Designer Quantum Systems Out of Equilibrium
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Non-Equilibrium Duality and Universal Phenomena for Low-Dimensional Driven Open Quantum Systems

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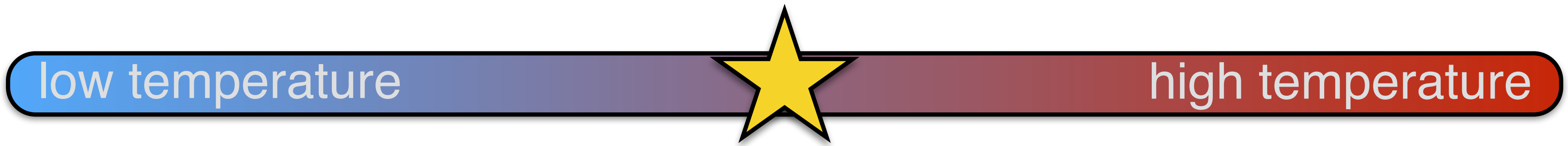
€€€ :

Universality in low dimensions: 2D



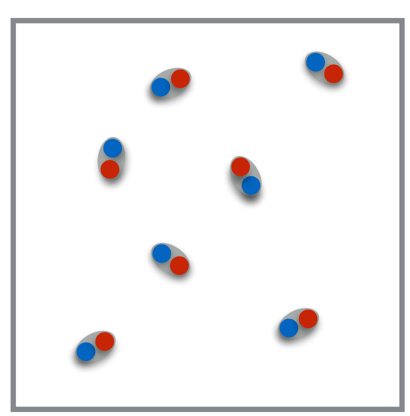
- correlations

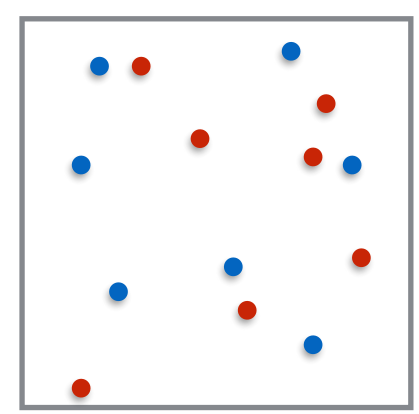
$$\langle \phi(r)\phi^*(0)\rangle \sim r^{-\alpha} \qquad\qquad \sim e^{-r/\xi}$$

- superfluidity

$$\rho_s \neq 0 \qquad\qquad \rho_s = 0$$

- KT transition: unbinding of vortex-antivortex pairs

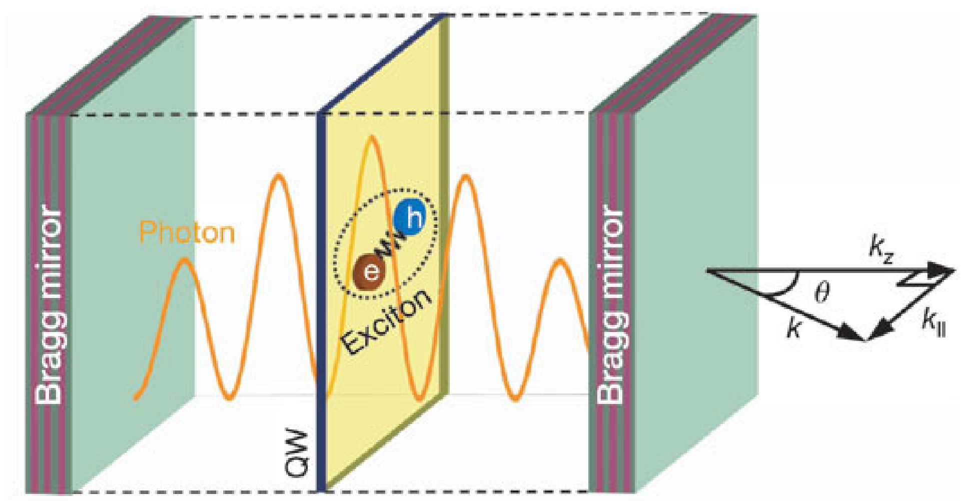




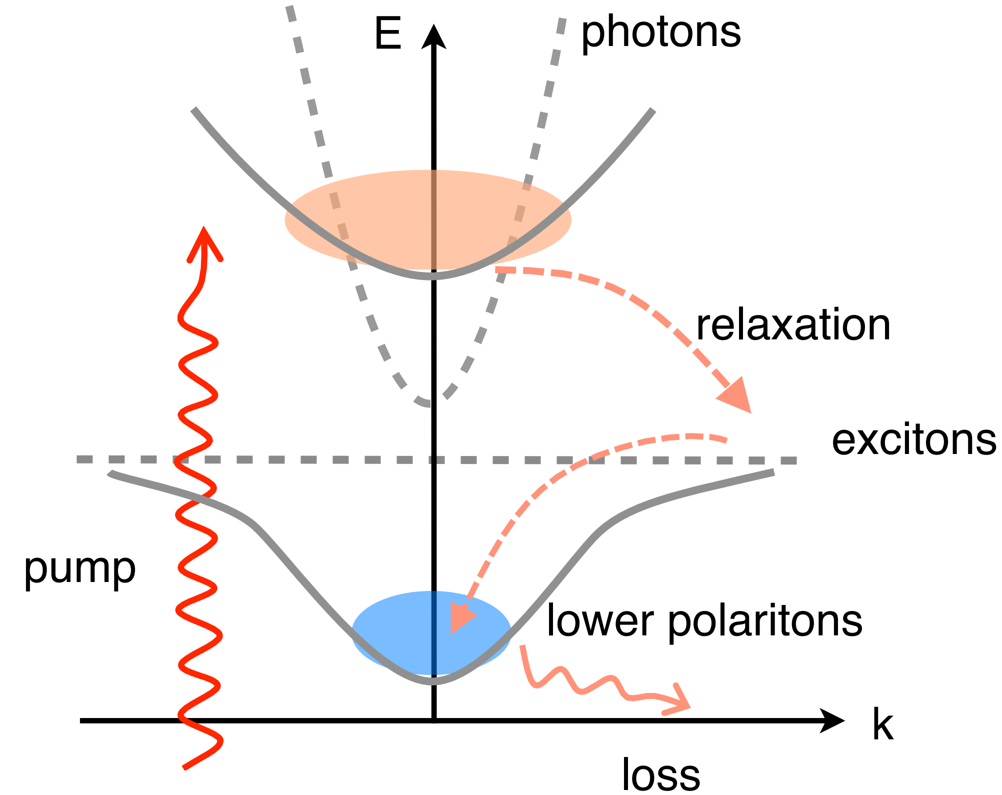
… also for out-of-equilibrium systems?
… new universal phenomena tied to non-equilibrium?

Experimental Platform: Exciton-Polariton Systems

Kasprzak et al., Nature 2006



Imamoglu et al., PRA 1996



- phenomenological description: stochastic driven-dissipative Gross-Pitaevskii-Eq

$$i\partial_t\phi = \left[-\frac{\nabla^2}{2m} - \mu + i(\gamma_p - \gamma_l) + (\lambda - i\kappa)\,|\phi|^2\right]\phi + \zeta$$

propagation

pump & loss rates

elastic collisions

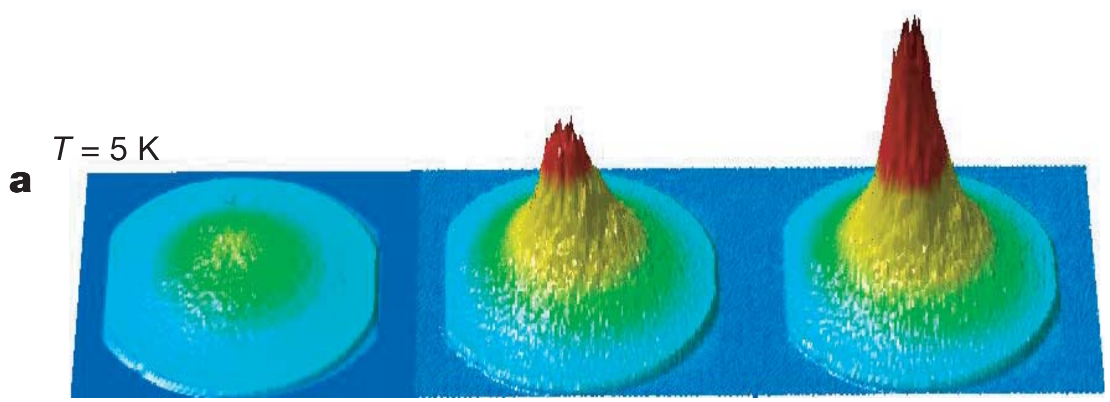
two-body loss

$$\langle\zeta^*(t,\mathbf{x})\zeta(t',\mathbf{x}')\rangle = \gamma\delta(t-t')\delta(\mathbf{x}-\mathbf{x}')$$

microscopic derivation and linear fluctuation analysis:
Szymanska, Keeling, Littlewood PRL (04, 06); PRB (07));
Wouters, Carusotto PRL (07,10)

Experimental Platform: Exciton-Polariton Systems

- Bose condensation seen despite non-equilibrium conditions



Kasprzak et al., Nature 2006

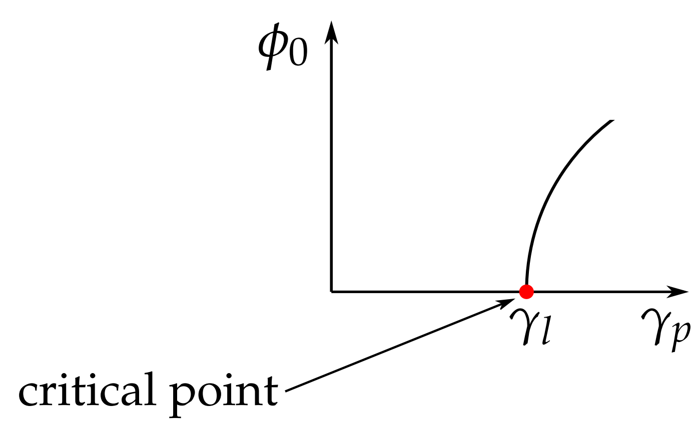
stationary state!

- stochastic driven-dissipative Gross-Pitaevskii-Eq

$$\cancel{i\partial_t\phi} = \left[-\cancel{\frac{\nabla^2}{2m}} - \mu + i(\gamma_p - \gamma_l) + (\lambda - i\kappa)\,|\phi|^2\right]\phi + \cancel{\zeta}$$

Szymanska, Keeling, Littlewood PRL (04, 06); PRB (07)); Wouters, Carusotto PRL (07,10)

- mean field
  - neglect noise
  - homogeneous solution $\phi(\mathbf{x},t) = \phi_0$



- naively, just as Bose condensation in equilibrium!
- Q: What is “non-equilibrium” about it?

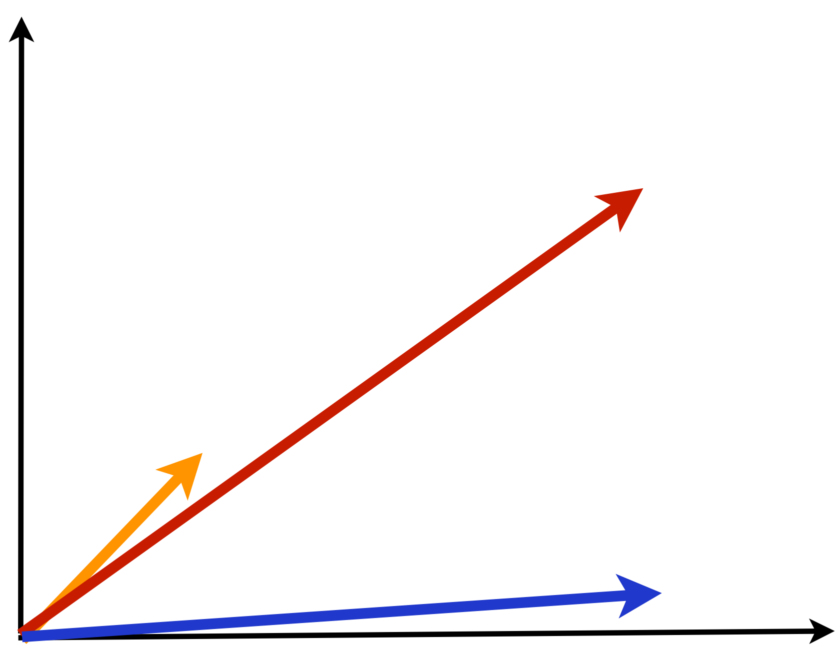
“What is non-equilibrium about it?”

