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UNIVERSITY OF INNSBRUCK

Generating and Analyzing Models with Three-Body Hardcore Constraint

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IQOQI
AUSTRIAN ACADEMY OF SCIENCES

SFB
Coherent Control of Quantum Systems

Collaboration:
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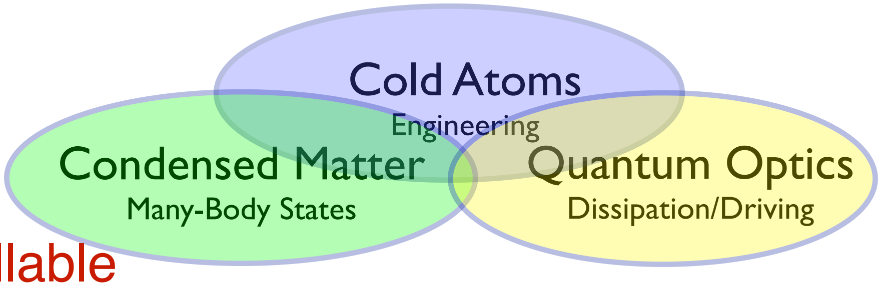
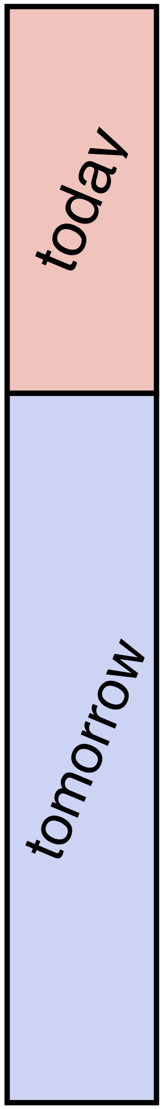
Andrew Daley (Innsbruck) Adrian Kantian (Innsbruck)
Peter Zoller (Innsbruck) Marcello Dalmonte (Bologna)

Lecture Overview



Main theme:
Dissipation can be turned into a favorable, controllable tool in cold atom many-body systems.



Part I: Dissipative Generation and Analysis of 3-Body Hardcore Models

- Mechanism
- Experimental prospects, ground state preparation
- Application I: phase diagram for attractive 3-hardcore bosons
- Application II: atomic color superfluid for 3-component fermions

- Collaboration: M. Baranov, A. J. Daley, M. Dalmonte, A. Kantian, J. Taylor, P. Zoller

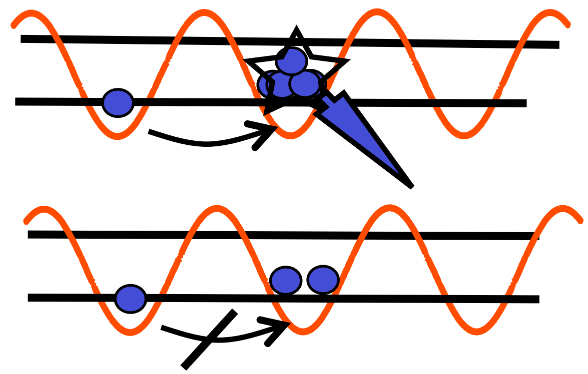
Part II: Quantum State Engineering in Driven Dissipative Many-Body Systems

- Proof of principle: Driven Dissipative BEC
- Application I: Nonequilibrium phase transition from competing unitary and dissipative dynamics
- Application II: Cooling into antiferromagnetic and d-wave states of fermions

- Collaboration: H. P. Büchler, A. Daley, A. Kantian, B. Kraus, A. Micheli, A. Tomadin, W. Yi, P. Zoller

Outline Part I

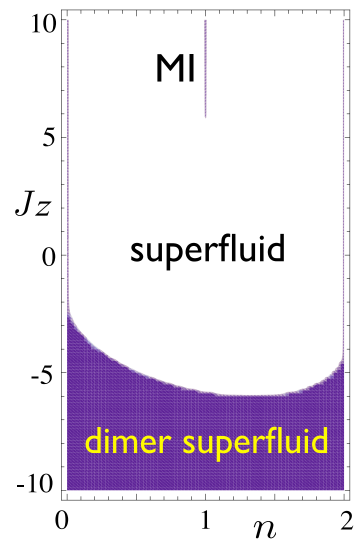
Dissipative generation of a three-body hardcore interaction



- Mini-tutorial: open quantum systems
- Mechanism
- Experimental prospect
- Ground state preparation

Phase diagram for three-body hardcore bosons

- First look: Dimer superfluid phase in Mean Field theory
- Construction of a Quantum Field Theory
- Beyond mean field results



Atomic colour superfluid of three-component fermions

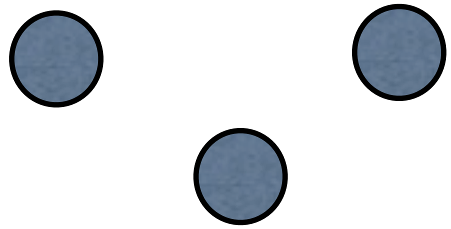
- Fermionic Lithium
- Phase Diagram

A. J. Daley, J. Taylor, SD, M. Baranov, P. Zoller, Phys. Rev. Lett. **102**, 040402 (2009)

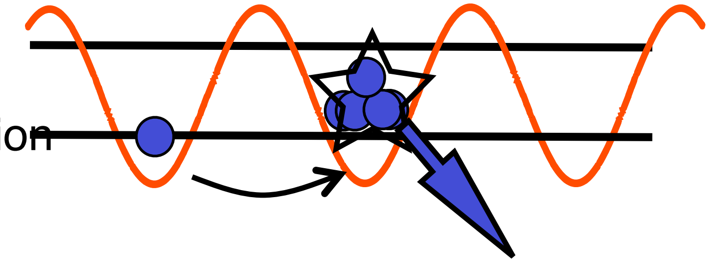
SD, M. Baranov, A. J. Daley, P. Zoller, to appear in Phys. Rev. Lett, arxiv:0910.1859 (2009); arxiv:0912.3192 (2009), arxiv:0912.3196 (2009)

A. Kantian, M. Dalmonte, SD, W. Hofstetter, P. Zoller, A. J. Daley, Phys. Rev. Lett. **103**, 240401 (2009)

Motivation

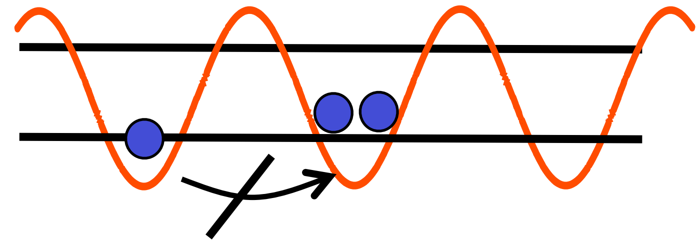


- 3-body loss processes (-)



  - ubiquitous, but typically undesirable inelastic 3 atom collision
  - inelastic 3 atom collision
  - molecule + atom ejected from lattice

- 3-body interactions (+)

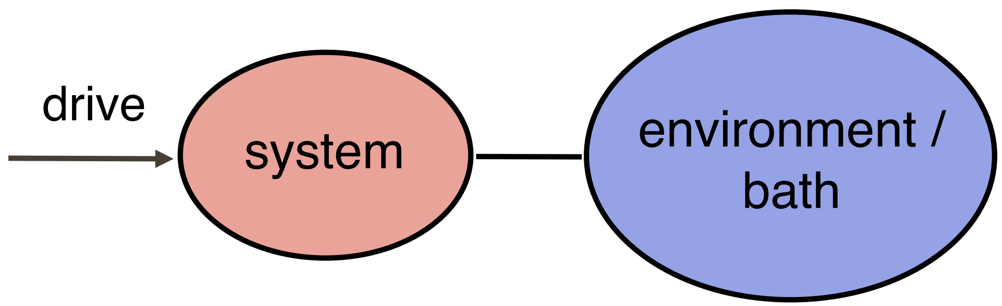


  - Stabilize bosonic system with attractive interactions
  - Generate Pfaffian-like states [Munich, M. Rizzi, J.I. Cirac, arXiv:0905.1247 (2009)]
  - Stabilize 3-component fermion system: atomic color superfluidity

$$i\gamma_3 \rightarrow \gamma_3$$

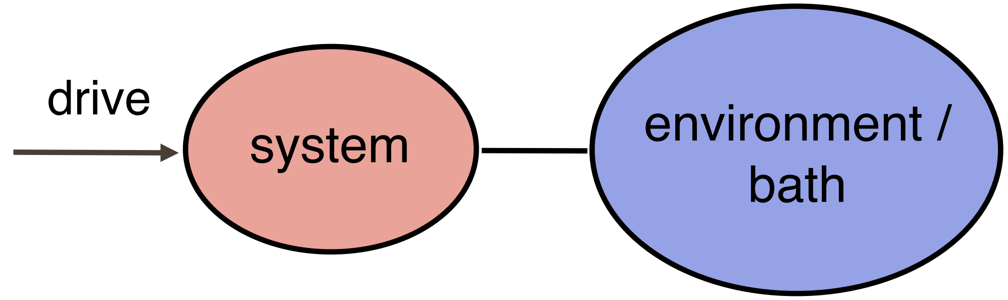
➡ We make use of strong 3-body loss to generate a 3-body hard-core constraint

Mini-Tutorial:
Open Quantum Systems



Open Quantum Systems

$$H = H_{\mathrm{S}} + H_B + H_{\mathrm{int}}$$



$$H_B = \int d\omega\, \omega b_\omega^\dagger b_\omega$$

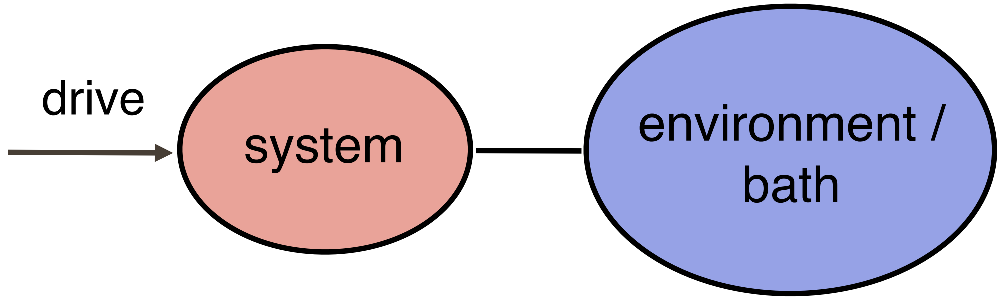
continuum bath of
harmonic oscillators

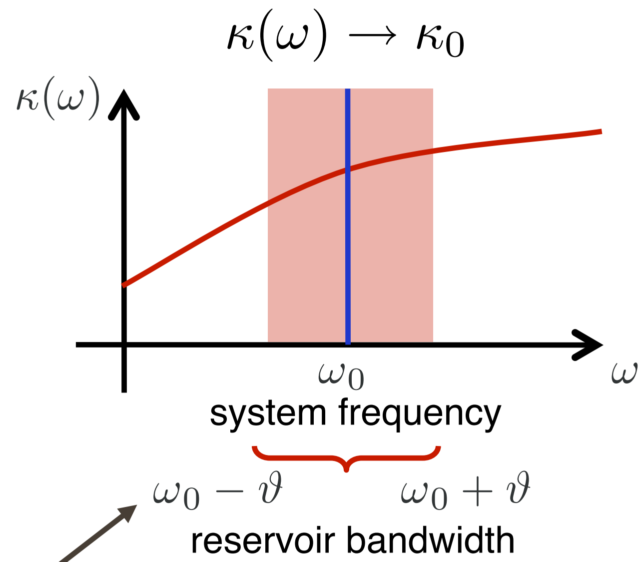
$$H_{\mathrm{int}} = i \int d\omega \kappa(\omega) \left[b_\omega^\dagger J - b_\omega J^\dagger \right]$$

quantum jump operators
polynomial in system
operators

linear bath operator coupling to the system

Open Quantum Systems





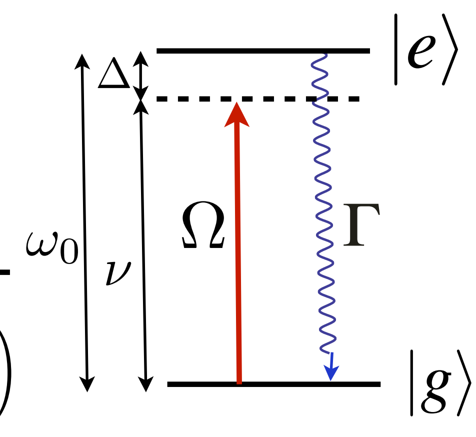
Three approximations:

(1) Born approximation: $\kappa(\omega)/\omega_0 \ll 1$

(2) Markov approximation: $\kappa(\omega) \approx$ const. $\Rightarrow \kappa(t-t') \sim \delta(t-t')$

(3) Rotating wave approximation: $\dfrac{\omega_0 - \nu}{\omega_0 + \nu} \ll 1$

$\omega_0 - \nu = \Delta$ detuning



in this example:

