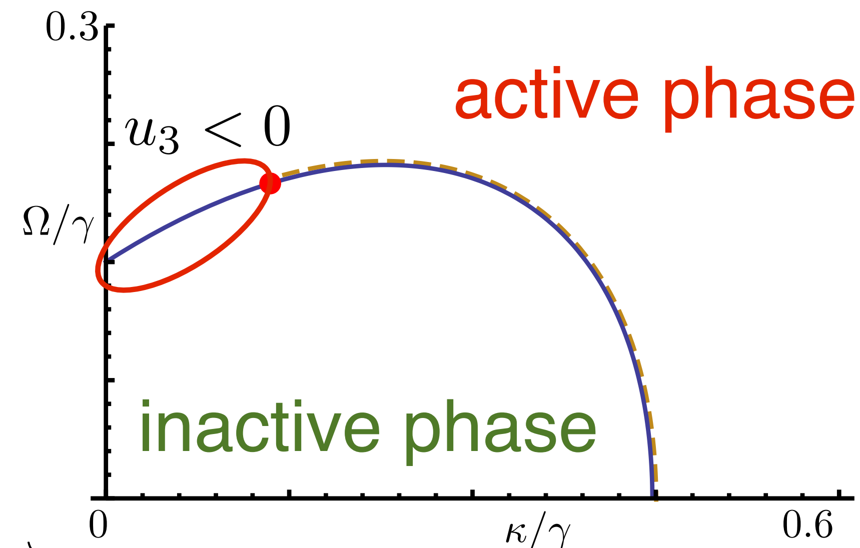
optimal path distribution

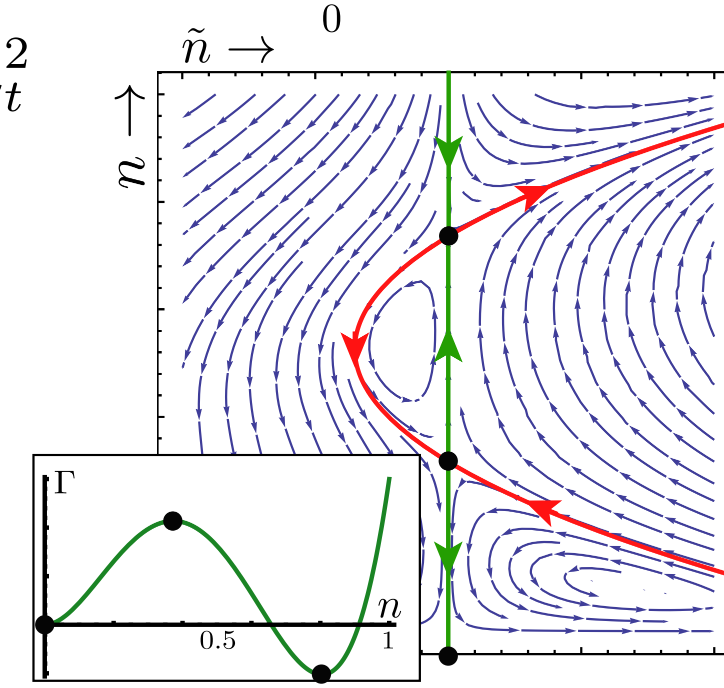
- Phase boundaries: minima of OPA potential