- rewrite driven-dissipative Gross-Pitaevski equation

$$i\partial_t \phi_c = \frac{\delta \mathcal{H}_c}{\delta \phi_c^*} - i\frac{\delta \mathcal{H}_d}{\delta \phi_c^*} + \xi$$

$$\mathcal{H}_\alpha = \int d^d x [r_\alpha |\phi_c|^2 + K_\alpha |\nabla \phi_c|^2 + \lambda_\alpha |\phi_c^* \phi_c|^4], \quad \alpha = c, d$$

- couplings located in the complex plane:



example: two-body processes

$\mathrm{Re}\lambda$ elastic two-body collisions

$\mathrm{Im}\lambda$ inelastic two-body losses

“What is non-equilibrium about it?”: Field theory

- Representation of stochastic Langevin dynamics as MSRJD functional integral

$$Z = \int \mathcal{D}[\phi_c, \phi_c^*, \phi_q, \phi_q^*] \, e^{iS[\phi_c, \phi_c^*, \phi_q, \phi_q^*]}$$

$$S = \int_{t,\mathbf{x}} \left\{ \phi_q^* \frac{\delta \bar{S}[\phi_c]}{\delta \phi_c^*} + c.c. + i2\gamma \phi_q^* \phi_q \right\} \qquad \bar{S} = \int_{t,\mathbf{x}} \{ \phi_c^* i\partial_t \phi_c - \mathcal{H}_c + i\mathcal{H}_d \}$$

- Equilibrium conditions signalled by presence of symmetry under:

H. K. Janssen (1976); C. Aron et al, J Stat. Mech (2011)

$$\mathcal{T}_\beta \phi_c(t, \mathbf{x}) = \phi_c^*(-t, \mathbf{x}),$$

$$\mathcal{T}_\beta \phi_q(t, \mathbf{x}) = \phi_q^*(-t, \mathbf{x}) + \frac{i}{2T} \partial_t \phi_c^*(-t, \mathbf{x})$$

generalisation to quantum systems (Keldysh functional integral)

L. Sieberer, A. Chiochetta, U. Tauber, A. Gambassi, SD, PRB (2015)

- Implication 1 [equivalence]: (classical) fluctuation-dissipation

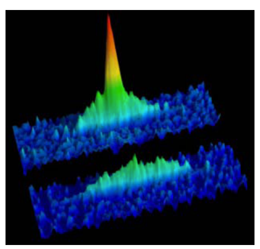
$$\underbrace{\langle \phi_c(\omega, \mathbf{q}) \phi_c^*(\omega, \mathbf{q}) \rangle}_{\text{correlations}} = \frac{2T}{\omega} \underbrace{\left[\langle \phi_c(\omega, \mathbf{q}) \phi_q^*(\omega, \mathbf{q}) - \langle \phi_c(\omega, \mathbf{q}) \phi_q^*(\omega, \mathbf{q}) \rangle \right.}_{\text{responses (imaginary part)}}$$

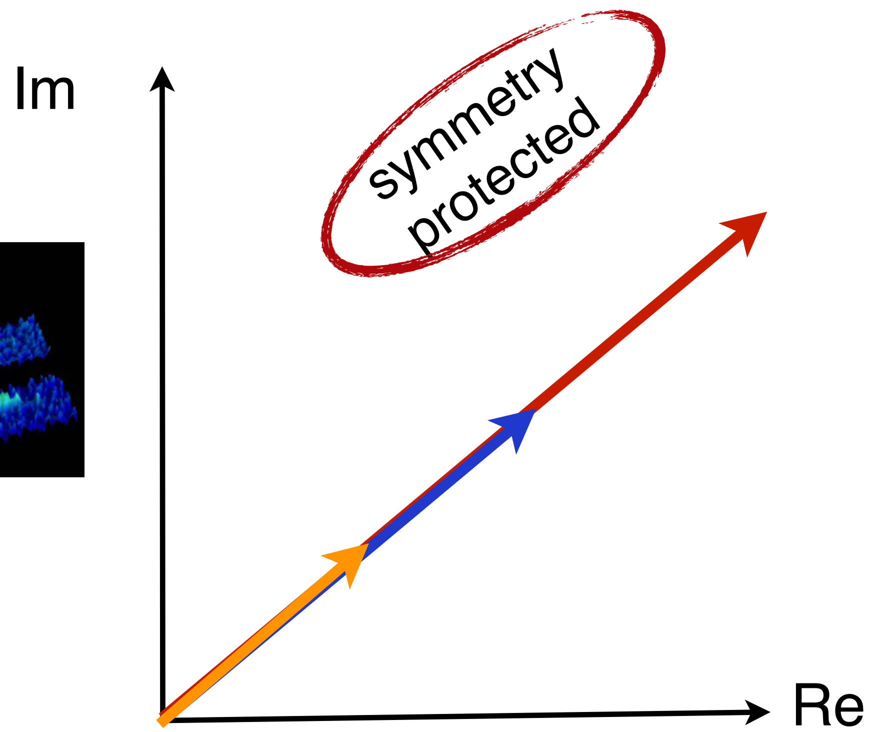
➡ equilibrium conditions as a symmetry

“What is non-equilibrium about it?”: Geometric interpretation

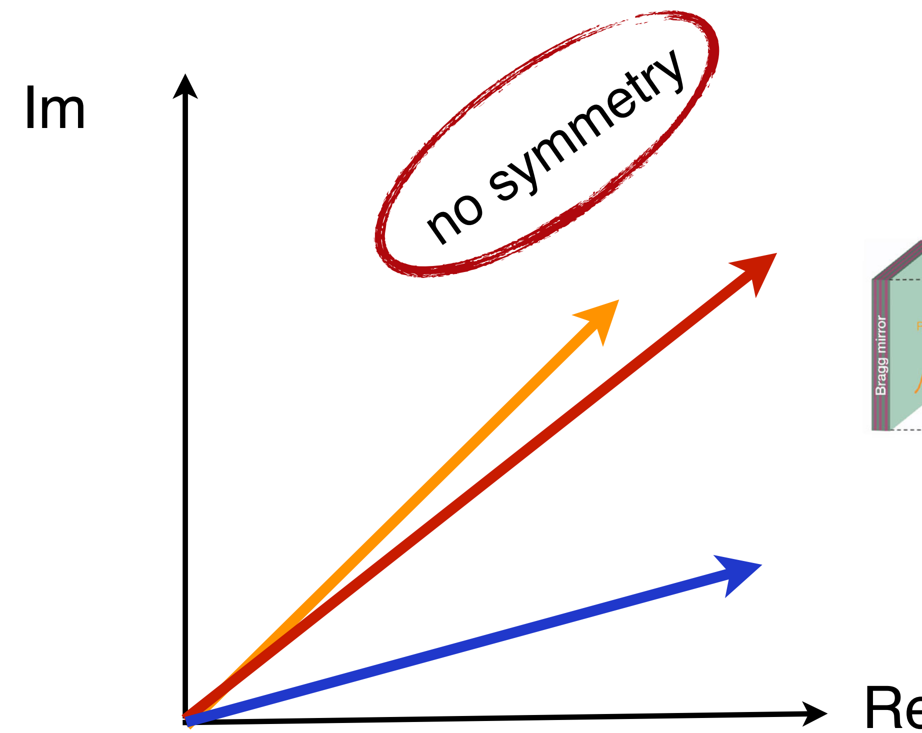
- Implication 2: geometric constraint

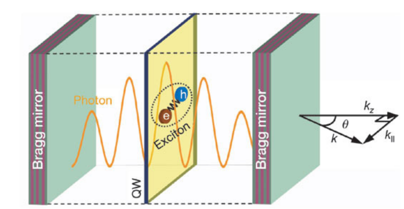
equilibrium dynamics





non-equilibrium dynamics





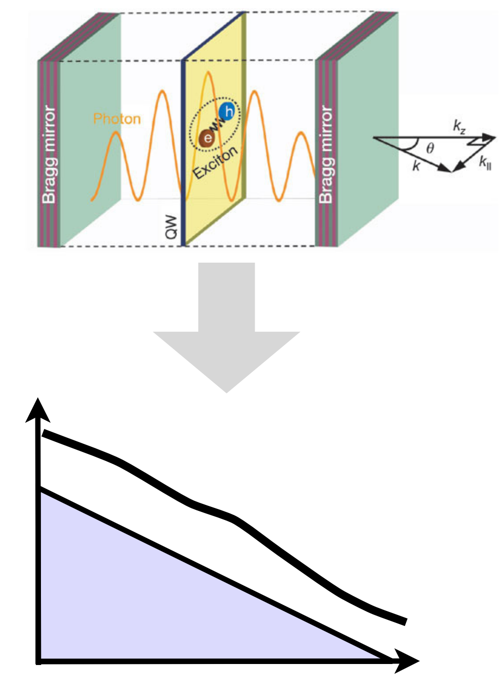
- coherent and dissipative dynamics may occur simultaneously
- but they are not independent

- coherent and driven-dissipative dynamics do occur simultaneously
- they result from different dynamical resources

➡ what are the physical consequences of the spread in the complex plane?

Review: L. Sieberer, M. Buchhold, SD, *Keldysh Field Theory for Driven Open Quantum Systems*, Reports on Progress in Physics (2016)

Outline



- mapping of the driven-dissipative GPE to KPZ-type equation
- fundamental difference to conventional context:

KPZ variable: condensate phase, compact

➡ weak non-equilibrium drive: two competing scales

- smooth non-equilibrium fluctuations -> emergent KPZ length scale L_*
- non-equilibrium vortex physics -> emergent length scale L_v

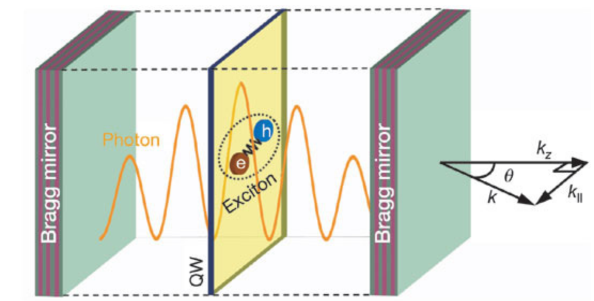
  - result: different order in 2D and 1D

➡ strong non-equilibrium drive: new first order phase transition (one dimension)

Low frequency phase dynamics

- driven-dissipative stochastic GPE

$$i\partial_t \phi = \left[-\frac{\nabla^2}{2m} - \mu + i(\gamma_p - \gamma_l) + (\lambda - i\kappa)\,|\phi|^2 \right] \phi + \zeta$$



- integrate out fast amplitude fluctuations: $\phi(\mathbf{x},t) = (M_0 + \chi(\mathbf{x},t))e^{i\theta(\mathbf{x},t)}$

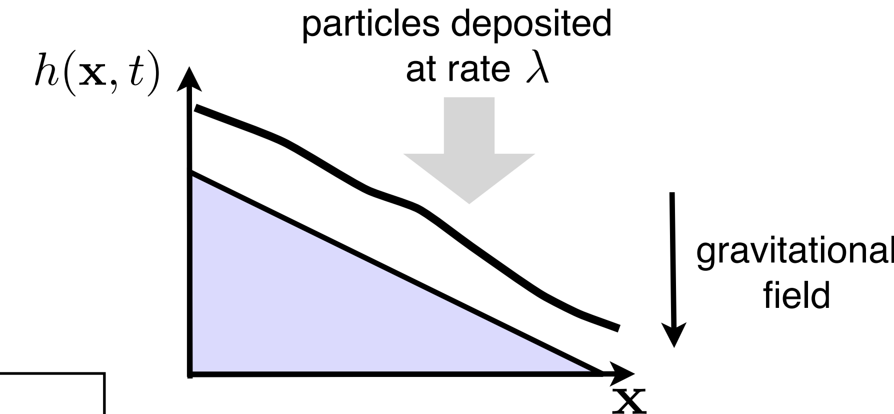
see also: G. Grinstein et al., PRL 1993

$$\partial_t \theta = D\nabla^2\theta + \lambda(\nabla\theta)^2 + \xi$$

phase diffusion   phase nonlinearity   Markov noise

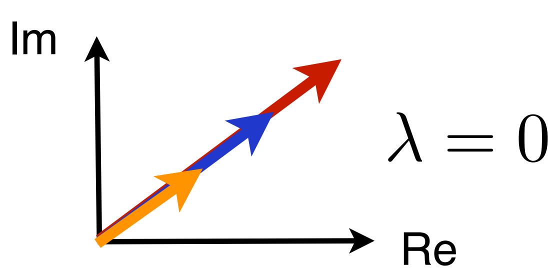
form of the KPZ equation

Kardar, Parisi, Zhang, PRL (1986)

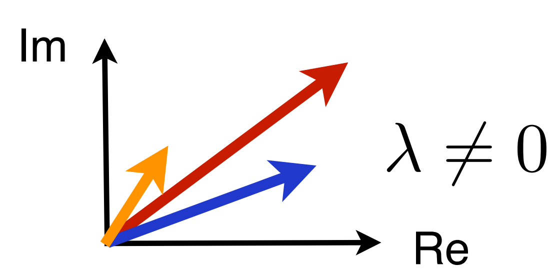


surface roughening, fire spreading, bacterial colony growth..