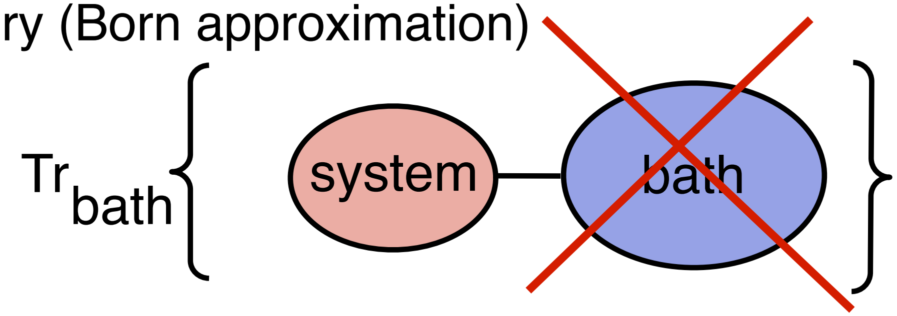
system Hamiltonian $H = (|e\rangle, |g\rangle) \begin{pmatrix} \Delta & \Omega \\ \Omega & 0 \end{pmatrix} \begin{pmatrix} \langle e| \\ \langle g| \end{pmatrix}$

jump operator $J_\alpha = |g\rangle\langle e| = \sigma^-$

Open Quantum Systems

$$\partial_t \rho_{\rm tot} = -i[H_S + H_B + H_{\rm int}, \rho_{\rm tot}]$$

➡ Eliminate bath degrees of freedom in second order time-dependent perturbation theory (Born approximation)



quantum jump operators

effective system dynamics from Master Equation (zero temperature bath)

$$\partial_t \rho = -i[H_S, \rho] + \underbrace{\kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger - \tfrac{1}{2}\{J_\alpha^\dagger J_\alpha, \rho\}}_{\mathcal{L}[\rho]}$$

$\mathcal{L}[\rho]$ Liouvillian operator in Lindblad form

- Structure: second order perturbation theory
- mnemonic: norm conservation $\partial_t \mathrm{tr}\rho = 0$
- but: $\partial_t \mathrm{tr}\rho^2 \neq 0$

pure state: $\mathrm{tr}\rho = \mathrm{tr}\rho^2 = 1$

$\Rightarrow \mathrm{tr}\rho^2$ -- “purity”

➡ Purity is not conserved

➡ go for $\partial_t \mathrm{tr}\rho^2 < 0$

Open Quantum Systems

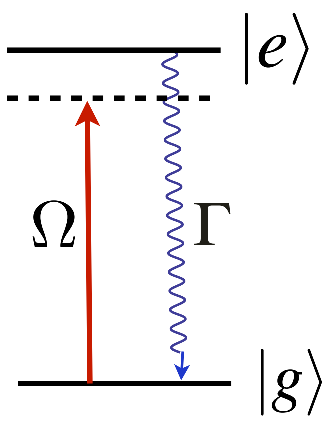
- Stochastic Interpretation: Quantum Jumps

$$\partial_t \rho = -i[H,\rho] + \kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger - \tfrac{1}{2}\{J_\alpha^\dagger J_\alpha, \rho\}$$

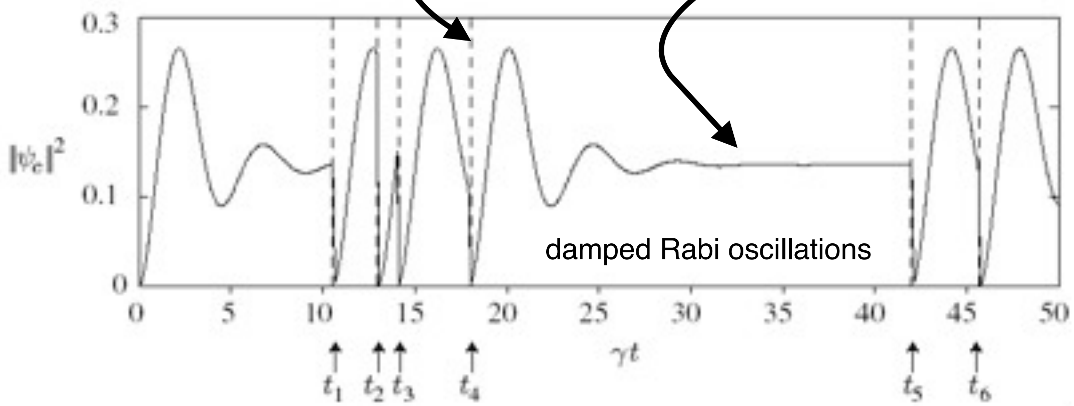
$$= -i[H_{\mathrm{eff}},\rho]^* + \kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger \qquad H_{\mathrm{eff}} = H - i\kappa/2 \sum_\alpha J_\alpha^\dagger J_\alpha$$

quantum jump operators

decay



$$J_\alpha = |g\rangle\langle e| = \sigma^-$$

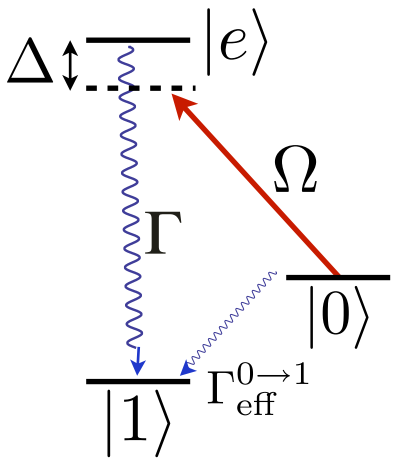


time evolution of upper state population of driven dissipative two-level system (single run)

- Averaging over "quantum trajectories" generates all correlation functions

$$[A,B]^* := AB - B^\dagger A^\dagger$$

Example: optical pumping



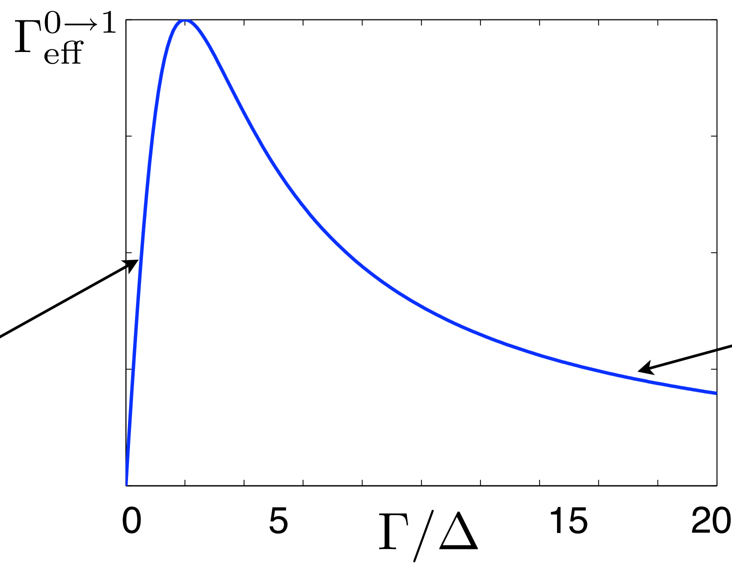
master equation in Lindblad form

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H,\rho] + \mathcal{L}[\rho]$$

with $$H = \frac{\Omega}{2}(|0\rangle\langle e| + |e\rangle\langle 0|) - \Delta|e\rangle\langle e|$$

$$\mathcal{L}[\rho] = \Gamma\left(J\rho J^\dagger - \frac{1}{2}(J^\dagger J\rho + \rho J^\dagger J)\right) \qquad J = |1\rangle\langle e|$$

pumping rate $\Gamma_{\mathrm{eff}}^{0\to 1} = \dfrac{\Omega^2}{4\Delta^2+\Gamma^2}\Gamma$ (for $\Omega \ll \Gamma, \Delta$)



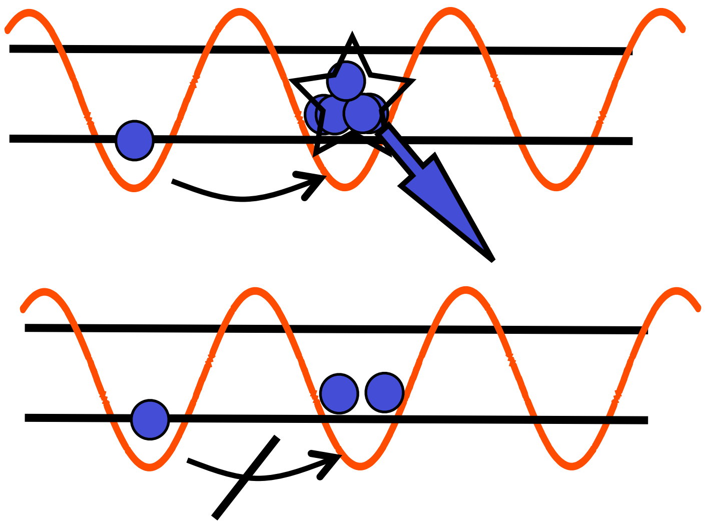
$\Gamma_{\mathrm{eff}}^{0\to 1} \approx \dfrac{\Omega^2}{4\Delta^2}\Gamma$

$\Gamma_{\mathrm{eff}}^{0\to 1} \approx \dfrac{\Omega^2}{\Gamma}$

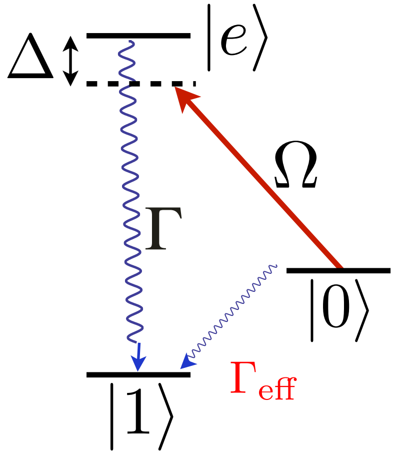
Zeno regime: system frozen in $|0\rangle$

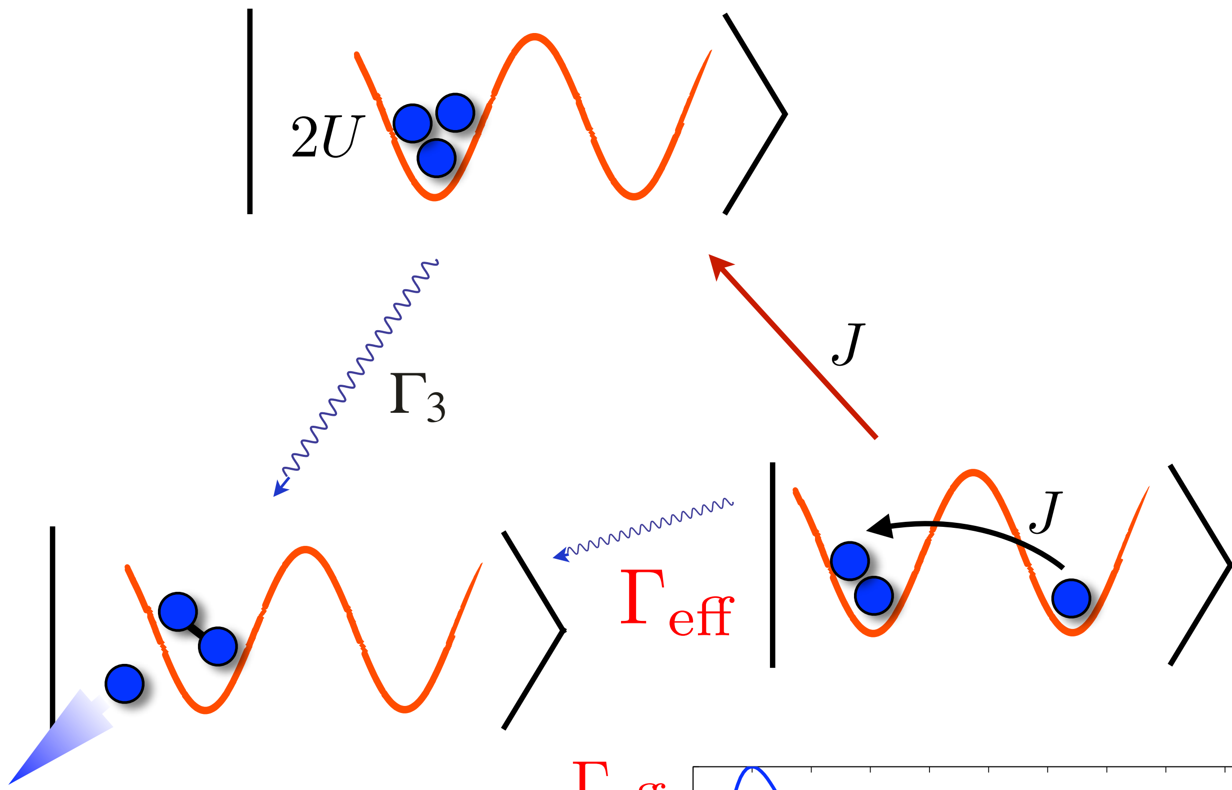
- The effective loss rate 0 -> 1 becomes small again for large Γ