$$W(n) = \int_0^n \Gamma'/\Xi$$

different from equilibrium prediction

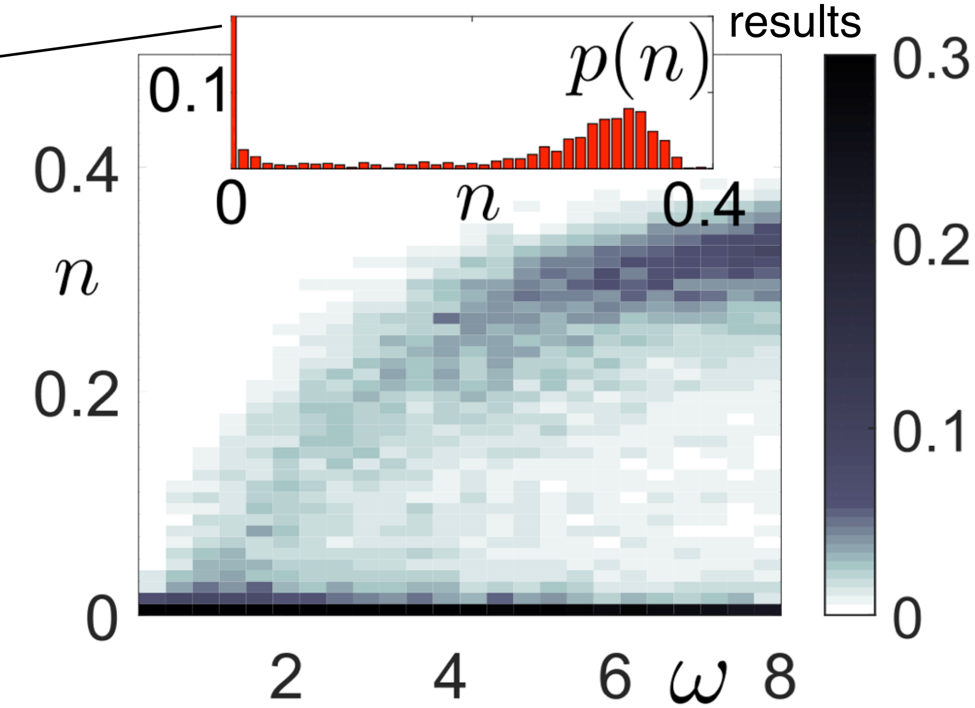
$$W(n) = \Gamma(n)/T$$



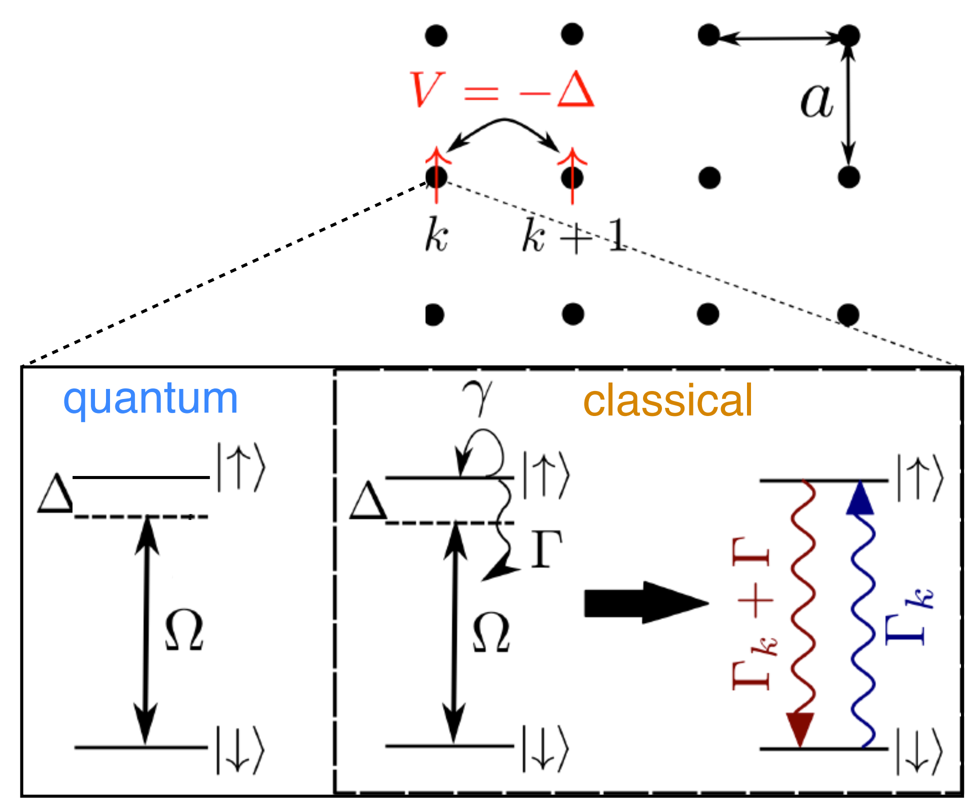


optimal phase space trajectories

numerical results



Experimental implementation



- Rydberg atoms on a lattice

- Effective two level system $|\downarrow\rangle \longleftrightarrow |\uparrow\rangle$

  ground state Rydberg state

- Neighboring Rydberg states repel each other

$$V \sim \frac{C_0}{a^\alpha} \qquad \alpha = 6$$

for Rydberg s-states

- Quantum branching/coagulation:

  coherent pump laser, detuning $\Delta = -V$

- Classical branching/coagulation:

  pump laser with strong phase noise $\gamma \gg \Omega$

  Walls, Milburn, PRA (1985)

- Incoherent decay:

  spontaneous emission Γ

coherent + incoherent

enables

$$|iai\rangle \longrightarrow \begin{cases} |aai\rangle \\ |iaa\rangle \end{cases}$$

suppresses

$$|aia\rangle \longrightarrow |aaa\rangle$$

negligible at small n

RG irrelevant

Keldysh functional integral

- quantum master equation: $\partial_t \rho = -i[H,\rho] + \mathcal{D}[\rho] = \mathcal{L}[\rho]$