- spin wave becomes non-linear
- nonlinearity: single-parameter measure of non-equilibrium strength (ruled out in equilibrium by symmetry)

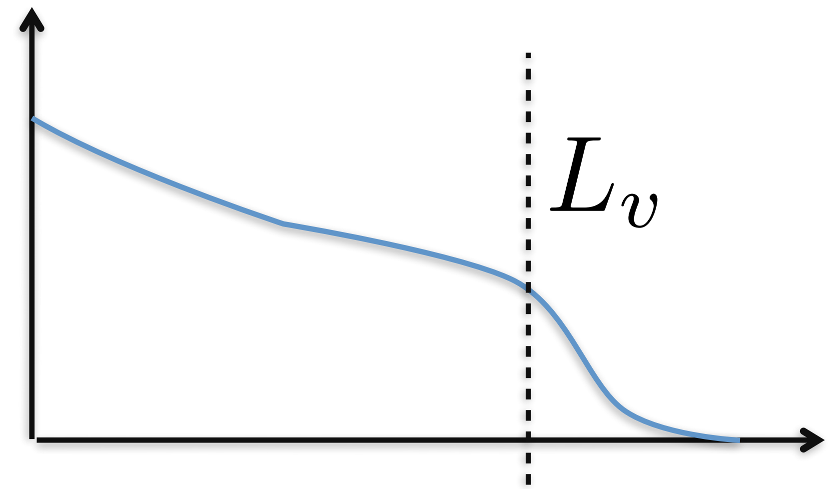


equilibrium



non-equilibrium

2 Dimensions



E. Altman, L. Sieberer, L. Chen, SD, J. Toner, PRX (2015)
G. Wachtel, L. Sieberer, SD, E. Altman, PRB (2016)
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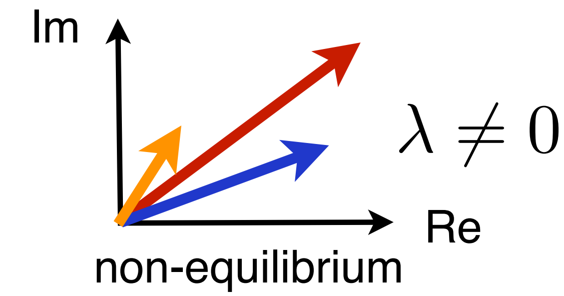
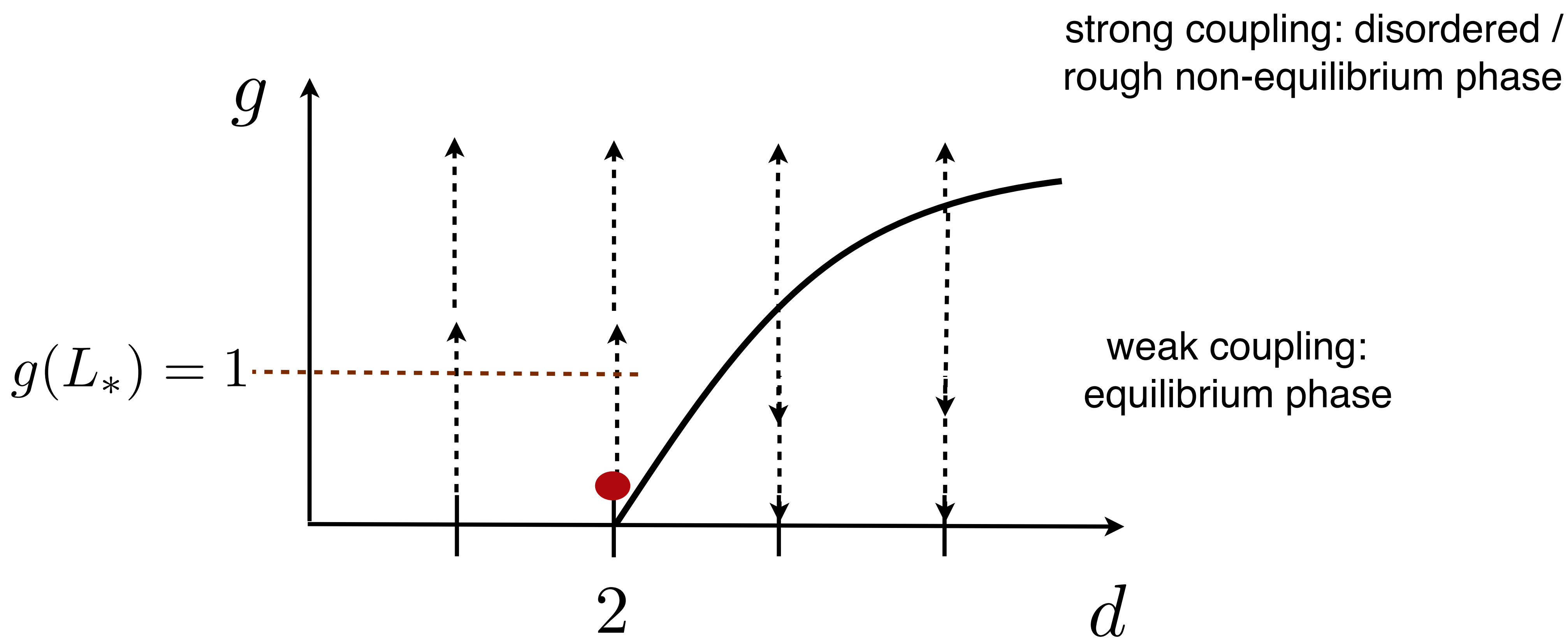
Physical implication I: Smooth KPZ fluctuations



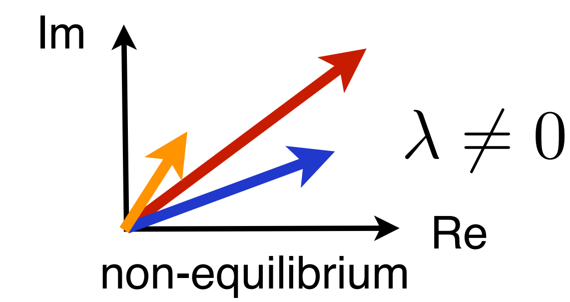
- RG flow of the effective dimensionless KPZ coupling parameter

$$g^2 = \frac{\lambda^2 \Delta}{D^3}$$



- general trend: non-equilibrium effects in systems with soft mode are
  - enhanced in d = 1,2
  - softened in d = 3 (below a threshold)

Physical implication I: Smooth KPZ fluctuations



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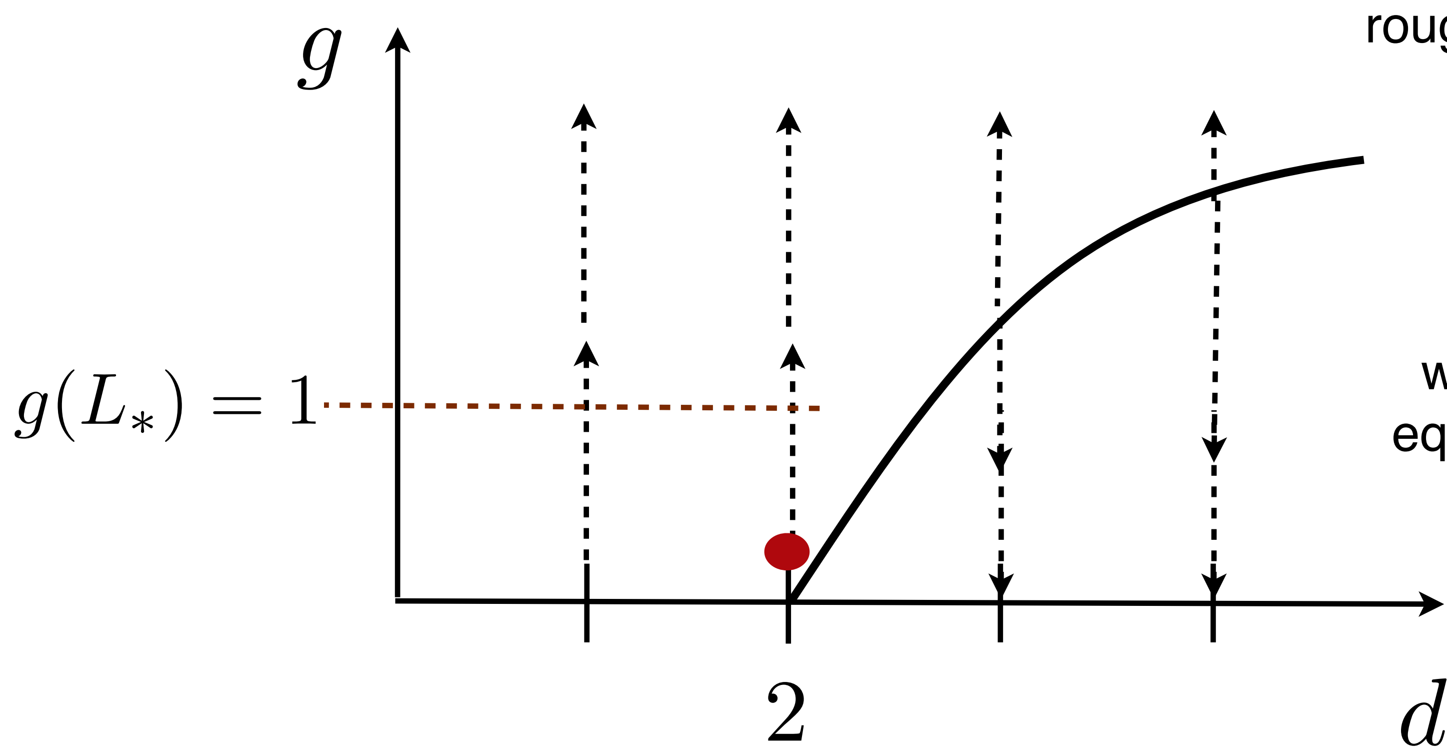


strong coupling: disordered / rough non-equilibrium phase

weak coupling: equilibrium phase

- 2D: implication: a length scale is generated

$$L_* = a_0 e^{\frac{16\pi}{g^2}}$$

microscopic (healing) length

- exponentially large for
  - weak nonequilibrium λ
  - small noise level Δ

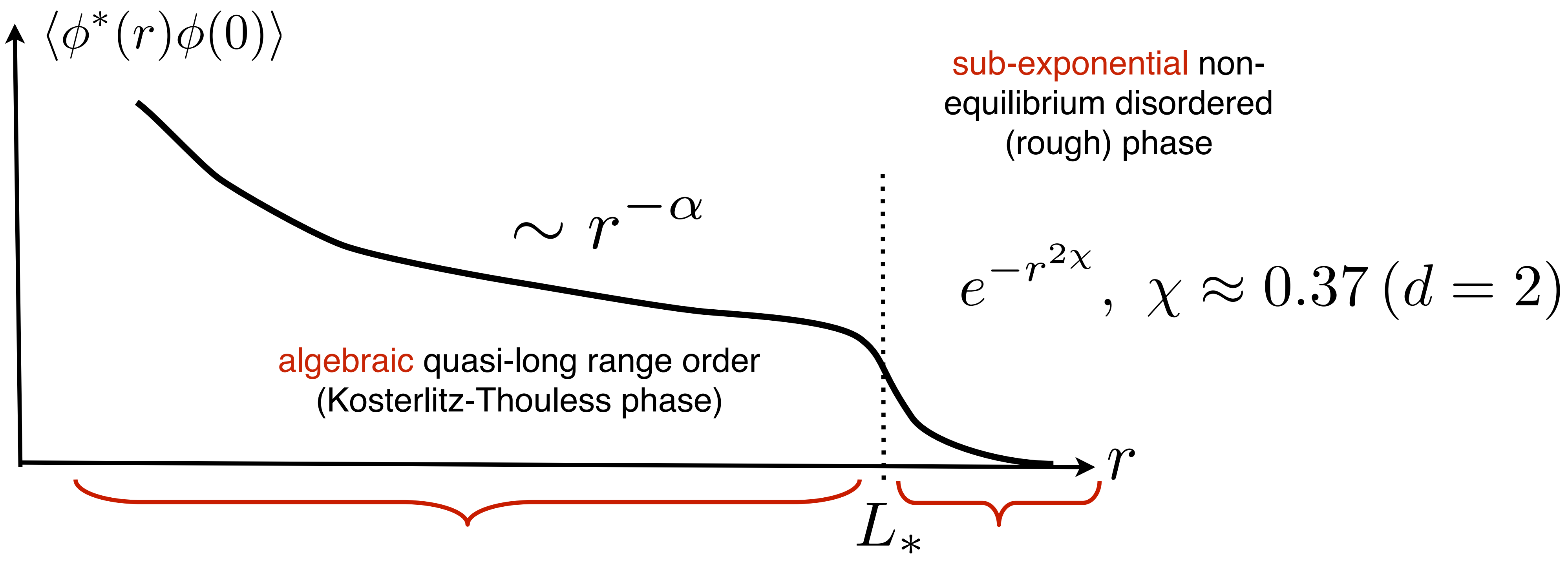
Physical implications I: Absence of quasi-LRO

- long-range behavior of two-point/ spatial coherence function:

$$\langle \phi^*(r)\phi(0)\rangle \approx n_0 e^{-\langle [\theta(\mathbf{x})-\theta(0)]^2\rangle}$$

leading order cumulant expansion

- generated length scale distinguishes two regimes: $L_* = a_0 e^{\frac{16\pi}{g^2}}$



➡ algebraic order absent in any two-dimensional driven open system at the largest distances

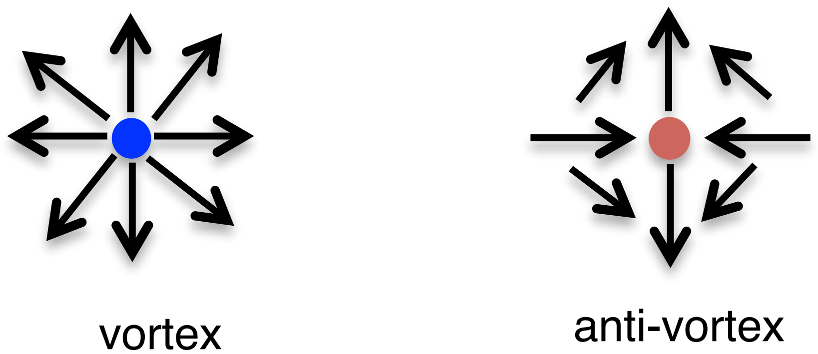
➡ but crossover scale exponentially large for small deviations from equilibrium (cf. Marzena's talk)

Physical implications II: Non-equilibrium Kosterlitz-Thouless

- KPZ equation for phase variable

$$\partial_t \theta = D\nabla^2\theta + \lambda(\nabla\theta)^2 + \xi$$

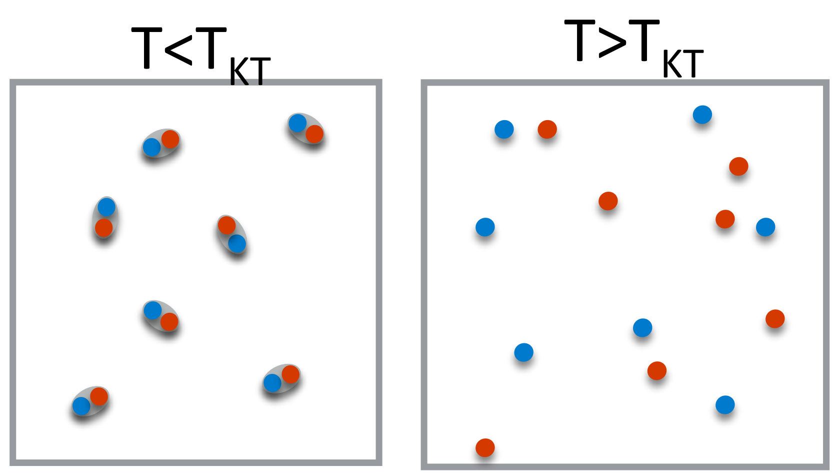
- compact nature of phase allows for vortex defects in 2D!