3-body interactions via 3-body loss

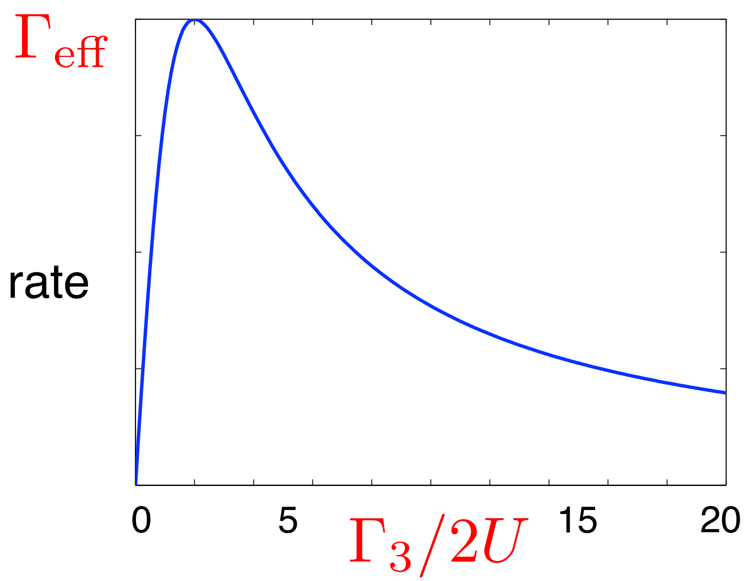


Analogy to three-body loss





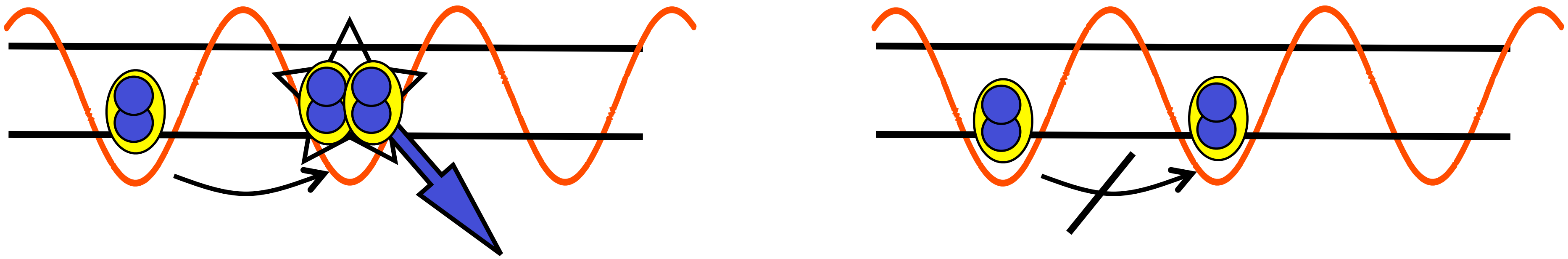
detuning	Δ	$\longleftrightarrow$	$2U$	onsite interaction energy
Rabi frequency	Ω	$\longleftrightarrow$	J	tunnel coupling
decay rate	Γ	$\longleftrightarrow$	Γ_3	three-body recombination rate



Operating a lossy lattice system in the Zeno regime stabilizes against effective particle loss

Related work

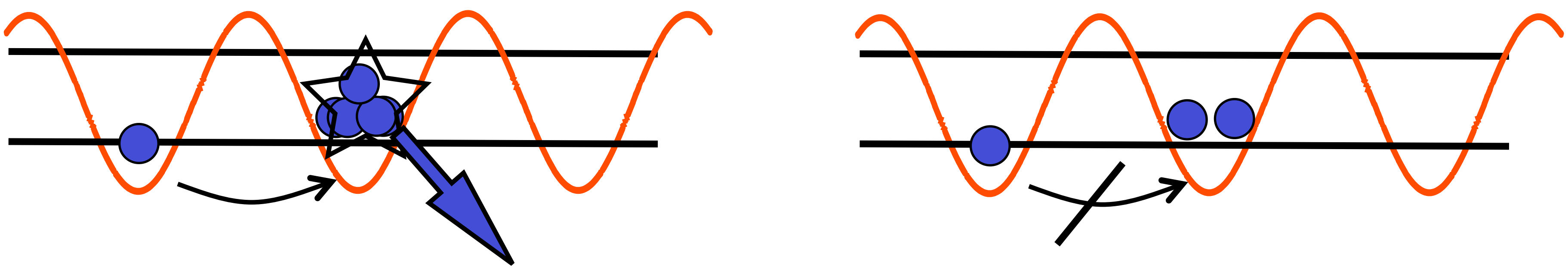
Effective 2-body interactions:



- Experimental observation for 2-body interactions (Feshbach molecules)

  N. Syassen et al., Science 320, 1329 (2008)
  J. J. Garcia-Ripoll et al., New J. Phys. ***11****, 013053 (2009)*
  S. Dürr et al., Phys. Rev. A 79, 023614 (2009)

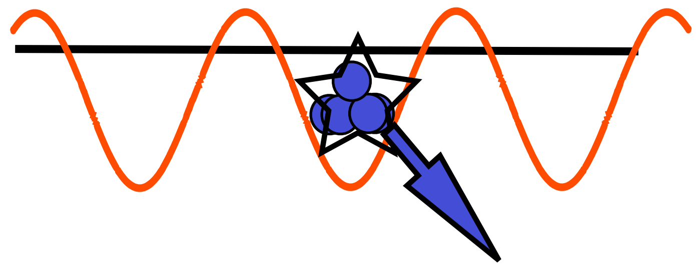
Effective 3-body interactions:



Microscopic Model: Interactions via Loss

- Model: Bosons on the optical lattice with three-body recombination
- Hamiltonian: $H = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$
- Three-body recombination: loss from lattice to continuum of unbound states
- Model on-site three-body loss: Master Equation

  in Lindblad form



couples density matrix sectors with n+3, n particles

$$\dot{\rho} = -i[H, \rho] + \frac{\gamma_3}{12} \sum_i 2\hat{b}_i^3 \rho \hat{b}_i^{\dagger 3} - \{\hat{b}_i^{\dagger\,3} \hat{b}_i^3, \rho\}$$

three-body loss rate

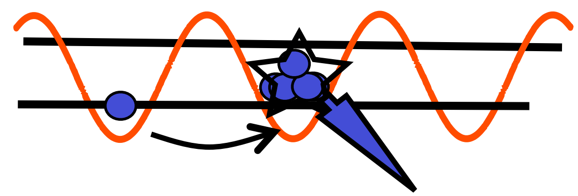
- zero temperature approximation: binding energy of deeply bound molecule much larger than lattice depth

Microscopic Model: Interactions via Loss

- Rewrite the Master Equation as

non-particle number conserving: couples sectors with n +3, n particles in the density matrix

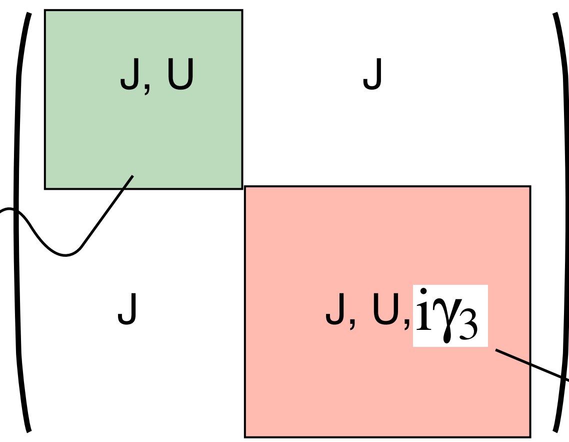
$$\dot{\rho} = -i\left(H_{eff}\rho - \rho H_{eff}^{\dagger}\right) + \frac{\gamma_3}{12}\sum_i 2\hat{b}_i^3 \rho (\hat{b}_i^{\dagger})^3$$



particle number conserving (but norm decays)

$$H_{eff} = H - i\frac{\gamma_3}{12}\sum_i (\hat{b}_i^{\dagger})^3 \hat{b}_i^3$$

=



up to double occupancy

triple and higher occupancy

➡ Consider the limit $\gamma_3 \gg U, J$

Microscopic Model: Interactions via Loss

- Second order Perturbation Theory
  - Define projector P onto subspace with at most 2 atoms per site (Q=1-P)

$$H_{P,\,\mathrm{eff}} \approx PHP + \frac{2i}{\gamma_3} PHQHP = PHP - \frac{i\Gamma}{2}\sum_j P c_j^\dagger c_j P$$

$$c_j = b_j^2 \sum_{\langle k|j\rangle} b_k / \sqrt{2}$$

$$\begin{pmatrix} PHP & PHQ \\ QHP & QHQ \end{pmatrix}$$

$$PHP = -J\sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) \quad \& \; b_i^{\dagger 3} \equiv 0$$

➡ Three-body hardcore constraint due to: dynamic suppression of triple onsite occupation (analogous Quantum Zeno Effect)

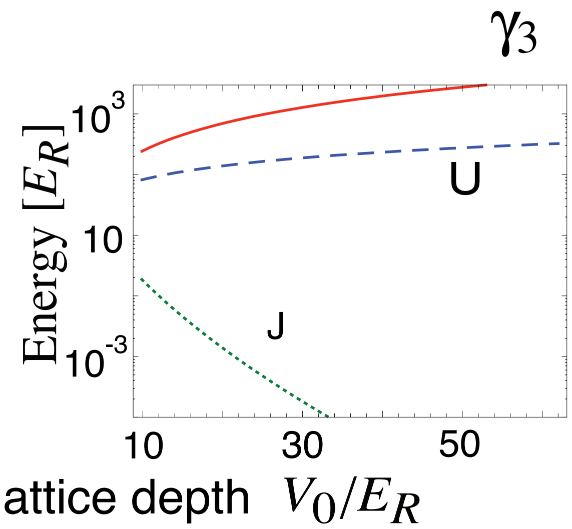
➡ Small decay constant in P subspace: $\Gamma = 12\dfrac{J^2}{\gamma_3}$

➡ Realization of a Hubbard-Hamiltonian with three-body hard-core constraint on time scales $\tau = 1/\Gamma$

Physical Realization in Cold Atomic Gases

- Estimate Loss rate: Integrate free space recombination rate over Wannier function
  - short length scale collisions not modified by lattice
- Cesium close to a zero crossing of the scattering length (e.g. Naegerl et al.)

parameter estimate



- Preparation of the ground state of PHP:
  - Nonequilibrium problem: role of residual heating effects
  - Approach: Exact numerical time evolution of full Master Equation in 1D; combine DMRG method with stochastic simulation of ME
  - Find optimal experimental sequence to avoid heating

Ground State Preparation

Quantum Trajectories: Stochastic Simulation

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] - \Gamma \sum_\alpha c_\alpha \rho c_\alpha^\dagger$$

- Evolve stochastic trajectories (states)

$$H_{\text{eff}} = H - i\frac{\Gamma}{2} \sum_\alpha c_\alpha^\dagger c_\alpha$$

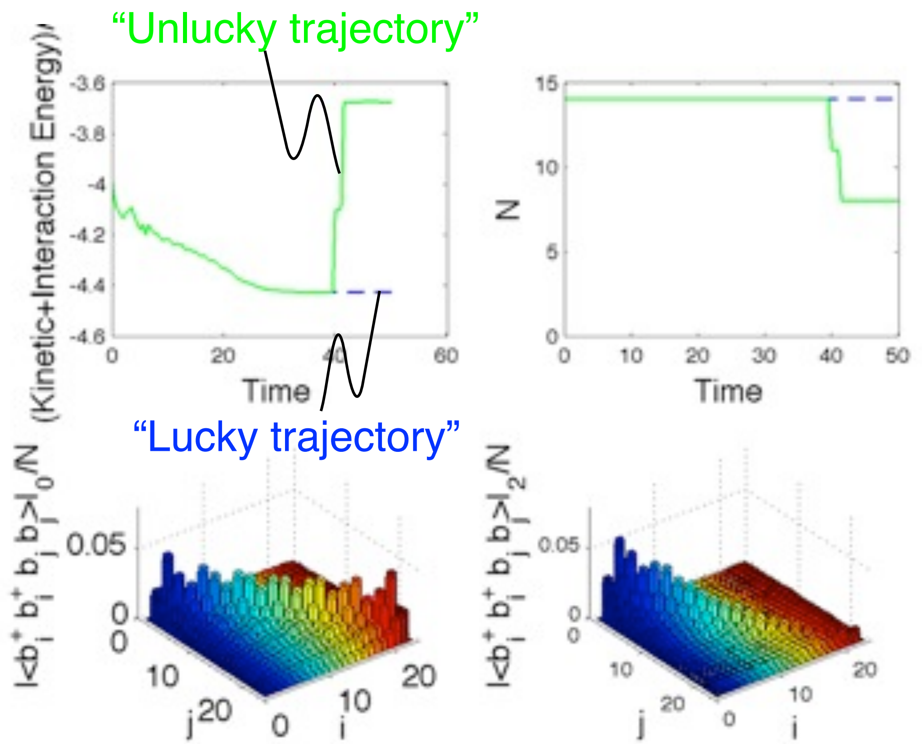
- Quantum Jumps

$$|\psi\rangle = \frac{c_m|\psi\rangle}{||c_m|\psi\rangle||}$$

  - Norm decays below random threshold
  - Jump operator chosen randomly

Features:

- Evolution of individual trajectories
- Expectation values by stochastic average

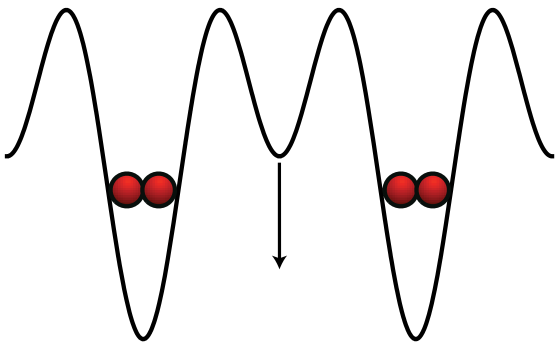


"Lucky trajectory": long range order

"Unlucky trajectory"