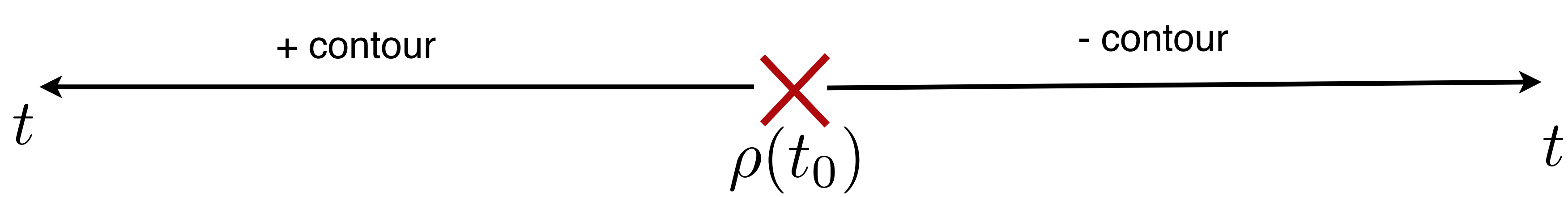
- equivalent Keldysh functional integral:

$$Z = \int \mathcal{D}(\Phi_+, \Phi_-) \mathrm{e}^{\mathrm{i}(S_M[\Phi_+,\Phi_-]}$$

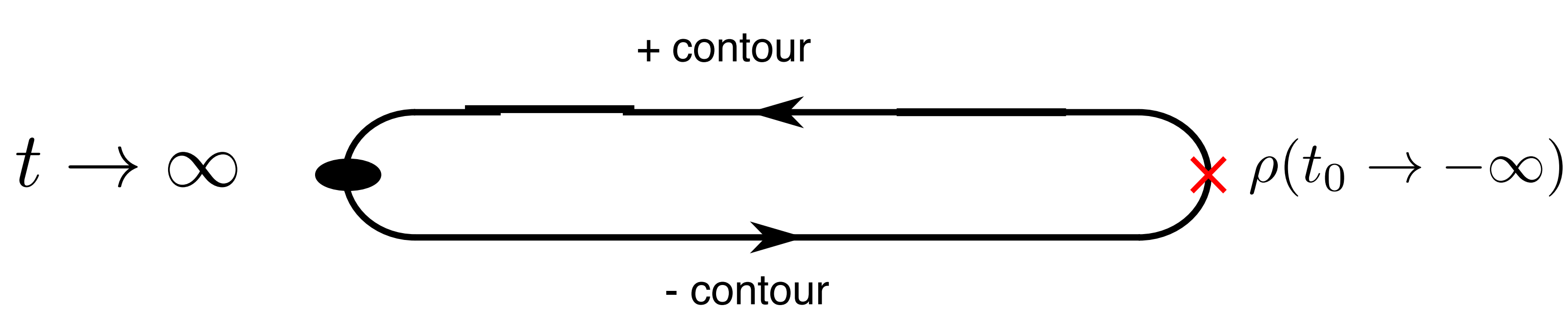
$$\Phi_\pm = \begin{pmatrix} \phi_\pm \\ \phi^*_\pm \end{pmatrix}$$

$$S_M[\Phi_+, \Phi_-] = \int dt (\phi^*_+ i\partial_t \phi_+ - \phi^*_- i\partial_t \phi_- - i\mathcal{L}[\Phi_+, \Phi_-])$$

- two fields: track left/right action of operators



- partition function: $Z = \mathrm{tr}\rho(t)$



Keldysh functional integral

- quantum master equation:

$$\partial_t \rho = -i[H,\rho] + \mathcal{D}[\rho] = \mathcal{L}[\rho]$$

$$= -i(H\rho - \rho H) + \kappa \sum_i (L_i \rho L_i^\dagger - \tfrac{1}{2} L_i^\dagger L_i \rho - \tfrac{1}{2} \rho L_i^\dagger L_i)$$

- equivalent Keldysh functional integral:

$$Z = \int \mathcal{D}(\Phi_+, \Phi_-) e^{\mathrm{i}(S_M[\Phi_+,\Phi_-]} \qquad \Phi_\pm = \begin{pmatrix} \phi_\pm \\ \phi_\pm^* \end{pmatrix}$$

$$S_M[\Phi_+, \Phi_-] = \int dt (\phi_+^* i\partial_t \phi_+ - \phi_-^* i\partial_t \phi_- - i\mathcal{L}[\Phi_+, \Phi_-])$$

$$\boxed{\mathcal{L}[\Phi_+, \Phi_-] = -i\left(H_+ - H_-\right) - \kappa \sum_i \left(L_{i,+} L_{i,-}^\dagger - \tfrac{1}{2} L_{i,+}^\dagger L_{i,+} - \tfrac{1}{2} L_{i,-}^\dagger L_{i,-}\right)}$$

$$H_\pm = H(\Phi_\pm) \text{ etc.}$$

➡ recognize Lindblad structure

➡ simple translation table (for normal ordered Liouvillian)

- operator right of density matrix -> - contour
- operator left of density matrix -> + contour