- in 2D equilibrium: perfect analogy between vortices and electric charges
  - log(r) interactions, $1/(\epsilon r)$ forces
  - dielectric constant ϵ^{-1}= superfluid stiffness $\mathbf{P} = (\varepsilon - 1)\,\mathbf{E}_{\rm ext}$

superfluid = dipole gas

$\epsilon^{-1} > 0$



normal fluid = plasma
metallic screening

$\epsilon^{-1} \to 0$

➡ how is this scenario modified in the driven system?

Duality approach

- KPZ equation for phase variable

$$\partial_t \theta = D\nabla^2\theta + \lambda(\nabla\theta)^2 + \xi$$

- phase compactness = local discrete gauge invariance of $\psi_{t,\mathbf{x}} = \sqrt{\rho_{t,\mathbf{x}}}e^{i\theta_{t,\mathbf{x}}}$

$$\theta_{t,\mathbf{x}} \mapsto \theta_{t,\mathbf{x}} + 2\pi n_{t,\mathbf{x}} \qquad \theta_{t,\mathbf{x}} \in [0, 2\pi), \quad n_{t,\mathbf{x}} \in \mathbf{Z}$$

➡ needs to be taught to the KPZ equation:

- deterministic part: lattice regularization

$$\partial_t \theta_{\mathbf{x}} = \underbrace{-\sum_{\mathbf{a}} \left[D\sin(\theta_{\mathbf{x}} - \theta_{\mathbf{x}+\mathbf{a}}) + \frac{\lambda}{2}\left(\cos(\theta_{\mathbf{x}} - \theta_{\mathbf{x}+\mathbf{a}}) - 1\right)\right]}_{=: \mathcal{L}[\theta]_{t,\mathbf{x}}} + \eta_{\mathbf{x}}$$

unit lattice direction

deterministic

noise

Duality approach

- KPZ equation for phase variable

$$\partial_t \theta = D\nabla^2\theta + \lambda(\nabla\theta)^2 + \xi$$

- phase compactness = local discrete gauge invariance of $\psi_{t,\mathbf{x}} = \sqrt{\rho_{t,\mathbf{x}}}e^{i\theta_{t,\mathbf{x}}}$

$$\theta_{t,\mathbf{x}} \mapsto \theta_{t,\mathbf{x}} + 2\pi n_{t,\mathbf{x}} \qquad \theta_{t,\mathbf{x}} \in [0, 2\pi), \quad n_{t,\mathbf{x}} \in \mathbf{Z}$$

- temporal part: stochastic update

$$\theta_{t+\epsilon,\mathbf{x}} = \theta_{t,\mathbf{x}} + \epsilon\left(\mathcal{L}[\theta]_{t,\mathbf{x}} + \eta_{t,\mathbf{x}}\right) + 2\pi n_{t,\mathbf{x}}$$

  - NB: phase can jump, continuum limit eps -> 0 ill defined, derivatives discrete

- stochastic difference equation -> discrete dynamical functional integral:

$$\boxed{Z = \sum_{\{\tilde{n}_{t,\mathbf{x}}\}} \int \mathcal{D}[\theta] e^{iS[\theta,\tilde{n}]}}$$

discrete noise -> manifest gauge invariance

$$Z = \int \mathcal{D}[\tilde{\theta}]\mathcal{D}[\theta] e^{iS[\theta,\tilde{\theta}]}$$

vs. continuous variable

$$S = \sum_{t,\mathbf{x}} \tilde{n}_{t,\mathbf{x}} \left[-\Delta_t\theta_{t,\mathbf{x}} + \epsilon\left(\mathcal{L}[\theta]_{t,\mathbf{x}} + i\Delta\tilde{n}_{t,\mathbf{x}}\right)\right]$$

Duality approach

- discrete gauge invariant dynamical functional integral

$$\boxed{Z = \sum_{\{\tilde{n}_{t,\mathbf{x}}\}} \int \mathcal{D}[\theta] e^{iS[\theta,\tilde{n}]}}$$

$$S = \sum_{t,\mathbf{x}} \tilde{n}_{t,\mathbf{x}} \left[-\Delta_t \theta_{t,\mathbf{x}} + \epsilon \left(\mathcal{L}[\theta]_{t,\mathbf{x}} + i\Delta\tilde{n}_{t,\mathbf{x}} \right) \right]$$

- introduce Fourier conjugate variables, use continuity equations to parameterise in terms of gauge fields, Poisson transform
- dual description:

$$Z \propto \underbrace{\sum_{\substack{\{n_{vX},\tilde{n}_{vX}, \\ \mathbf{J}_{vX},\tilde{\mathbf{J}}_{vX}\}}}}_{\substack{\text{vortex density} \\ \text{and current}}} \underbrace{\int \mathcal{D}[\phi,\tilde{\phi},\mathbf{A},\tilde{\mathbf{A}}]}_{\substack{\text{smooth spin wave fluctuations} \\ \text{(equivalent KPZ equation)}}} e^{iS[\phi,\tilde{\phi},\mathbf{A},\tilde{\mathbf{A}},n_v,\tilde{n}_v,\mathbf{J}_v,\tilde{\mathbf{J}}_v]}$$

- interpretation: study the associated Langevin equations

Electrodynamic Duality

- Langevin equations = modified nonlinear noisy Maxwell equations

- formulated in electric and magnetic fields alone:

$$\mathbf{E} = -\nabla\phi - \mathbf{A}, \qquad \tilde{\mathbf{E}} = -\nabla\phi - \partial_t\tilde{\mathbf{A}},$$
$$\mathbf{B} = D\nabla\times\mathbf{A} \qquad \tilde{\mathbf{B}} = \nabla\times\tilde{\mathbf{A}}$$

fixed by modified gauge invariance

irrotational flow

$$\nabla\cdot\mathbf{E} = 2\pi n_v$$

modified continuity eq

$\partial_t \to 1/D$

$$\nabla\times\mathbf{E} + \frac{1}{D}\mathbf{B} = 0$$

phase dynamics

$$\nabla\times\mathbf{B} - \frac{\partial\mathbf{E}}{\partial t} = 2\pi\mathbf{J}_v - \hat{\mathbf{z}}\times\nabla\left(\frac{\lambda}{2}E^2 + \bar{\zeta}\right)$$

$$\nabla\cdot\mathbf{B} = 0$$

vortex density & current

KPZ non-linearity and noise

phenomenologically added vortex dynamics

$$\frac{d\mathbf{r}_i}{dt} = \mu n_i\mathbf{E}(t,\mathbf{r}_i) + \boldsymbol{\xi}_i$$

- reproducing KPZ: identify $\mathbf{E} \equiv \hat{\mathbf{z}}\times\nabla\theta$ & integrate out magnetic field, neglect vortices

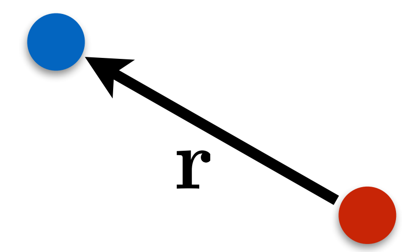
$$\partial_t\theta = D\nabla^2\theta + \lambda(\nabla\theta)^2 + \xi$$

- next: integrate out gapless electric field degrees of freedom = phase fluctuations
  - equilibrium $\lambda = 0$: exactly
  - non-equilibrium: perturbatively in λ

A single vortex-antivortex pair

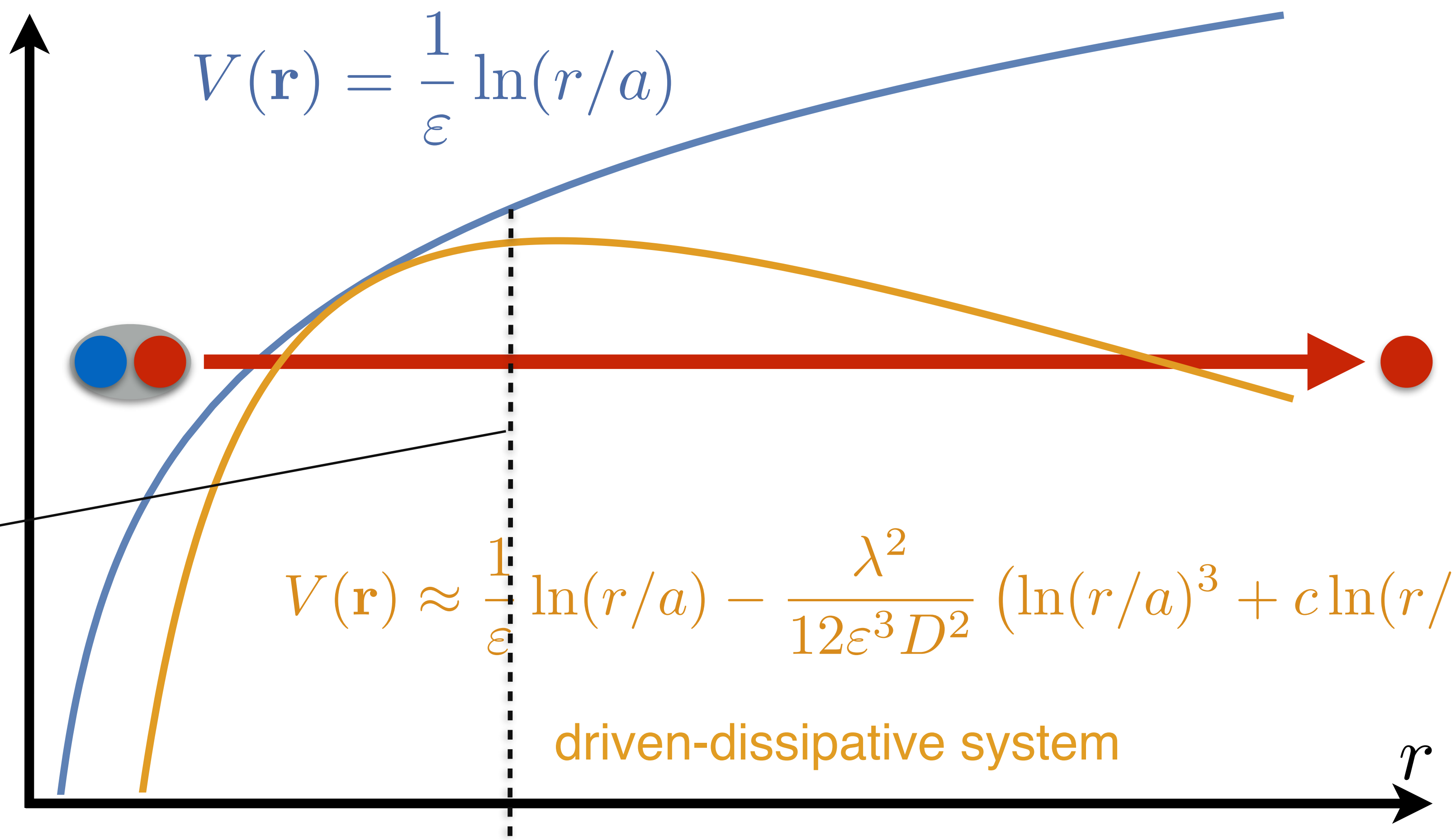
- equation of motion for a single vortex-antivortex pair

$$\frac{d\mathbf{r}}{dt} = -\mu \nabla V(r) + \boldsymbol{\xi}$$



equilibrium: Coulomb potential (2D)

$$V(\mathbf{r}) = \frac{1}{\varepsilon} \ln(r/a)$$



length scale:

$$L_v = a_0 e^{\frac{2D}{\lambda}}$$

see also: I Aranson et al., PRB (1998) two-vortex problem

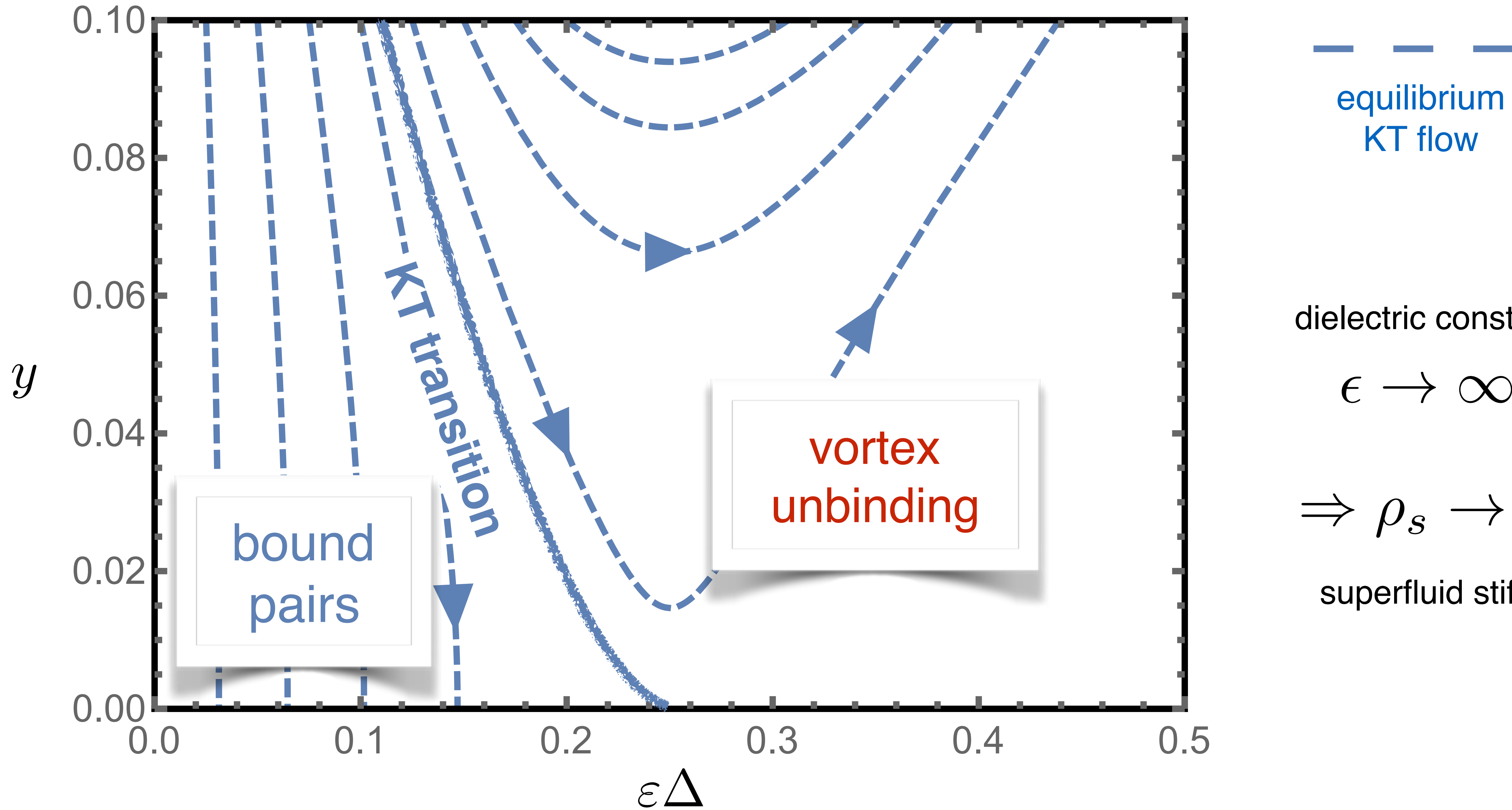
$$V(\mathbf{r}) \approx \frac{1}{\varepsilon} \ln(r/a) - \frac{\lambda^2}{12\varepsilon^3 D^2} \left(\ln(r/a)^3 + c \ln(r/a)^2 \right)$$

driven-dissipative system

➡ noise-activated unbinding for a single pair (at exp small rate)

Many pairs: Modified Kosterlitz-Thouless RG flow

$$\frac{d\varepsilon}{d\ell} = \frac{2\pi^2 y^2}{T} \qquad \frac{dy}{d\ell} = \left[2 - \frac{1}{2\varepsilon T} + \cancel{\frac{\lambda^2}{4\varepsilon^2 D^2}\left(\frac{1}{4}+\ell\right)}\right] y \qquad \frac{dT}{d\ell} = \cancel{\frac{\lambda^2 T}{2\varepsilon^2 D^2}\left(\frac{1}{4}+\ell\right)}$$



dielectric constant

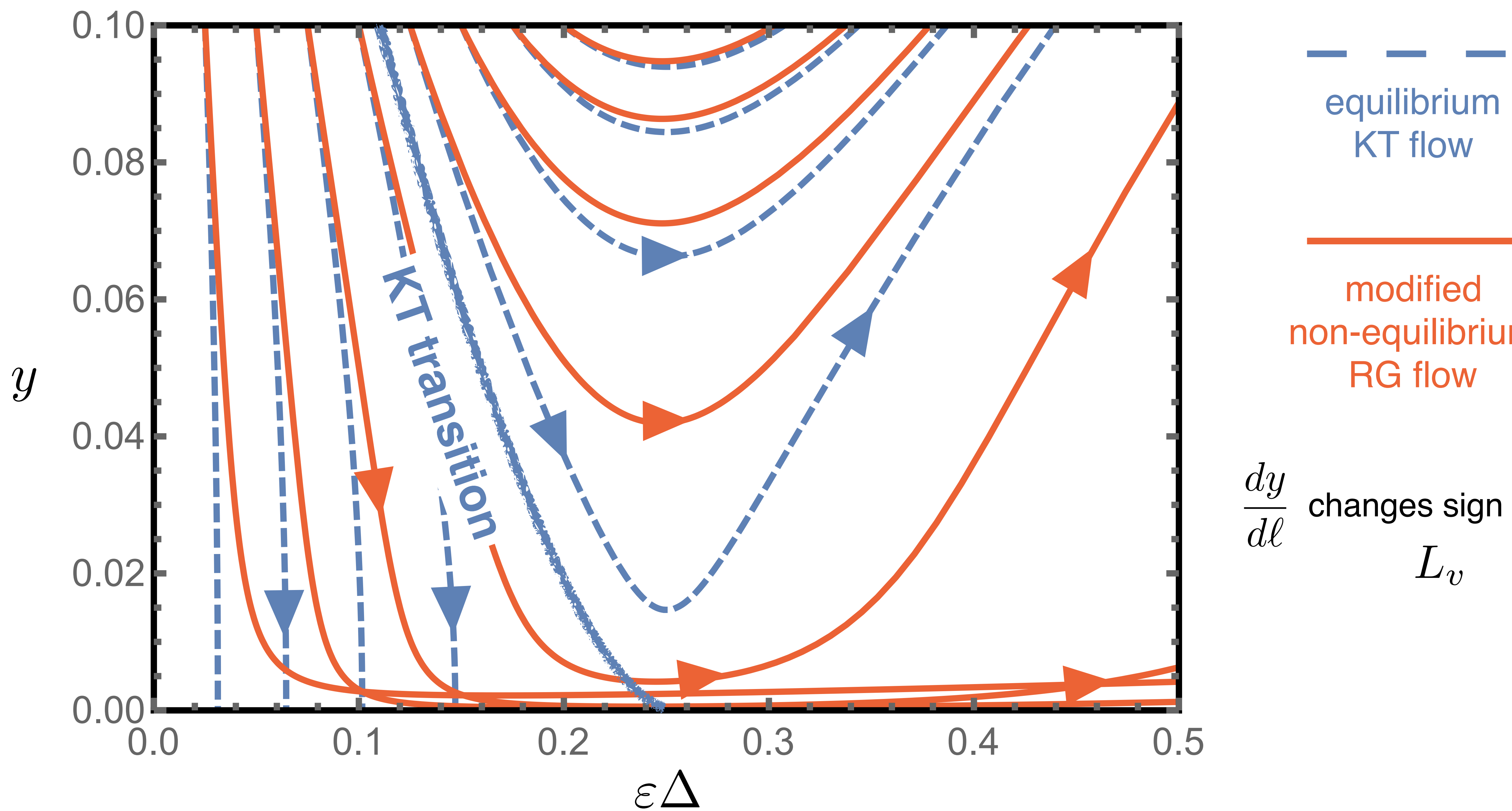
$$\epsilon \to \infty$$

$$\Rightarrow \rho_s \to 0$$

superfluid stiffness

Many pairs: Modified Kosterlitz-Thouless RG flow

$$\frac{d\varepsilon}{d\ell} = \frac{2\pi^2 y^2}{T} \qquad \frac{dy}{d\ell} = \left[2 - \frac{1}{2\varepsilon T} + \frac{\lambda^2}{4\varepsilon^2 D^2}\left(\frac{1}{4} + \ell\right)\right] y \qquad \frac{dT}{d\ell} = \frac{\lambda^2 T}{2\varepsilon^2 D^2}\left(\frac{1}{4} + \ell\right)$$



$\frac{dy}{d\ell}$ changes sign at a scale L_v

➡ vortex unbinding for any value of the noise strength

E. Altman, L. Sieberer, L, Chen, SD, J. Toner, PRX (2015)
L. Sieberer, G. Wachtel, E. Altman, SD, PRB (2016)
G. Wachtel, L. Sieberer, SD, E. Altman, PRB (2016)

Summary: 2D

- two emergent length scales in complementary approaches:

$$L_* = a_0 e^{\frac{16\pi}{g^2}}$$

KPZ length

$$L_v = a_0 e^{\frac{2D}{\lambda}}$$

vortex length

- scaling for the relevant fixed points

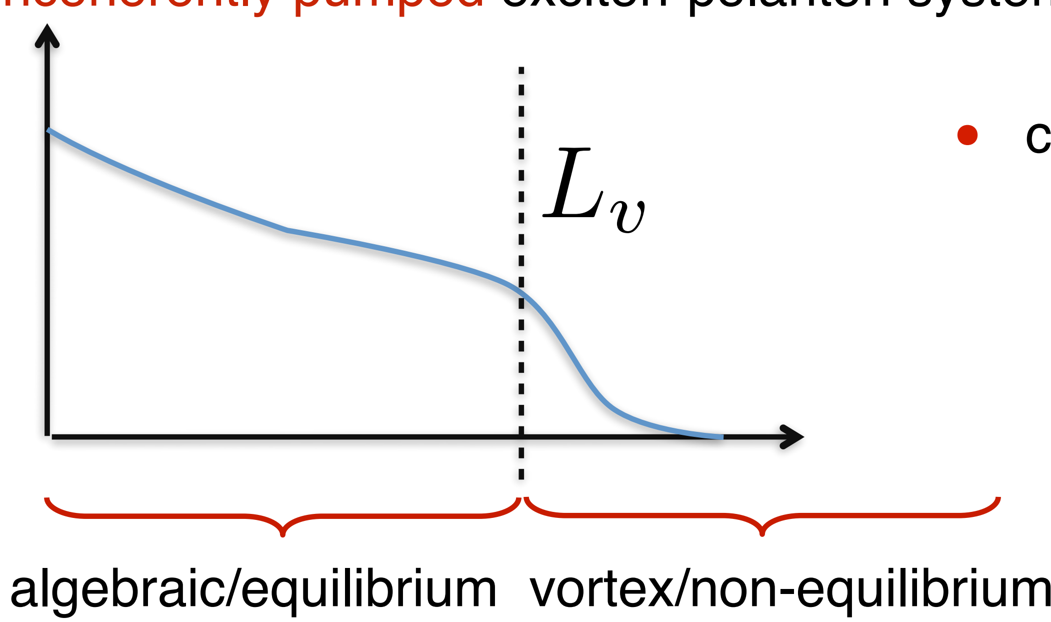
$$\langle \phi^*(r)\phi(0)\rangle \sim e^{-r^{2\chi}}, \quad \chi = 0.4$$

KPZ fixed point

$$\langle \phi^*(r)\phi(0)\rangle \sim e^{-r}$$

free vortex/disordered fixed point

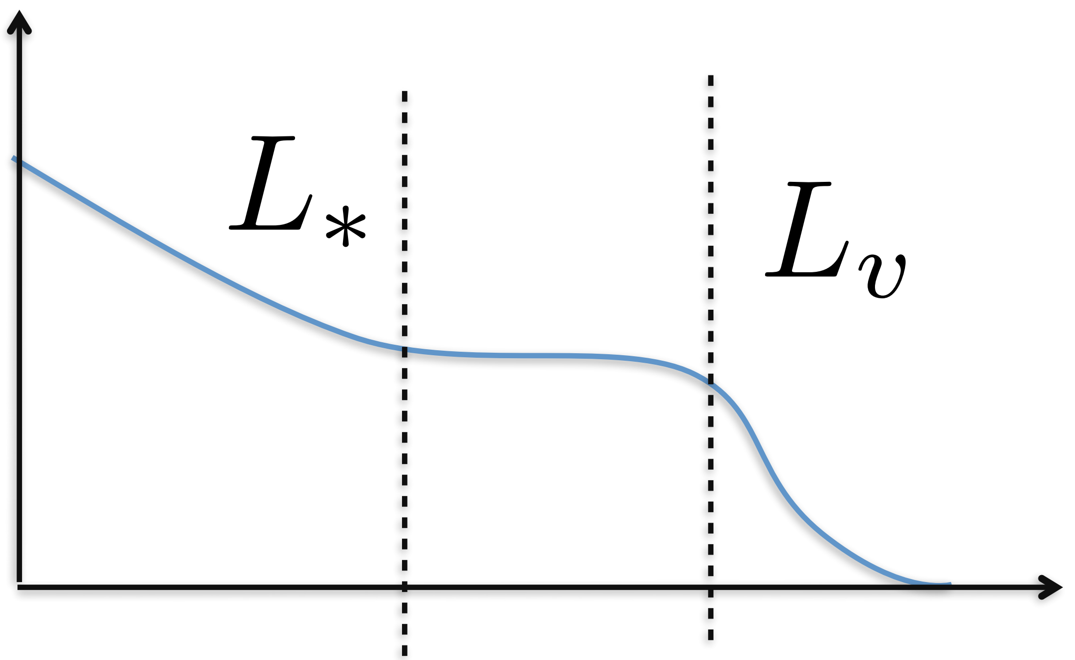
- for incoherently pumped exciton-polariton systems, $L_v \ll L_*$



- caveats for observability:
  - length scales exponentially large
  - assumes stationary states (unknown non-universal vortex dynamics)

coherently pumped: see Marzena's talk!