Ground State Preparation

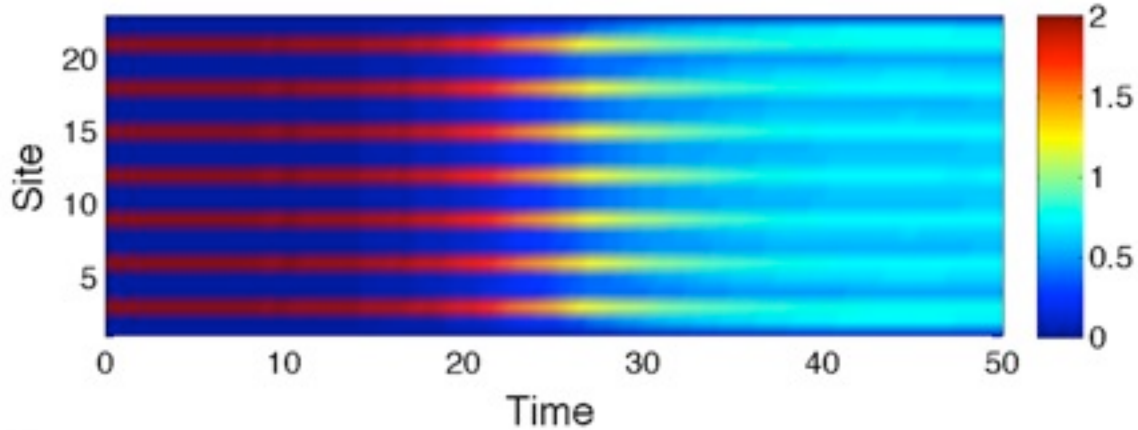
Ramping down a superlattice

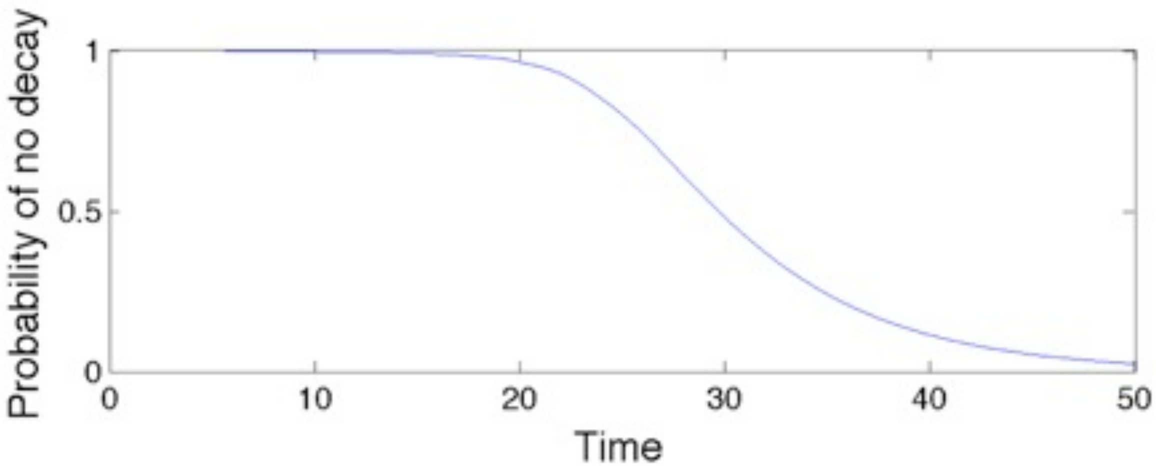


Ramp: Superlattice, V/J=30 to V/J=0,
N=M=20; U/J =-8

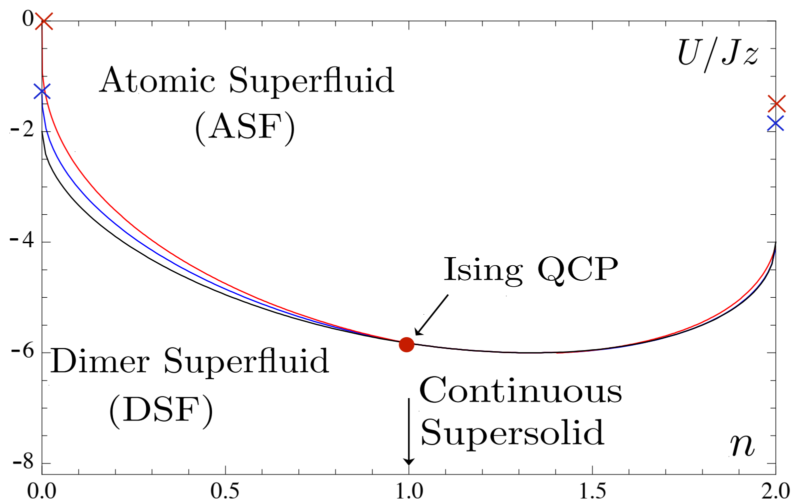
Buildup of long-range order in “lucky” case

$$\Gamma = 250J$$





Phase Diagram for Three-Body Hardcore Bosons



microscopic

interactions

thermodynamic

interactions
condensatio
n

long distance

interactions
condensatio
n
spin waves

Physics of the projected Hamiltonian

- The constrained Bose-Hubbard Hamiltonian stabilizes attractive two-body interactions

$$PHP = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) \quad \& \; \boxed{b_i^{\dagger 3} \equiv 0}$$

$$\boxed{U < 0}$$

- Qualitative picture for ground state: Mean Field Theory
  - homogenous Gutzwiller Ansatz for projected on-site Hilbert space

$$|\Psi\rangle = \prod_i |\Psi\rangle_i \qquad |\Psi\rangle_i = f_0|0\rangle + f_1|1\rangle + f_2|2\rangle \qquad f_\alpha = r_\alpha e^{i\phi_\alpha}$$

  - Gutzwiller energy

$$\Phi = \phi_2 + \phi_0 - 2\phi_1$$

$$E(r_\alpha, \phi_\alpha) = U r_2^2 - JZ r_1^2 \left[r_0^2 + 2\sqrt{2} r_2 r_0 \cos\Phi + 2 r_2^2 \right]$$

Mean Field Phase Diagram

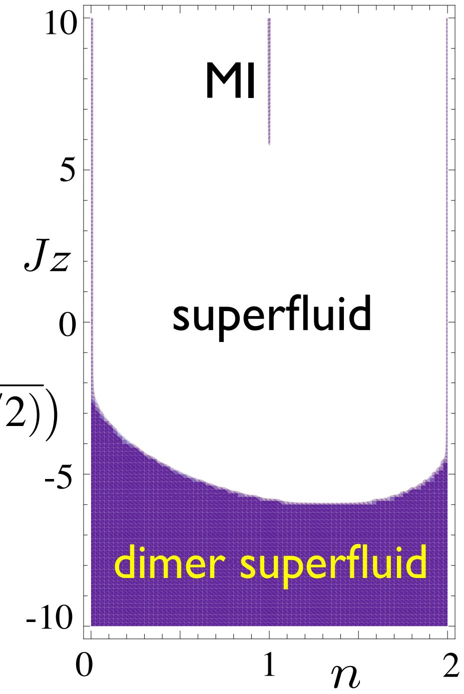
- Consider correlation functions:

$\langle \hat{b} \rangle$ - Atomic SF order parameter

$\langle \hat{b}^2 \rangle$ - Dimer SF order parameter

critical interaction strength:

$$\frac{U_c}{Jz} = -2\left(1 + n/2 + 2\sqrt{n(1-n/2)}\right)$$



- Symmetry breaking patterns:

$$\langle \hat{b} \rangle \neq 0, \quad \langle \hat{b}^2 \rangle \neq 0$$ - Conventional SF

$$\langle \hat{b} \rangle \neq 0, \quad \langle \hat{b}^2 \rangle = 0$$ - NO! phase locking in GW energy

$$\langle \hat{b} \rangle = 0, \quad \langle \hat{b}^2 \rangle \neq 0$$

- "Dimer SF"

$$E(r_\alpha, \phi_\alpha) = U r_2^2 - JZ r_1^2 \left[r_0^2 + 2\sqrt{2} r_2 r_0 \cos\Phi + 2 r_2^2 \right]$$

- Phase transition reminiscent of Ising (cf Radzihovsky& '03; Stoof, Sachdev& '03):

$$\langle \hat{b} \rangle \sim \exp i\theta \qquad \langle \hat{b}^2 \rangle \sim \exp 2i\theta$$

➡ Spontaneous breaking of Z_2 symmetry $\theta \rightarrow \theta + \pi$ of the DSF order parameter

➡ Second order within MFT

Beyond Mean Field Physics?

$$PHP = -J\sum_{\langle i,j\rangle}\hat{b}_i^\dagger\hat{b}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)\ \ \&\ b_i^{\dagger 3} \equiv 0$$

- The classical Gutzwiller mean field theory leaves open questions on various scales

microscopic: $\varepsilon_M = -\frac{1}{Ma^2}$

thermodynamic: $n = \frac{k_F^3}{3\pi^2}, T$

long: $k_{ld} \gg n^{1/3}, T^{1/2}, \varepsilon_M^{1/2}$

- A quantum field theory can be constructed:

  - Constrained model can be mapped exactly on coupled boson theory with polynomial interactions. The two bosonic degrees of freedom find a natural interpretation in terms of “atoms” and “dimers”
  - This should be seen as a requantization of Gutzwiller mean field theory
  - The theory is conveniently analyzed in terms of the Effective Action: conventional symmetry principles are supplemented with a new constraint principle

$PHP \longrightarrow$

$X_i = 1 - \hat{n}_{1,i} - \hat{n}_{2,i}$ “atoms” “dimers”

$$H = (U - 2\mu)\sum_i \hat{n}_{2,i} - \mu\sum_i \hat{n}_{1,i} - J\sum_{\langle i,j\rangle}\left[t_{1,i}^\dagger X_i X_j t_{1,j} + \sqrt{2}(t_{2,i}^\dagger t_{1,i} X_j t_{1,j}^\dagger + t_{1,i}^\dagger X_i t_{1,j}^\dagger t_{2,j}) + 2t_{2,i}^\dagger t_{2,j} t_{1,j}^\dagger t_{1,i}\right]$$

➡This Hamiltonian contains interesting quantitative and qualitative effects

✓Tied to interactions

✓Tied to the constraint

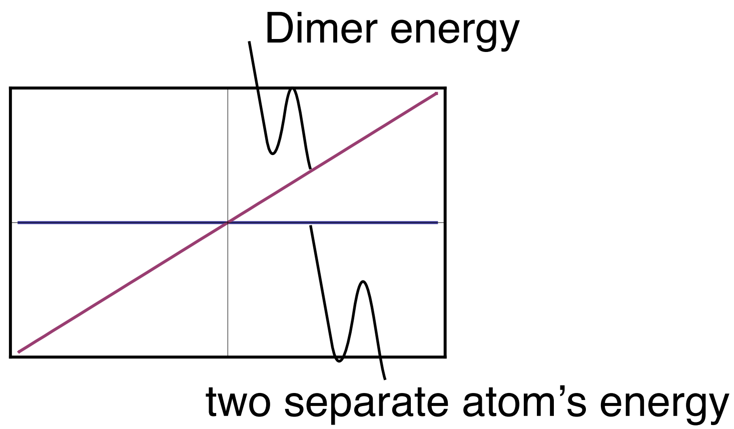
The requantized Gutzwiller model

- Hamiltonian to cubic order is of Feshbach type:

  - quadratic part:

$$H_{\text{pot}} = \sum_i (U - 2\mu) n_{2,i} - \mu n_{1,i}$$

detuning from atom level



  - leading interaction:

$$H_{\text{kin}} = -J \sum_{\langle i,j \rangle} \left[t^\dagger_{1,i} t_{1,j} + \sqrt{2} (t^\dagger_{2,i} t_{1,i} t_{1,j} + t^\dagger_{1,i} t^\dagger_{1,j} t_{2,j}) \right]$$

(bilocal) dimer splitting into atoms

- Compare to standard Feshbach models:

detuning $\sim 1/U$

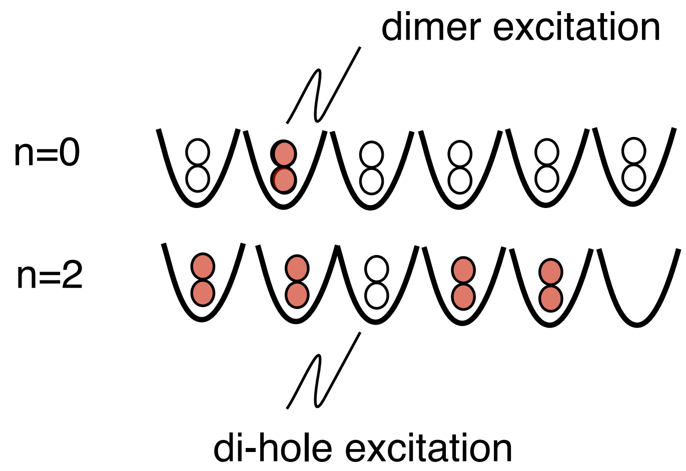
here: detuning $\sim U$

➡ we can expect resonant (strong coupling) phenomenology at weak coupling

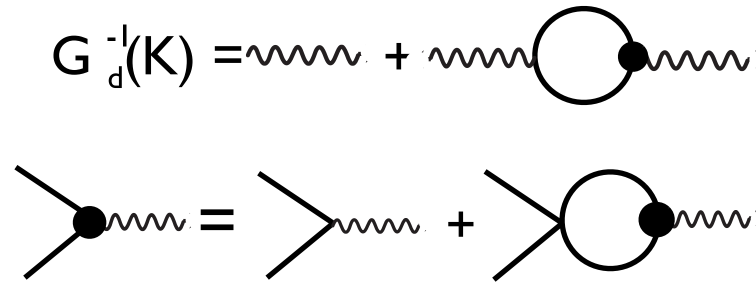
$$H_{\text{kin}} = -J \sum_{\langle i,j \rangle} \left[t^\dagger_{1,i}(1-n_{1,i}-n_{2,i})(1-n_{1,j}-n_{2,j})t_{1,j} + \sqrt{2}(t^\dagger_{2,i}t_{1,i}(1-n_{1,j}-n_{2,j})t_{1,j} + t^\dagger_{1,i}(1-n_{1,i}-n_{2,i})t^\dagger_{1,j}t_{2,j}) + 2t^\dagger_{2,i}t_{2,j}t^\dagger_{1,j}t_{1,i} \right]$$

Vacuum Problems

- The physics at n=0 and n=2 are closely connected:
  - “vacuum”: no spontaneous symmetry breaking
  - low lying excitations:
    - n=0: atoms and dimers on the physical vacuum
    - n=2: holes and di-holes on the fully packed lattice



- Two-body problems can be solved exactly
  - Bound state formation: $G_d^{-1}(\omega = \mathbf{q} = 0) = 0$



$$\frac{1}{a_n|\tilde{U}| + b_n} = \int \frac{d^d q}{(2\pi)^d} \frac{1}{-\tilde{E}_b + 2/d\sum_\lambda (1 - \cos \mathbf{q}\mathbf{e}_\lambda)}$$

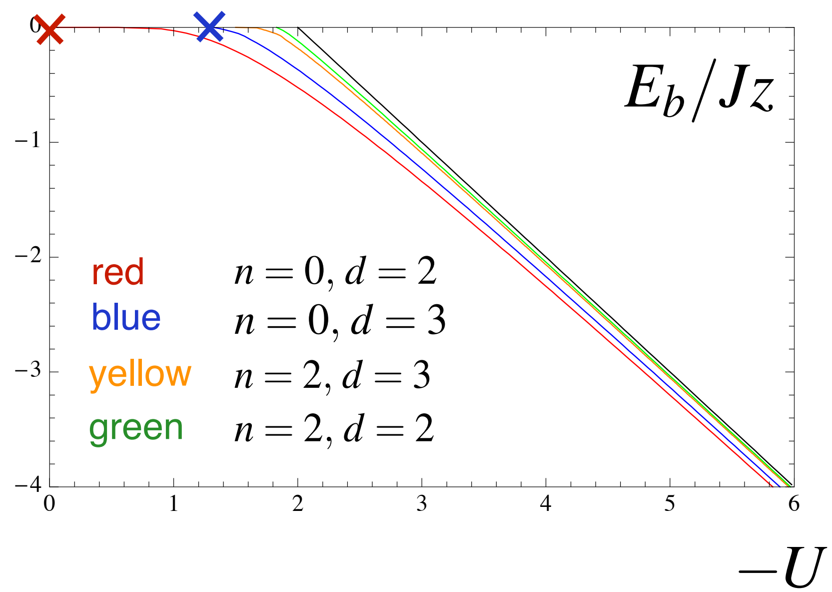
$$n = 0: \quad a_0 = 1, \; b_0 = 0$$

➡ reproduces Schrödinger Equation: benchmark

➡ Square root expansion of constraint fails

$$n = 2: \quad a_2 = 4, \; b_2 = -6 + 3\tilde{E}_b$$

➡ di-hole-bound state formation at finite U in 2D



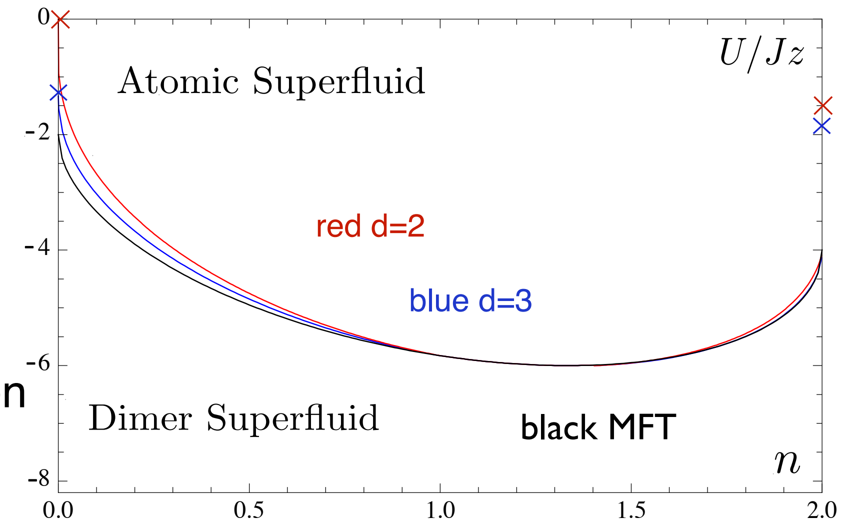
ASF - DSF Phase Border

- Goal: Effects of quantum fluctuations on phase border

- Result:
  - strong shifts only observed for low densities

- Understanding:
  - dominant fluctuations: associated to bound state formation
  - two scales: bound state formation and atom criticality

shifts of the phase border



(i) low density: coincidence of scales
➡ strong shifts, nonanalytic nonuniversal behavior

e.g. d=3: $$\frac{U_c}{Jz} \approx \frac{U_c(n=0)}{Jz} - \sqrt{\frac{\theta |U_c(n=0)|}{2Jz\sigma}}, \quad \sigma \approx 0.53$$

condensate angle

(ii) maximum density: mismatch of scales, di-hole bound state forms prior to atom criticality
➡ mean field like behavior

- Note: No particle-hole symmetry!

Symmetry Enhancement

- Perturbative limit U >> J: expect dimer hardcore model
- Interpret EFT as a spin model in external field:

$$H_{\text{eff}} = -2t \sum_{\langle i,j \rangle} \left(s_i^x s_j^x + s_i^y s_j^y + \lambda s_i^z s_j^z \right)$$

- Leading (second) order perturbation theory:

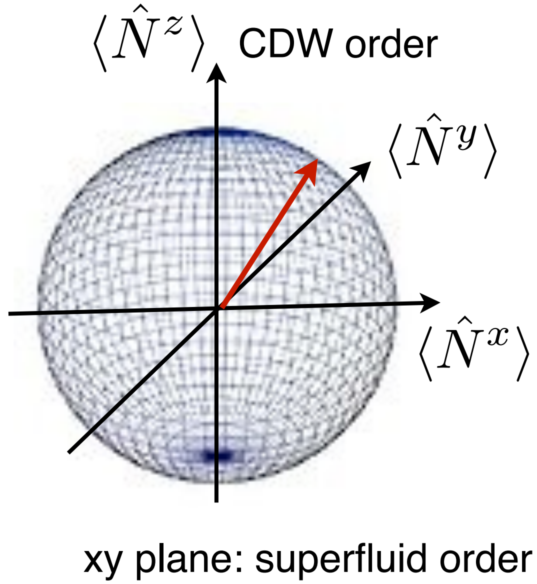
$$\boxed{\lambda = \frac{v}{2t} = 1}$$

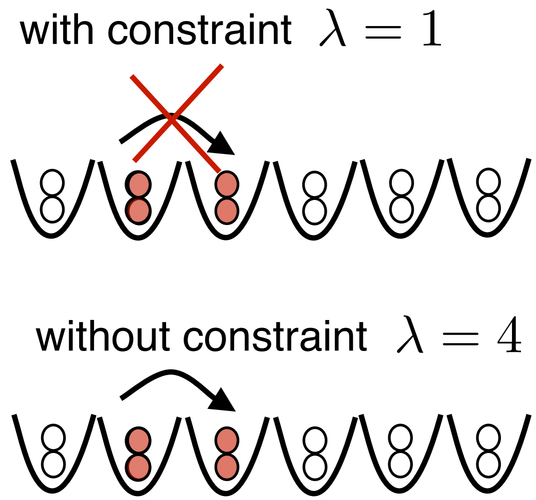
➡ Isotropic Heisenberg model (half filling n=1):

- Emergent symmetry: SO(3) rotations vs. SO(2) sim U(1)
- Bicritical point with Neel vector order parameter

$$\hat{N}^\alpha = \sum_j (-)^j s_i^\alpha$$

- charge density wave and superfluid exactly degenerate
  - CDW: Translation symmetry breaking
  - DSF: Phase symmetry breaking
- physically distinct orders can be freely rotated into each other: "continuous supersolid"





➡ The symmetry enhancement is unique to the 3-body hardcore constraint

Signatures of “continuous supersolid”

- Next (fourth) order perturbation theory: Superfluid preferred

$$\lambda = 1 - 8(z-1)(J/|U|)^2 < 1$$

- Proximity to bicritical point governs physics in strong coupling

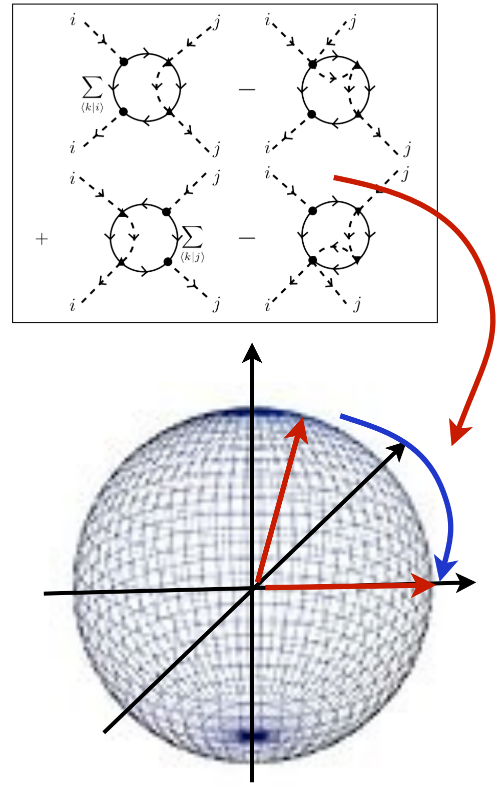
(1) Second collective (pseudo) Goldstone mode

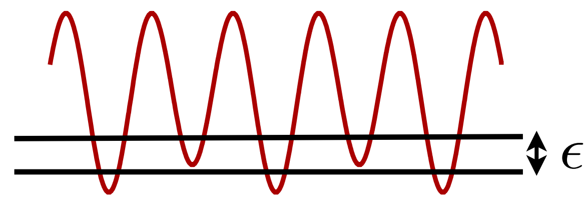
$$\omega(\mathbf{q}) = tz\big((\lambda\varepsilon_{\mathbf{q}} + 1)(1 - \varepsilon_{\mathbf{q}})\big)^{1/2}$$

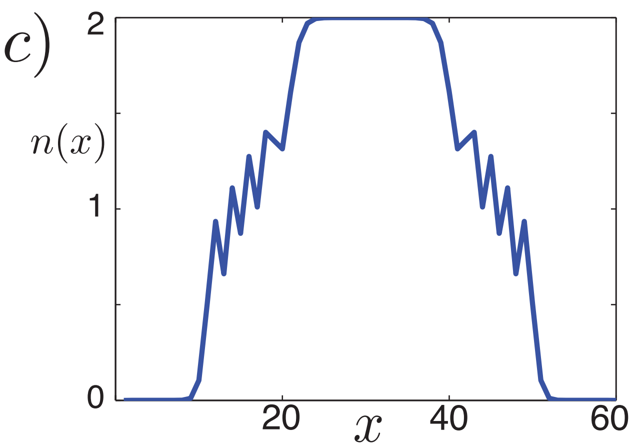
(2) Use weak superlattice to rotate Neel order parameter

$$\epsilon/tz = \Delta/tz = 1 - \lambda \approx 8(z-1)(J/U)^2$$

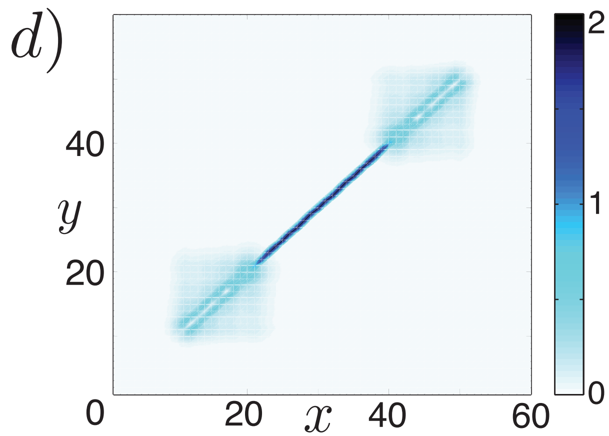
(3) Simulation of 1D experiment in a trap (t-DMRG)



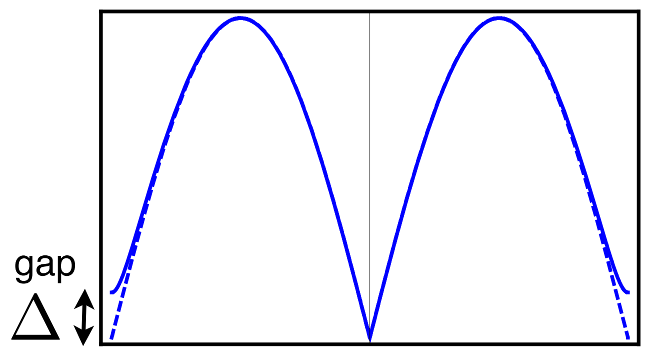




density profile: Onset of CDW



DSF order in textured regions



Second (pseudo) Goldstone mode

Signatures of "continuous supersolid"

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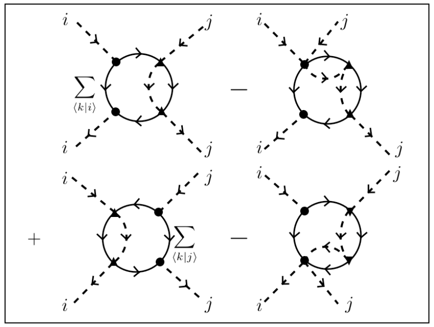