1 Dimension



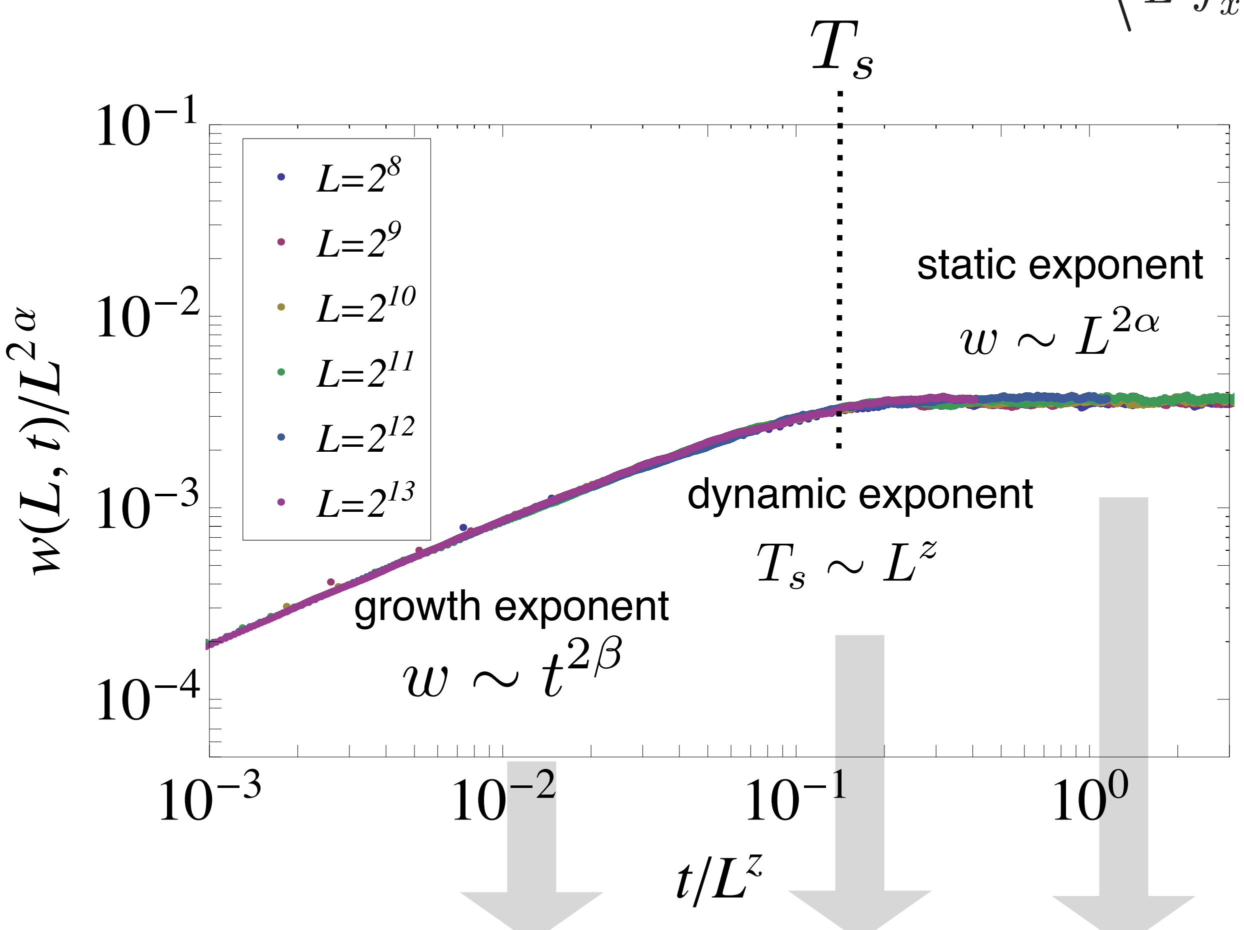
L. He, L. Sieberer, E. Altman, SD, PRB (2015)
L. He, L. Sieberer, SD, arxiv (2016)

L. He, L. Sieberer, E. Altman, SD, PRB (2015)
see also: K. Yi, V. Gladilin, M. Wouters, PRB (2015)

KPZ exponents & new scaling regime

- direct numerical solution of driven-dissipative GPE in one dimension

- observable: phase correlations $w(L,t) \equiv \left\langle \frac{1}{L}\int_x \theta^2(x,t) - \left(\frac{1}{L}\int_x \theta(x,t)\right)^2 \right\rangle$



	$\beta \approx 1/3$	$z \approx 3/2$	$\alpha \approx 1/2$
vs. eq.:	$\beta = 1/4$	$z = 2$	$\alpha = 1/2$

dynamic correlations needed to certify non-equilibrium!

➡ KPZ scaling fully confirmed in phase correlations

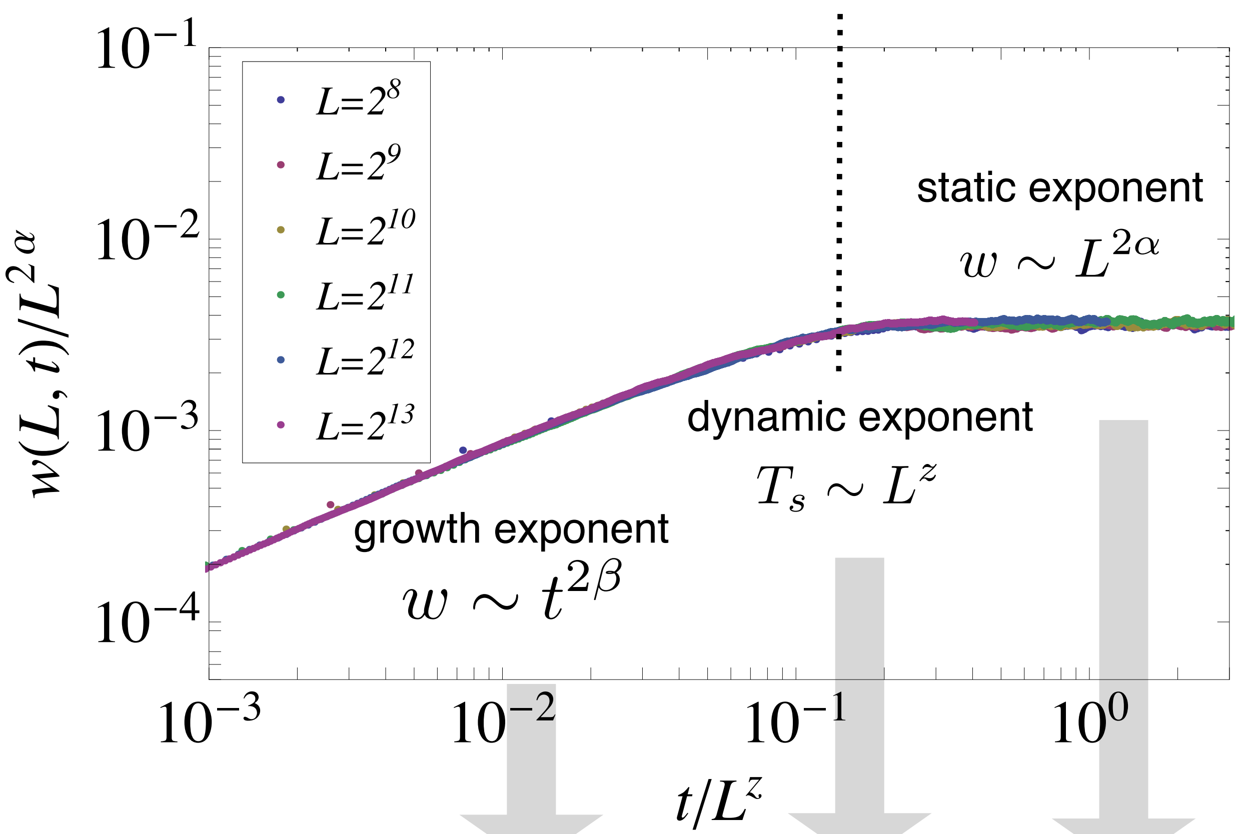
➡ experimentally accessible with “bad cavities” (lifetime 1ps, system size 150 mu m)

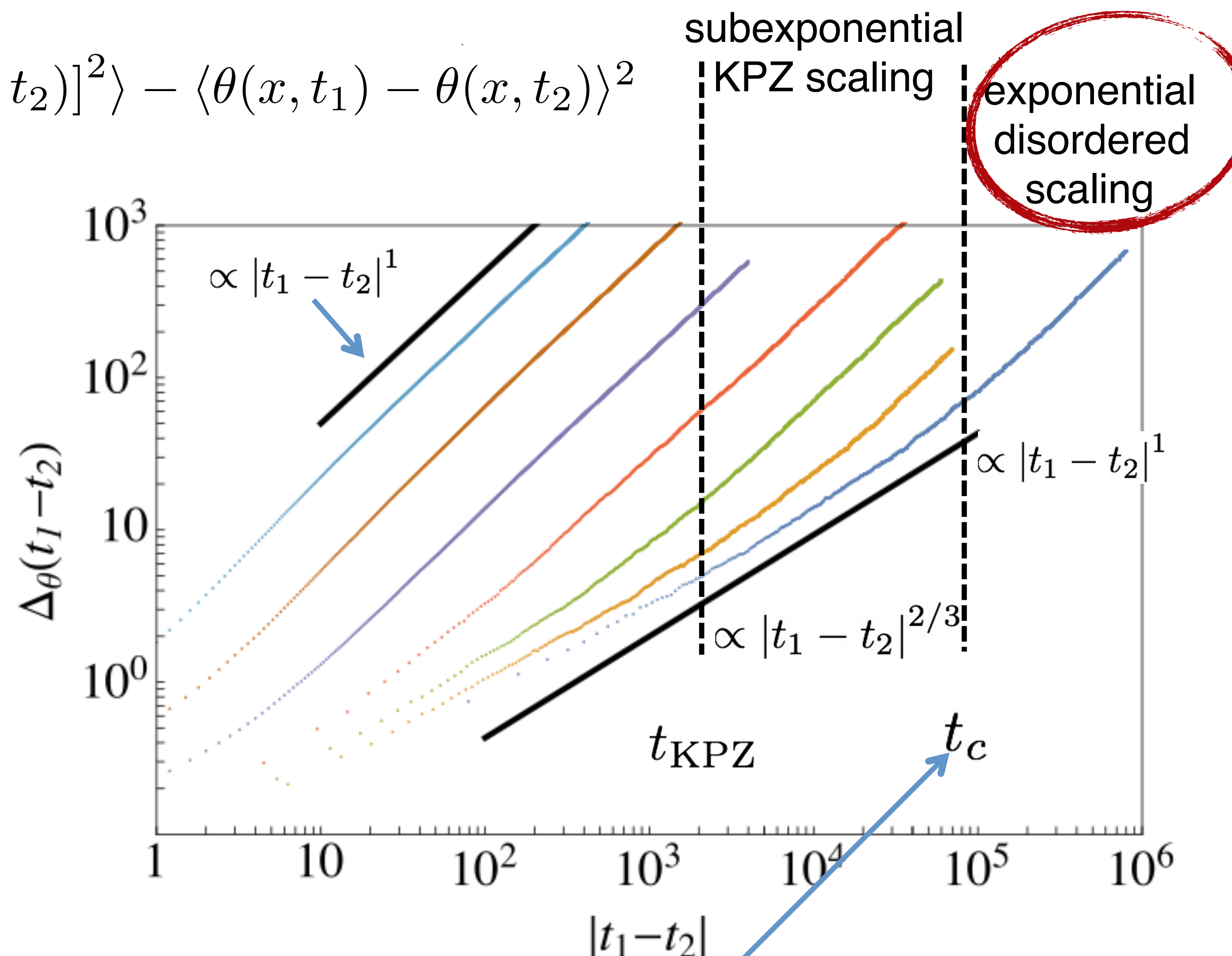
L. He, L. Sieberer, E. Altman, SD, PRB (2015)
see also: K. Yi, V. Gladilin, M. Wouters, PRB (2015)

KPZ exponents & new scaling regime

- direct numerical solution of driven-dissipative GPE in one dimension
- observable: temporal phase correlations

$$\Delta_\theta(t_1 - t_2) \equiv \frac{1}{L}\int_0^L dx \langle[\theta(x,t_1) - \theta(x,t_2)]^2\rangle - \langle\theta(x,t_1) - \theta(x,t_2)\rangle^2$$





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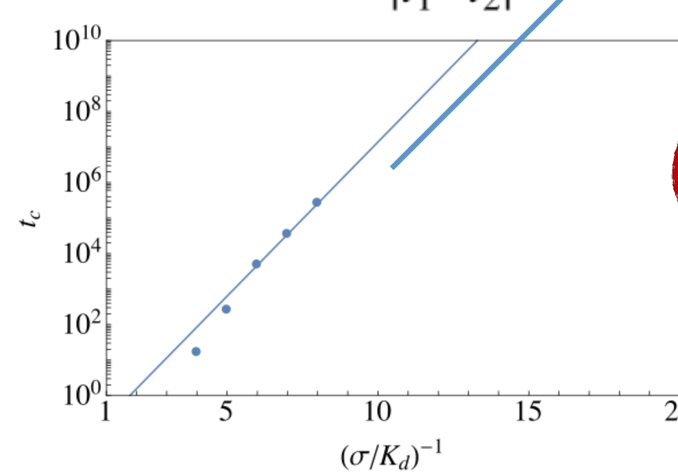