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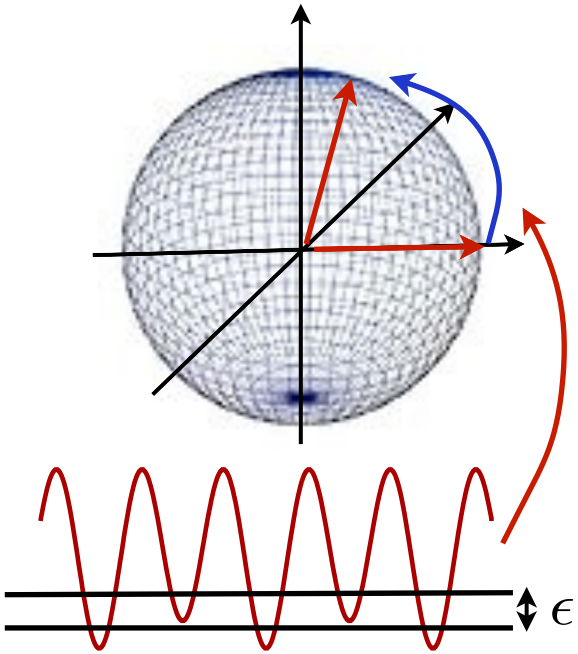

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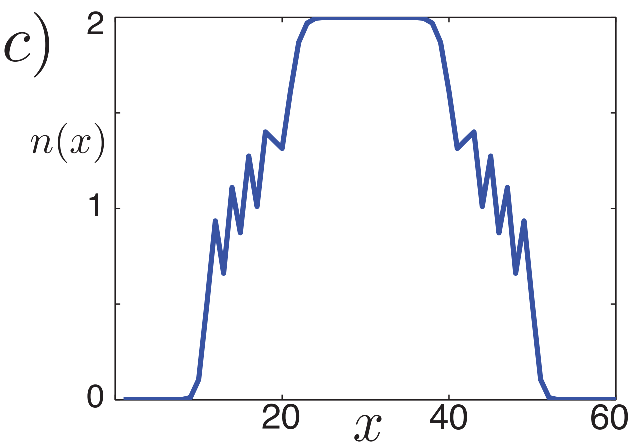
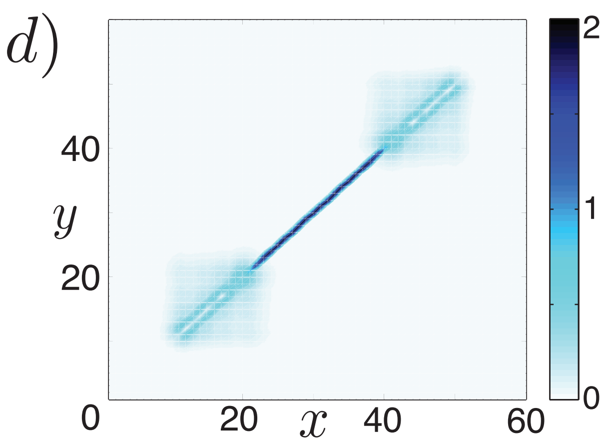
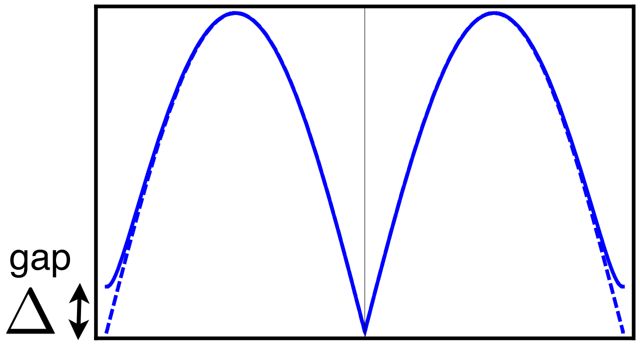


density profile: Onset of CDW



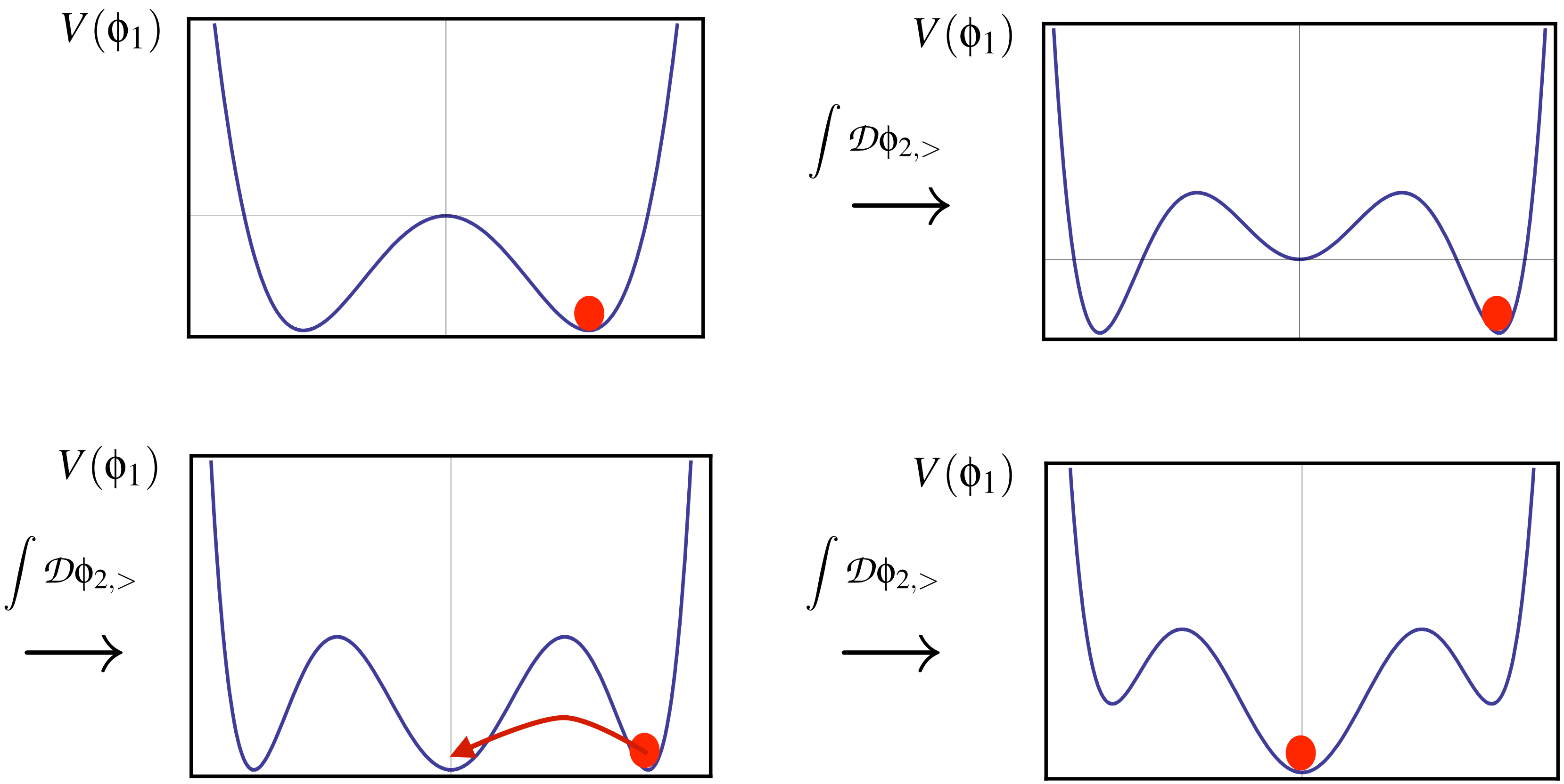
DSF order in textured regions



Second (pseudo) Goldstone mode

Infrared Limit: Nature of the Phase Transition

- Two near massless modes: Critical atomic field, dimer Goldstone mode
- Coleman-Weinberg phenomenon for coupled real fields: Radiatively induced first order PT



Infrared Limit: Nature of the Phase Transition

- Perform the continuum limit and integrate out massive modes:

pure Goldstone action

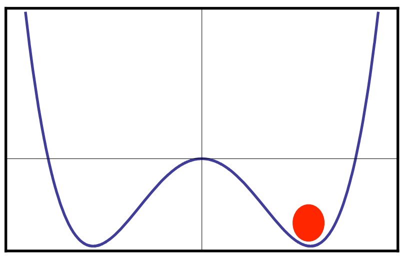
$$S[\vartheta,\phi] = S_{\mathrm{I}}[\phi] + S_{\mathrm{G}}[\vartheta] + S_{\mathrm{int}}[\vartheta,\phi]$$

pure Ising action

coupling term

$$S_{\mathrm{I}}[\phi] = \int \partial_\mu\phi\partial^\mu\phi + m^2\phi^2 + \lambda\phi^4$$

Ising field: Real part of atomic field



Ising potential landscape:
Z_2 symmetry breaking

$$\boxed{S_{\mathrm{int}}[\vartheta,\phi] = \mathrm{i}\kappa\int \partial_\tau\vartheta\,\phi^2}$$

Frey, Balents; Radzihovsky&

➡ Interactions persist to arbitrary long wavelength (cf. decoupling SW)

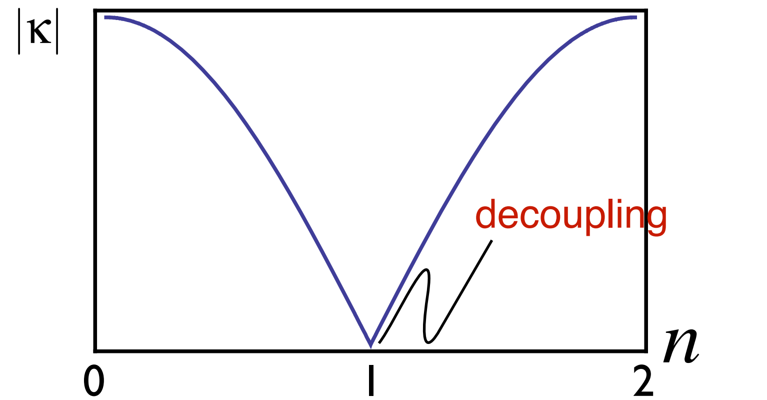
➡ $\kappa \neq 0$: Phase transition is driven first order by coupling of Ising and Goldstone mode

d+1 Ising Quantum Critical Point at n=1

- Plot the Ising-Goldstone coupling:

$$S_{\text{int}}[\vartheta,\phi] = i\kappa \int \partial_\tau \vartheta\, \phi^2$$

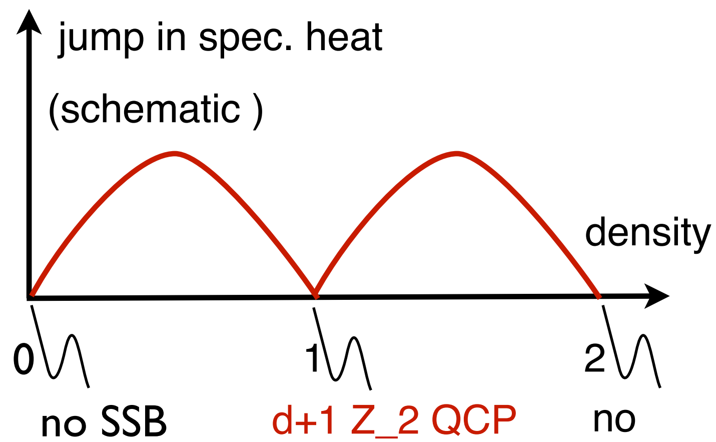
$$\Gamma \ni \int_{\vec{x},\tau} b^\dagger_{2,i}(-g_2\mu)b_{2,i}$$



- Symmetry argument:
  - dimer compressibility must have zero crossing
  - and is locked to other couplings by time-local gauge invariance and atom-dimer phase locking