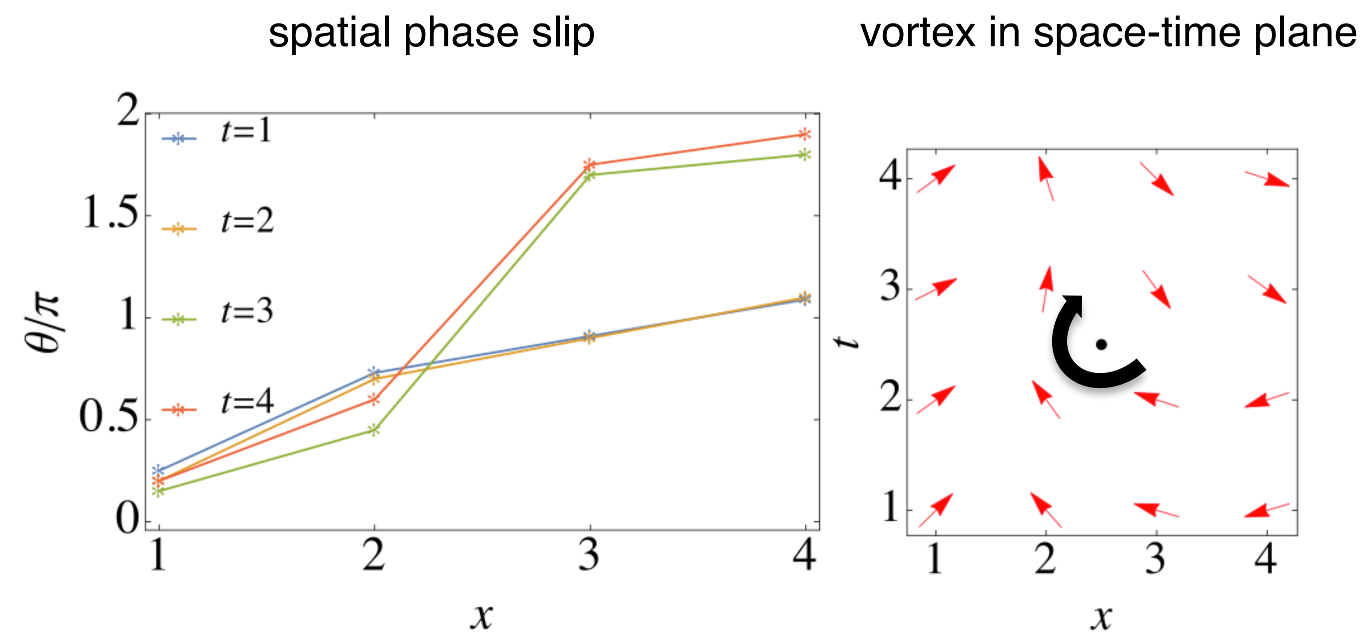
exponential dependence on noise level

$$t_c \propto e^{A\cdot\sigma^{-1}}$$

Space-time vortices in 1D XP condensate

- Physical origin: compactness of phase field

  topologically nontrivial phase field configurations on (1+1)D space-time plane

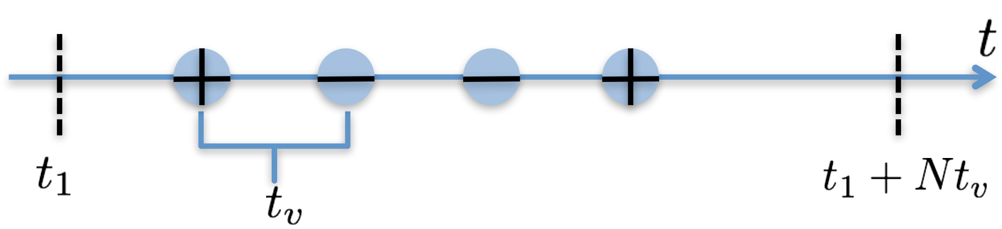


- unbound at infinitesimal noise level (weak non-equilibrium)
  - interaction potential: $(\partial_t + D\partial_x^2)^{-1} \sim (Dt)^{-1/2} e^{-x^2/(4Dt)}$

cf. 2D static equilibrium: $\nabla^{-2} \sim \log(|\mathbf{x}|)$

- explains qualitative features

  1. temporal scaling:
     - random uncorrelated charges $W_i = \pm 1$
     - phase field jumps by $\pm\delta\Theta$ when crossing vortex core



$$\Delta_\theta(t_1 - t_2) = |t_1 - t_2|\,(\delta\Theta)^2/t_v$$

➡ "disordered" scaling $\langle \psi^*(x,t')\psi(x,t)\rangle \sim e^{-c|t-t'|}$

Origin of exponential scaling with noise level

2. noise level dependence of crossover scale $t_v \propto e^{A\cdot\sigma^{-1}}$

- Onsager-Machlup functional integral: probability distribution for solutions $\theta(x,t)$ of cKPZ:

$$Z = \int \mathcal{D}[\theta] e^{-\mathcal{H}[\theta]/T} \qquad\qquad T = 4\sigma$$

$$\mathcal{H}[\theta] = \int_{t_0}^{t_1} dt \int_0^L dx \left[\partial_t\theta - D\partial_x^2\theta - \frac{\lambda}{2}\left(\partial_x\theta\right)^2\right]^2$$

- for $\lambda = 0$: maps to static 2D active smectic A liquid crystal problem Toner and Nelson, PRB (1984)

vortex core energy

- Arrhenius activation probability density $P_v \sim e^{-E_v/T} \sim (t_v x_v)^{-1}$
- vortex unbinding time scale

$$t_v \sim e^{\frac{(z+1)}{4z} E_v/\sigma}$$

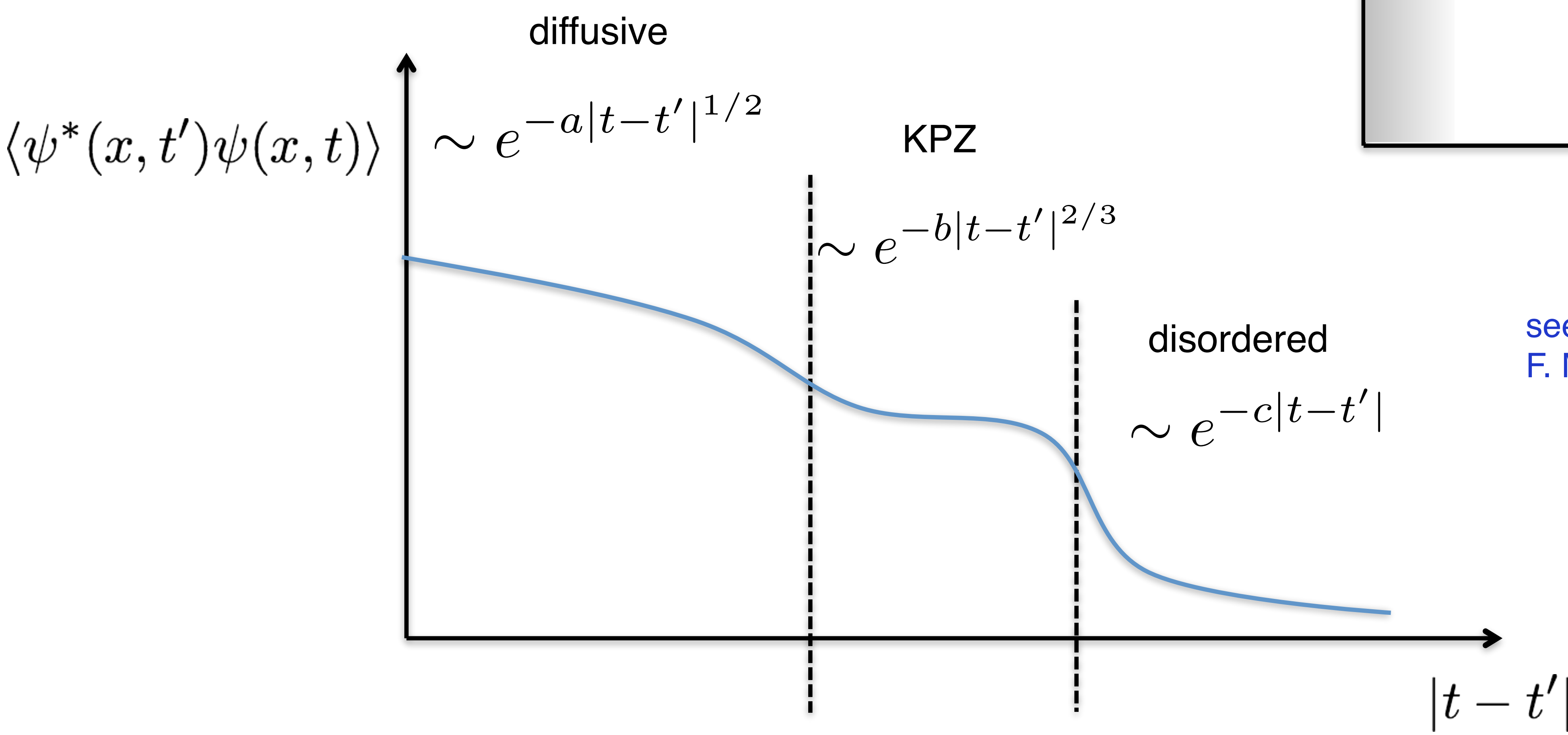
alternative at equilibrium: phase slips Langer, Ambegaokar, Phys. Rev. (1967); McCumber, Halperin, PRB (1970)

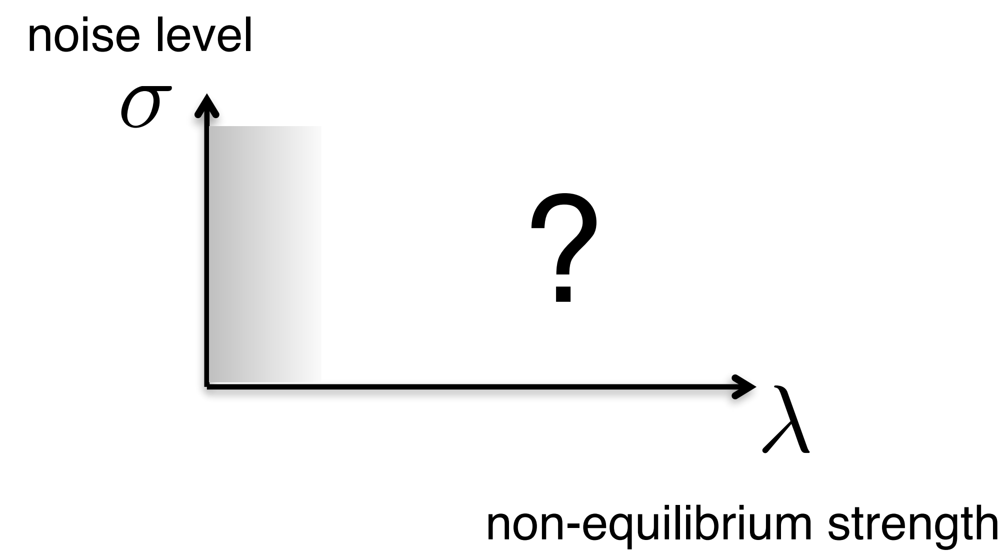
- for small $\lambda \neq 0$: finite upper bound $\mathcal{H}_{\lambda=0}[\theta_v] - \mathcal{H}_\lambda[\theta_v] > 0$

  ➡ exponential law not modified

Summary: 1D condensates at weak non-equilibrium

- temporal coherence function:





see also: R. Lauter, A. Mitra, F. Marquardt, arxiv (2016)

- crossover scales (weak noise sigma)

$$t_{\mathrm{KPZ}} \propto \sigma^{-2} \qquad t_c \propto e^{A\cdot\sigma^{-1}}$$

algebraic exponential

➡ KPZ scaling should be observable in exciton-polariton experiments in 1D

Strong non-equilibrium: Compact KPZ vortex turbulence

- deterministic dynamical instability in compact KPZ:

$$\partial_t \theta_i = -D\left(\sin\left(\theta_i - \theta_{i+1}\right) + \sin\left(\theta_i - \theta_{i-1}\right)\right) + \lambda\left(\left[\sin\left(\frac{\theta_i - \theta_{i+1}}{2}\right)\right]^2 + \left[\sin\left(\frac{\theta_i - \theta_{i-1}}{2}\right)\right]^2\right)$$

- EOM of phase differences between n.n. sites: $\Delta_i \equiv \theta_i - \theta_{i+1}$

$$\partial_t \Delta_i \simeq -3D\Delta_i + \frac{\lambda}{4}\left(\left(\Delta_{i-1}\right)^2 - \left(\Delta_{i+1}\right)^2\right)$$

decreases amplifies

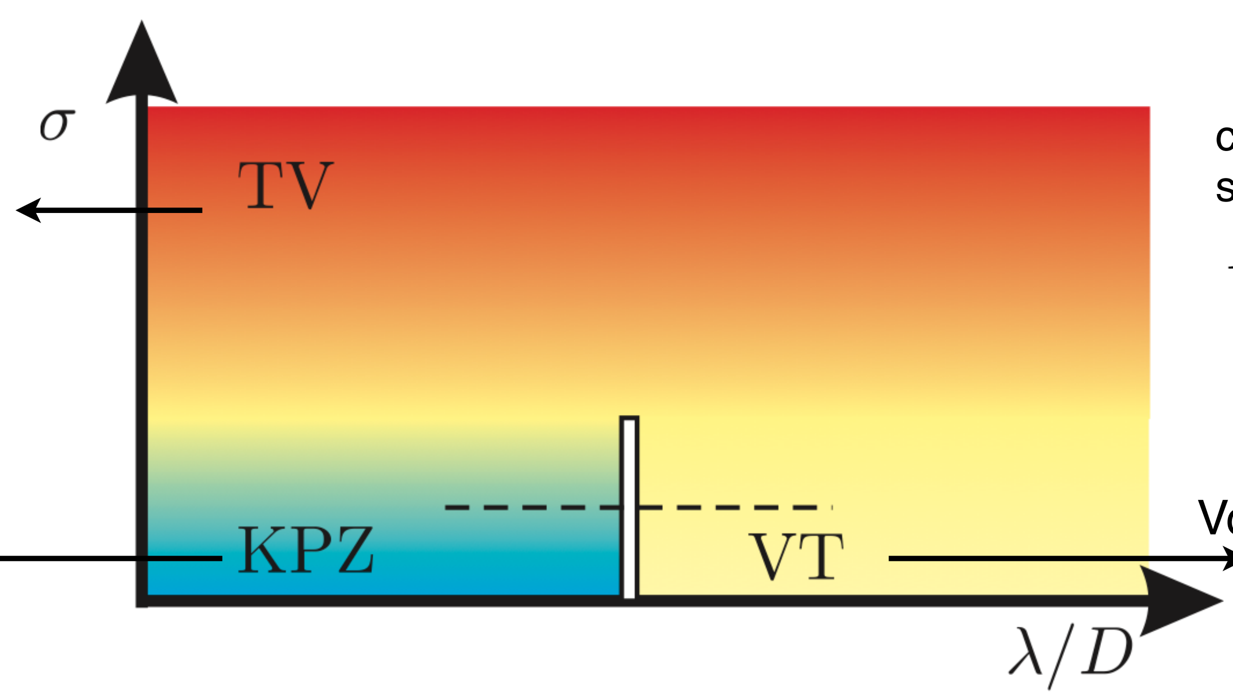
- $\lambda \gg D$ amplification even by small phase fluctuations
- continuous creation and annihilation of vortices --- “vortex turbulence”

chaotic solutions nonlinear dynamics:
e.g. Aranson et al., RMP (2002)