  ➡ emergent relativistic symmetry: isotropic d+1 dimensional model

  ➡ κ must have zero crossing: true quantum critical Ising transition

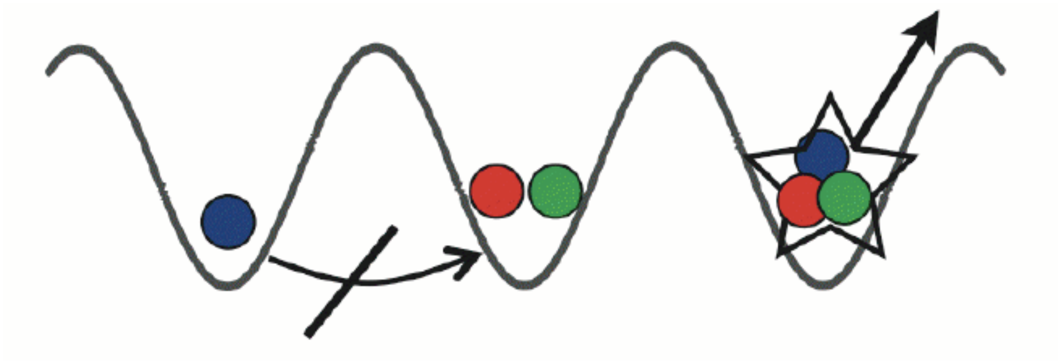


- Estimate correlation length: $\xi/a \sim \kappa^{-6} \sim |1-n|^{-6}$

  ➡ weakly first order, broad near critial domain

➡ Second order quantum critical behavior is a lattice+constraint effect

3-Body Hardcore 3-Component Fermions

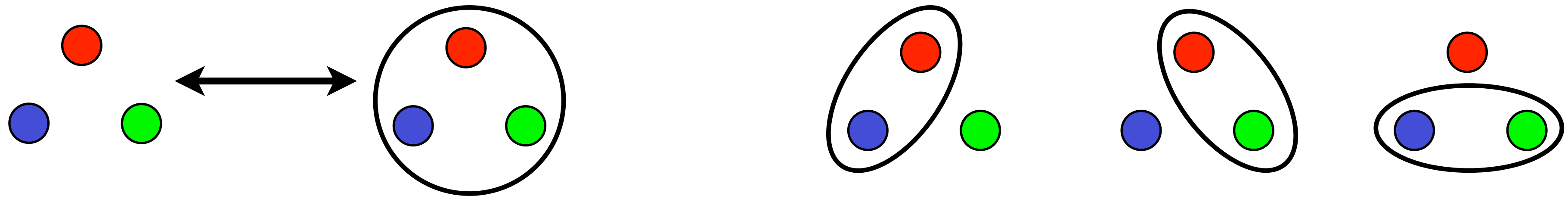


3-Component Fermions

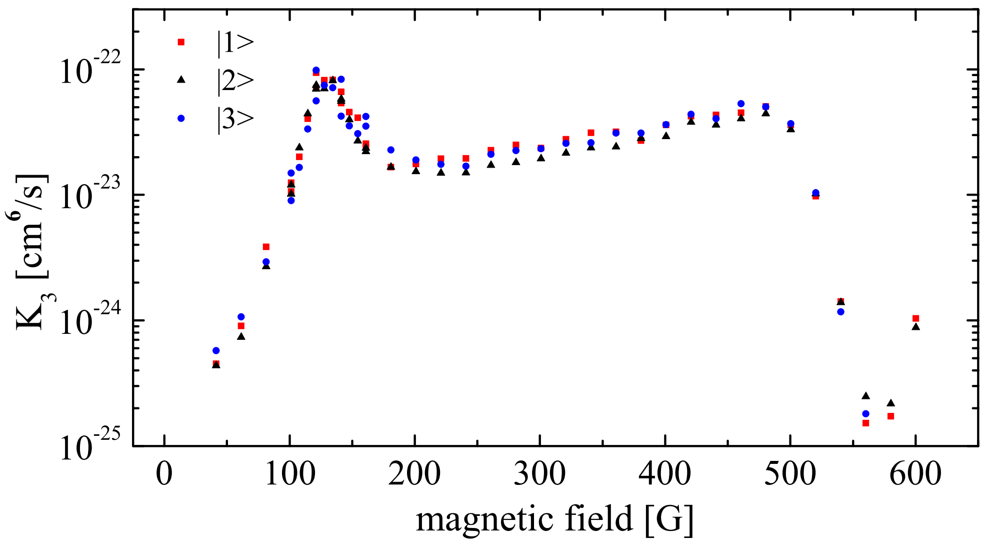
- Rich many-body physics (e.g. A. Rapp et al., PRL 07)

Trimers

Colour superfluid (BCS pairing)



- 3-species Fermi mixture
- e.g., Lithium-6: Very strong loss features (T. Ottenstein et al., PRL 2009)

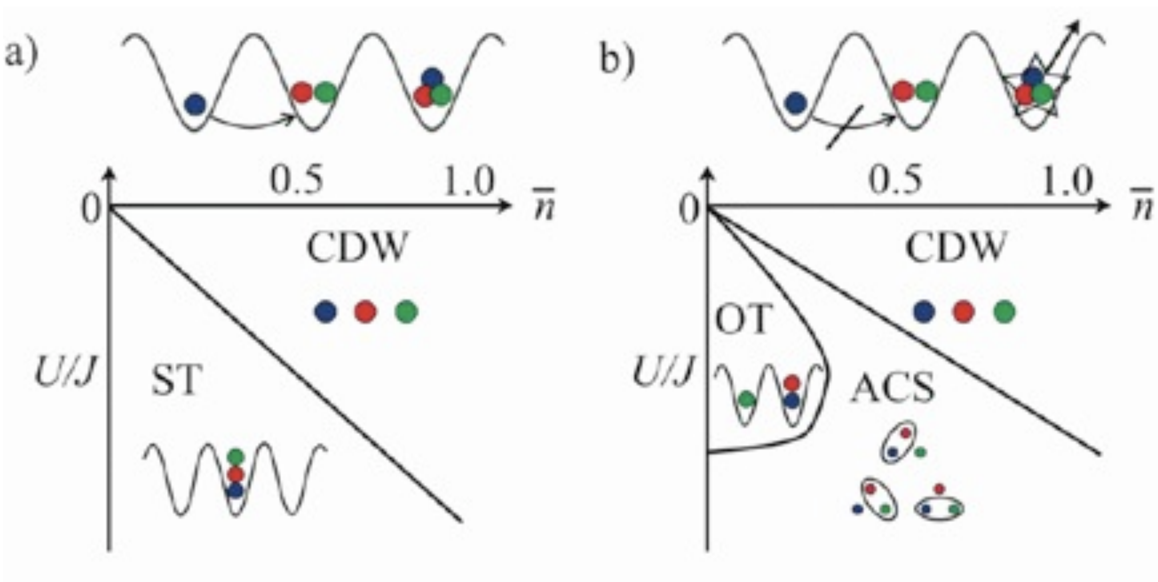


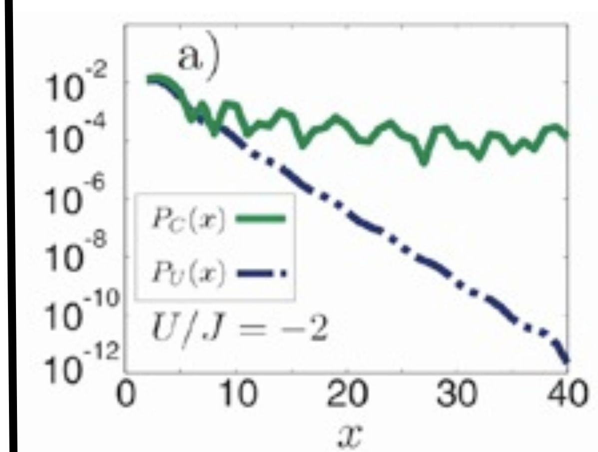
➡ Does the loss induced 3-body constraint stabilize the superfluid?

Phase Diagram

- Study the system in one dimension:
  - numerically: using DMRG
  - analytically: implementation of the constraint similar to the boson case, and subsequent bosonization techniques (weak coupling)

- Results for the attractive SU(3) symmetric case





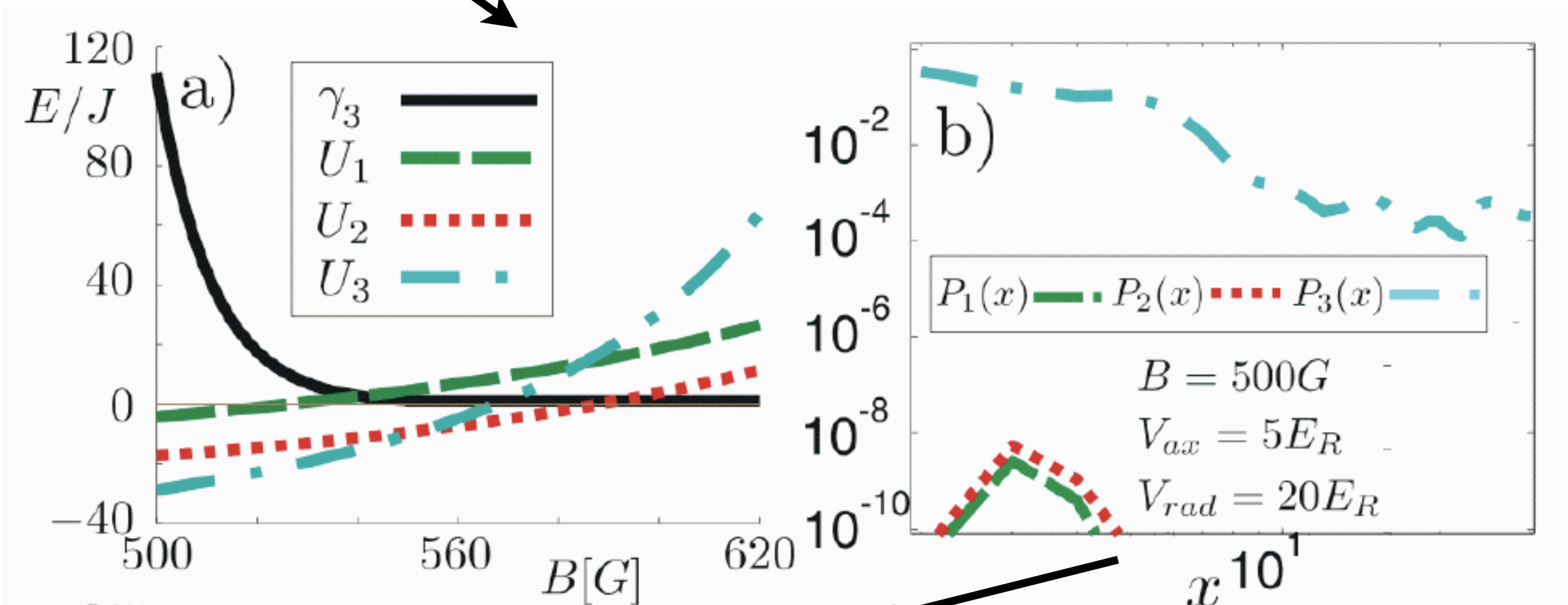
- Competition between CDW and onsite trions (ST) (Capponi et al.)
- no superfluid correlations

- Competition between CDW, atomic color superfluid, and Offsite Trions
- an extended BCS pairing region exists

- Pairing correlation functions without (blue) and with constraint (green): exponential vs. algebraic

The Lithium Case

- Strong breaking of SU(3) symmetry by different interactions between hyperfine states



- Pairing in one channel dominates

- With t-DMRG + Quantum Trajectories method, we can propose optimal experimental preparation sequence

Higher Dimensions

- In higher dimensions d=2,3 we can use the constraint formalism developed above for bosons
- Lithium Hamiltonian: species dependent chemical potentials, strongly anisotropic couplings

$$H = -J \sum_{\langle i,j\rangle,\alpha} c^{\dagger}_{i,\alpha} c_{j,\alpha} - \sum_{\alpha,i} \mu_{\alpha} \hat{n}_{i,\alpha} + \sum_{\alpha,i} U_{\alpha} \hat{n}_{i,\alpha+1} \hat{n}_{i,\alpha+2}$$

- Constraint: No onsite trions

$$\frac{1}{3!} \epsilon_{\alpha\beta\gamma} c^{\dagger}_{\alpha} c^{\dagger}_{\beta} c^{\dagger}_{\gamma} \equiv 0$$

- The residual states can be parameterized as

$$|0\rangle_i = b^{\dagger}_{0,i}|\mathrm{vac}\rangle$$

empty sites

$$|\alpha\rangle_i = t^{\dagger}_{\alpha,i}|\mathrm{vac}\rangle = c^{\dagger}_{\alpha,1}|0\rangle_i$$

single fermions

$$|\alpha_B\rangle_i = b^{\dagger}_{\alpha,i}|\mathrm{vac}\rangle = \tfrac{1}{2}\epsilon_{\alpha\beta\gamma} c^{\dagger}_{\beta,i} c^{\dagger}_{\gamma,i}|0\rangle_i = |\beta\gamma\rangle_i = |\alpha+1,\alpha+2\rangle_i$$

“molecules”

Constraint Hamiltonian

- Following the construction for bosons, the constraint Hamiltonian (low densities) reads

$$
\begin{aligned}
H &= -J \sum_{\langle i,j \rangle} \big[\mathbf{t}_i^\dagger X_i X_j \mathbf{t}_j + (\mathbf{t}_i^\dagger \times \mathbf{b}_i)(\mathbf{b}_j^\dagger \times \mathbf{t}_j) \\
&\qquad - (\mathbf{b}_j(\mathbf{t}_j^\dagger \times \mathbf{t}_i^\dagger X_i) + h.c.\big], \\
&\quad + \sum_i \big[- \vec{\mu}\hat{\mathbf{n}}_{f,i} + (-2\vec{\nu} + \mathbf{U})\hat{\mathbf{n}}_{b,i} \big] \\
X_i &= \mathbf{1} - \big(\sum_\alpha \hat{n}_{f,\alpha,i} + \hat{n}_{b,\alpha,i} \big) \\
\hat{n}_{f,\alpha,i} &= t_{\alpha,i}^\dagger t_{\alpha,i}, \quad \hat{n}_{b,\alpha,i} = b_{\alpha,i}^\dagger b_{\alpha,i}
\end{aligned}
$$

$$\nu_\alpha = (\mu_{\alpha+1} + \mu_{\alpha+2})/2$$

- The Hamiltonian is a Feshbach model generalized to include three species

Effective Low Energy Hamiltonian

- For Lithium, we are interested in equal and moderate densities, and the parameter is

$$n_\alpha = 1/6, \quad U_1 = -40J,\, U_2 = -20J,\, U_3 = -5J$$

- The interactions are large and attractive, and strongly separated from each other

- Simple energy considerations show that the most strongly interacting species pair up into molecules, while the third species remains unpaired