- Phase diagram for XP condensates

Thermally excited vortices (TV)

KPZ (conventional) dominated physics

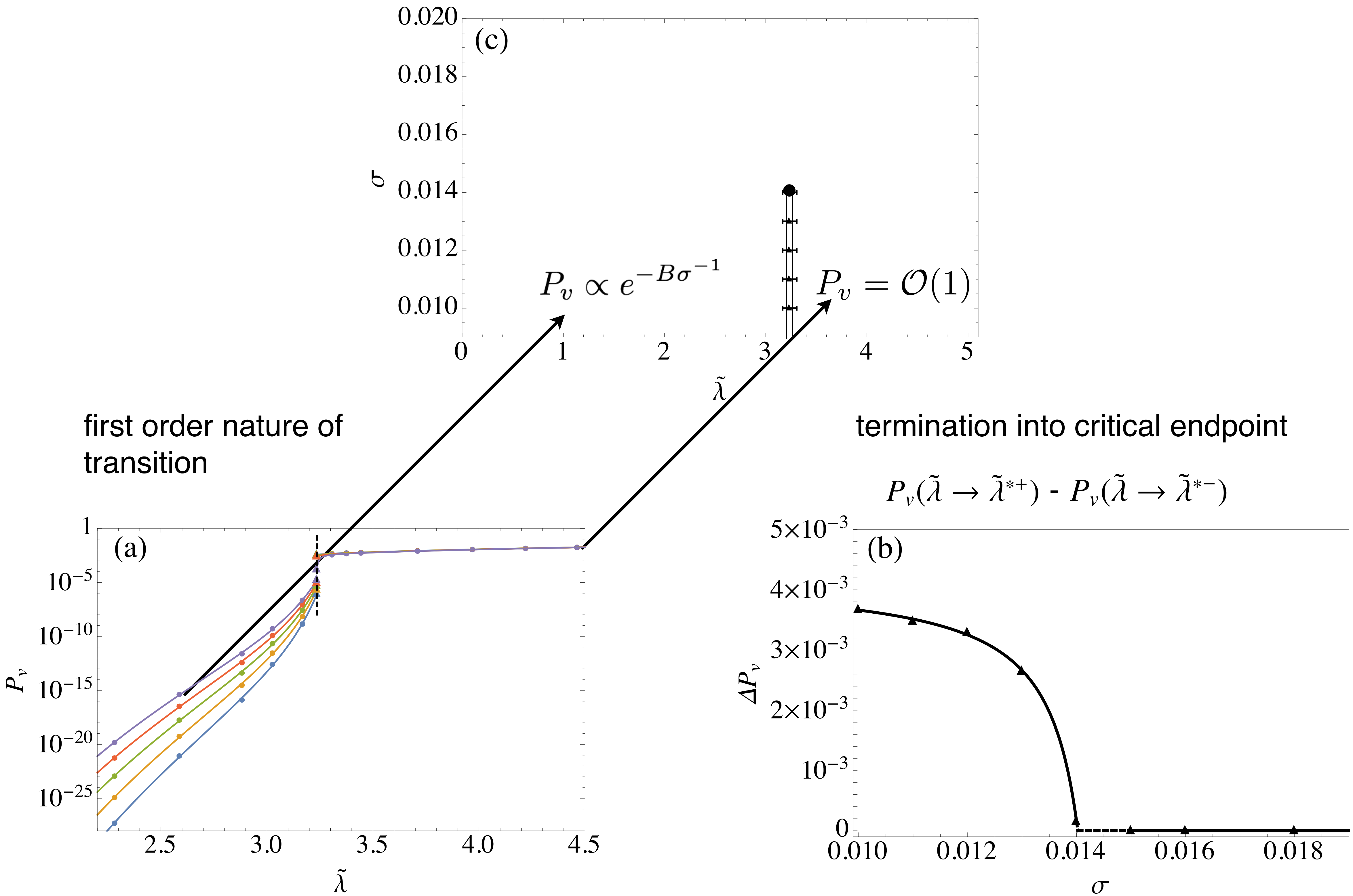


color code: vortex density on space-time plane P_v

Vortex turbulence (VT)

Strong non-equilibrium: Compact KPZ vortex turbulence

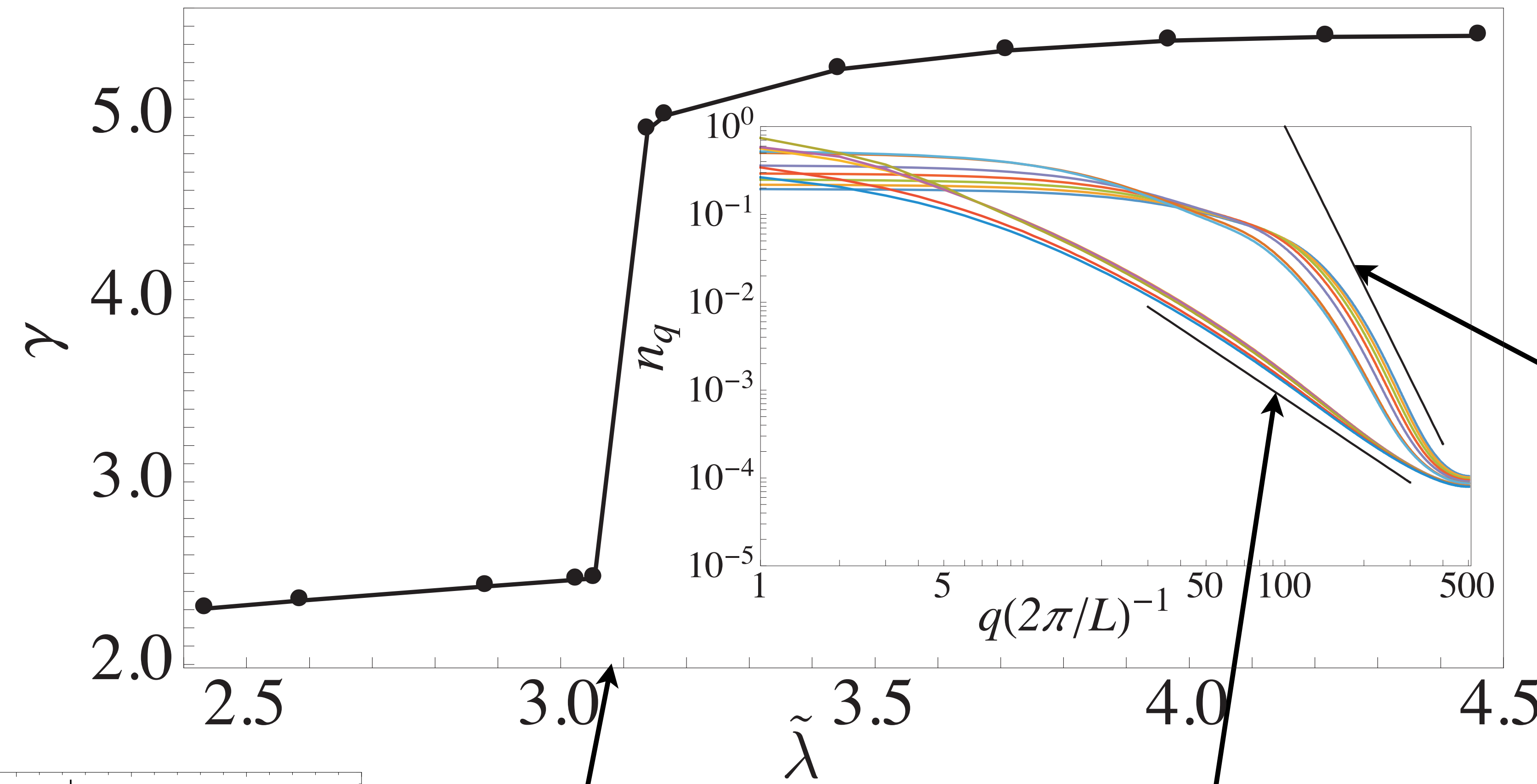
- quantitatively: transition in weak noise regime induced by nonequilibrium strength



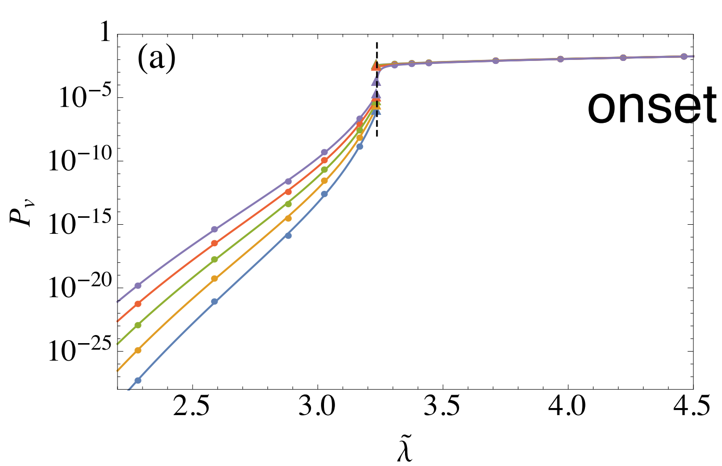
Compact KPZ vortex turbulence: Signatures

- scaling of the momentum distribution at intermediate momenta (full stochastic GPE)

$$n_q = \langle \psi^*(q)\psi(q)\rangle \sim q^{-\gamma}$$



scaling due to turbulent vortices: $\gamma \approx 5$



onset of vortex turbulence

scaling due to thermally activated vortices: $\gamma \approx 2$

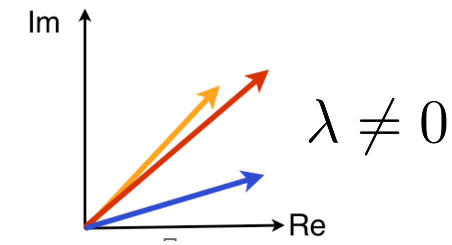
diffusion constant

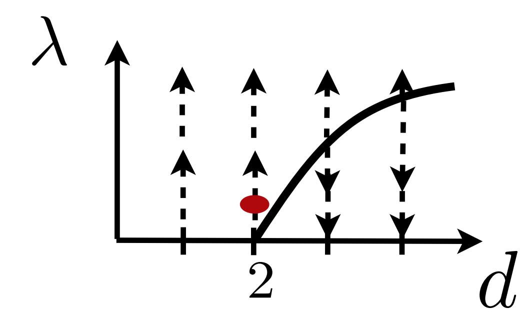
coherent propagation, inverse effective polariton mass

- experiments: vortex turbulence favored in systems with strong diffusion, $\lambda \sim K_d / K_c$
- flat band of 1D Lieb lattice realized with micropillar cavity arrays F. Baboux et al. PRL (2016)

Summary

Review: L. Sieberer, M. Buchhold, SD, *Keldysh Field Theory for Driven Open Quantum Systems*, Reports on Progress in Physics (2016)



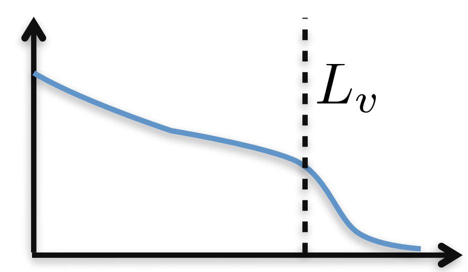


- low dimensional driven open quantum systems: non-equilibrium always relevant at large distances
- phase dynamics: compact KPZ
- compactness crucial

- weak non-equilibrium conditions
  - two intrinsic non-equilibrium length/time scales

- 2 dimensions:

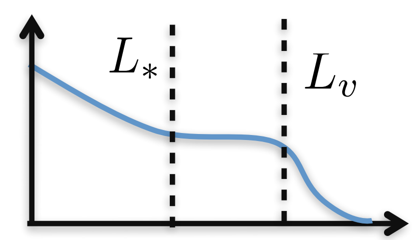
$$L_v \ll L_*$$



E. Altman, L. Sieberer, L. Chen, SD, J. Toner, PRX (2015)
L. Sieberer, G. Wachtel, E. Altman, SD, PRB (2016)
G. Wachtel, L. Sieberer, SD, E. Altman, PRB (2016)

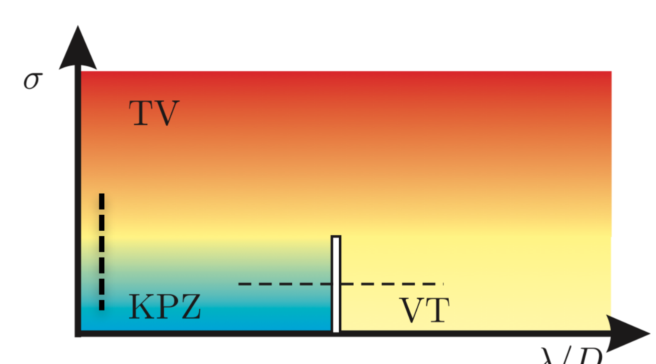
- 1 dimension:

$$L_v \gg L_*$$



L. He, L. Sieberer, E. Altman, SD, PRB (2015)
L. He, L. Sieberer, SD, arxiv (2016)

- strong non-equilibrium conditions
  - phase transition to vortex turbulent regime
  - challenge: analytical understanding via duality?



L. He, L. Sieberer, SD, arxiv (2016)
see also
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