- Using this separation of scales, we can show that the full constrained Feshbach Hamiltonian maps to a Fermi-Bose mixture

$$\begin{aligned} H &= H_{1F} + H_{BF} + H_{1B} \\ H_{1F} &= J \sum_{\langle i,j \rangle} \nabla_i t^\dagger_{1,i} \nabla_i t_{1,i} - \mu \sum_i \hat{n}_{1,f,i} \\ H_{1B} &= -t_1 \sum_{\langle i,j \rangle} \left[2b^\dagger_{1,i} X_i b_{1,j} X_j - \hat{n}_{1,B,i} \hat{n}_{1,B,i} \right] - \sum_i \mu_{1,B} \hat{n}_{1,B,i} \\ H_{BF} &\approx Jz \sum_i \hat{n}_{F,1,j} \hat{n}_{B,1,i} \end{aligned}$$

$$t_1 = \frac{2J^2}{U_1}$$

- There is a large fermion-boson repulsion ~ Jz, which originates from the constraint

Constraint Induced Phase Separation

- We calculate the stability of the canonical energy wrt density variations in mean field,

$$E(n_{1,F}, n_{1,B}) = J\tfrac{3}{5}(6\pi^2)^{2/3} n_{1,F}^{5/3} + tzn_{1,B}^2 + 2Jzn_{1,B}n_{1,F}$$

- Fermions contribute due to their kinetic energy, bosons due to interaction energy; the formula holds for small densities

- Stability matrix

$$M_{ab} = \frac{\partial^2 E}{\partial n_a \partial n_b} = \begin{pmatrix} \frac{2}{3}(6\pi^2)^{2/3} J n_F^{-1/3} & 2Jz \\ 2Jz & 2tz \end{pmatrix}$$

- The system is stable if $\det M \geq 0$

$$\approx 2.53, 5.06 \,\mathrm{in}\, d = 2, 3$$

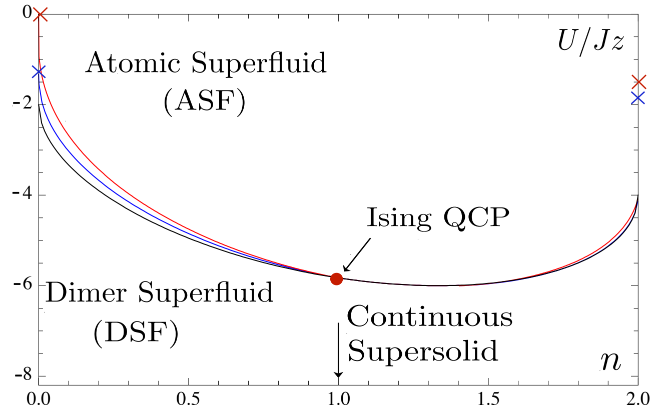
$$1 \quad \leq \quad \frac{(6\pi^2)^{2/3} Jtz}{3(Jz)^2}\, n_F^{-1/3} \approx \frac{2(6\pi^2)^{2/3} J}{3z|U_1|}\, n_F^{-1/3}$$

- For Lithium $J/|U_1| \approx 1/40$ and thus the system is unstable

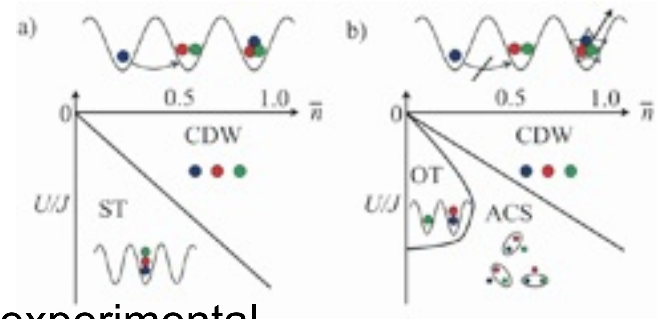
- The phase separated state does not feature the strong off diagonal term and is thus energetically favorable

Summary

- Generate a 3-body hard core constraint from ubiquitous, strong three-body loss
    - analogous Quantum Zeno Effect
    - ground state of constrained system reachable



- Beyond mean field effects in 3-body constrained bosons
    - requantized Gutzwiller theory allows to investigate effects tied to (i) interactions, (ii) 3-body constraint
    - quantum fluctuations shift the phase border for low densities
    - radiatively induced first order ASF-DSF transition terminates into Ising QCP
    - symmetry enhancement in strong coupling leads to “continuous supersolid”



- 3-component Fermions with 3-body constraint
    - strong loss makes them prime candidates (6Li)
    - 1D: “color superfluid” phase stabilized in SU(3) symmetric case
    - quantitative analysis for asymmetric Li case including proposal of experimental sequence
    - in higher dimensions, there are indications for a constraint induced phase separation

Quantum Field Theory: Why?

- Experiments are getting more quantitative and able to resolve subtle effects
  - T. Donner et al., Science 315, 1556 (2007): Critical exponents
  - A. Altmeyer et al., PRL. 98, 040401 (2007): Beyond mean field effects in BCS-BEC crossover
  - Y. Shin et al., Nature 451, 689 (2008): Phase diagram of imbalanced fermions
  - J. Stewart et al., Nature 454, 744 (2008): Dispersion relation of strongly interacting fermions
  - F. Gerbier et al., PRL 101, 155303 (2008): Quantitative benchmark of quantum simulators
- Gutzwiller mean field theory: classical field theory for the amplitudes $f_{\alpha,i}(t), \quad \sum_\alpha f^*_{\alpha,i} f_{\alpha,i} = 1$
- Questions on various scales:
  - Vacuum problem: Dimer bound state formation expected for attractive interaction
  - Condensation/Thermodynamics: phase border, superfluid stiffness/ Goldstone Theorem, EFT in strongly interacting limit
  - Infrared limit: Nature of the Phase transition

➡ Quantized version of the Gutzwiller mean field description desirable

➡ We identify quantitative and qualitative effects intimately connected to interactions

microscopic

$$\varepsilon_M = -\frac{1}{Ma^2}$$

thermodynamic

$$n = \frac{k_F^3}{3\pi^2}, T$$

long distance

$$k_{ld} \gg n^{1/3}, T^{1/2}, \varepsilon_M^{1/2}$$

Additional Material

Implementation of the Hard-Core Constraint

- Introduce operators to parameterize on-site Hilbert space (Auerbach, Altman '98)

$$t^{\dagger}_{\alpha,i}|\mathrm{vac}\rangle = |\alpha\rangle, \quad \alpha = 0,1,2$$

- They are not independent:

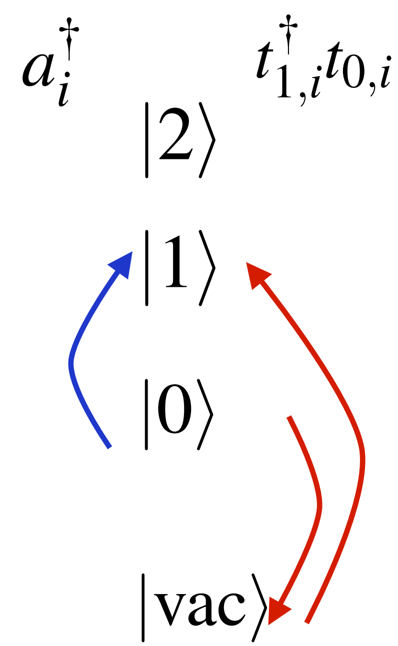
$$\sum_{\alpha} t^{\dagger}_{\alpha,i} t_{\alpha,i} = \mathbf{1}$$

- Representation of Hubbard operators:

$$a^{\dagger}_i = \sqrt{2}\, t^{\dagger}_{2,i} t_{1,i} + t^{\dagger}_{1,i} t_{0,i}$$

$$\hat{n}_i = 2 t^{\dagger}_{2,i} t_{2,i} + t^{\dagger}_{1,i} t_{1,i}$$

Action of operators



Implementation of the Hard-Core Constraint

- Hamiltonian:

$$H_{\text{pot}} = -\mu \sum_i 2t^\dagger_{2,i}t_{2,i} + t^\dagger_{1,i}t_{1,i} + U\sum_i t^\dagger_{2,i}t_{2,i}$$

$$H_{\text{kin}} = -J\sum_{\langle i,j\rangle}\left[t^\dagger_{1,i}t_{0,i}t^\dagger_{0,j}t_{1,j} + \sqrt{2}(t^\dagger_{2,i}t_{1,i}t^\dagger_{0,j}t_{1,j} + t^\dagger_{1,i}t_{0,i}t^\dagger_{1,j}t_{2,j}) + 2t^\dagger_{2,i}t^\dagger_{1,j}t_{1,i}t_{2,j}\right]$$

- Properties:
  - Mean field: Gutzwiller energy (classical theory)
  - interaction: quadratic
  - hopping: higher order
    - Role of interaction and hopping reversed
    - Strong coupling approach
  - One phase is redundant: absorb via *local* gauge transformation

$$t_{1,i} = \exp \mathrm{i}\varphi_{0,i}\,|t_{0,i}| \qquad t_{1,i} \to \exp -\mathrm{i}\varphi_{0,i}\,t_{1,i}, \quad t_{2,i} \to \exp -\mathrm{i}\varphi_{0,i}\,t_{2,i}$$

➡ e.g. t_0 can be chosen real

Implementation of the Hard-Core Constraint

- Resolve the relation between t-operators (zero density)

$$t^\dagger_{1,i} t_{0,i} = t^\dagger_{1,i}\sqrt{1 - t^\dagger_{1,i} t_{1,i} - t^\dagger_{2,i} t_{2,i}} \rightarrow t^\dagger_{1,i}(1 - t^\dagger_{1,i} t_{1,i} - t^\dagger_{2,i} t_{2,i})$$

  - justification: for projective operators one has from Taylor representation

$$X^2 = X \rightarrow f(X) = f(0)(1-X) + X f(1) \qquad X = 1 - t^\dagger_{1,i} t_{1,i} - t^\dagger_{2,i} t_{2,i}$$

- Now we can interpret the remaining operators as standard bosons:
  - on-site bosonic space $\mathcal{H}_i = \{|n\rangle^1_i |m\rangle^2_i\}, \quad n,m = 0,1,2,\ldots$
  - correct bosonic enhancement factors on physical subspace $\sqrt{n} = 0,1$
  - decompose into physical/unphysical space: $\mathcal{H}_i = \mathcal{P}_i \oplus U_i$

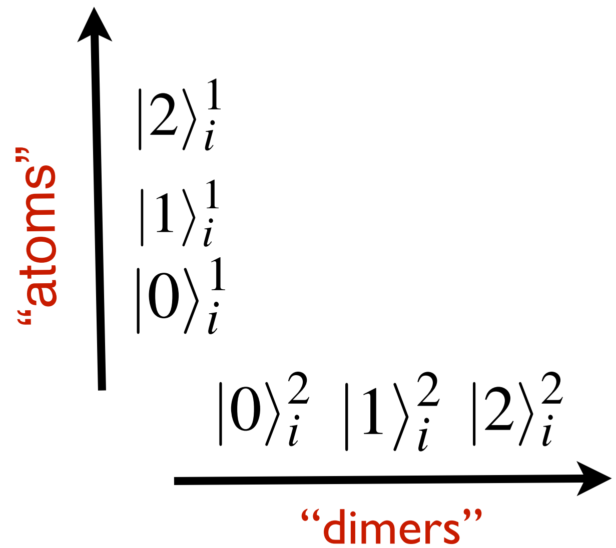
$$\mathcal{P}_i = \{|0\rangle^1_i |0\rangle^2_i, |1\rangle^1_i |0\rangle^2_i, |0\rangle^1_i |1\rangle^2_i\}$$

  - the Hamiltonian is an involution on P and U:

$$H = H_{PP} + H_{UU}$$

  - remaining degrees of freedom: “atoms” and “dimers”

➡ similarity to Hubbard-Stratonovich transformation



Implementation of the Hard-Core Constraint

- The partition sum does not mix U and P too:

$$Z = Tr\exp - \beta H = Tr_{PP}\exp - \beta H_{PP} + Tr_{UU}\exp - \beta H_{UU}$$

- Need to discriminate contributions from U and P: Work with Effective Action
  - Legendre transform of the Free energy $\quad W[J] = \log Z[J]$

$$\Gamma[\chi] = -W[J] + \int J^T \chi, \quad \chi \equiv \frac{\delta W[J]}{\delta J}$$

Quantum Equation of Motion for J=0

  - Has functional integral representation:

$$\exp - \Gamma[\chi] = \int \mathcal{D}\delta\chi \exp - S[\chi + \delta\chi] + \int J^T \delta\chi, \quad J = \frac{\delta\Gamma[\chi]}{\chi}$$

$$S[\chi = (t_1, t_2)] = \int d\tau \Big(\sum_i t^\dagger_{1,i}\partial_\tau t_{1,i} + t^\dagger_{2,i}\partial_\tau t_{2,i} + H[t_1, t_2] \Big)$$

- Usually: Effective Action shares all symmetries of S
- Here: symmetry principles are supplemented with a constraint principle

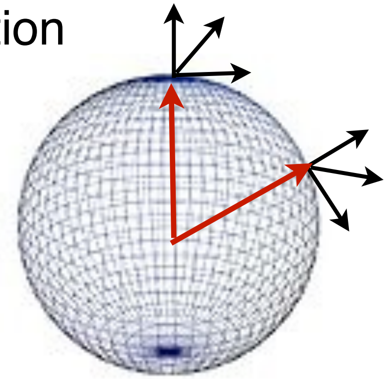
Condensation and Thermodynamics

- Physical vacuum is continuously connected to the finite density case:

Introduce new, expectationless operators by (complex) Euler rotation

$$\vec{t} = (t_0, t_1, t_2)^T$$

$$\vec{b} = R_\theta \, R_\varphi \, \vec{t}$$



- Hamiltonian in new coordinates takes form:

Quadratic part: Spin waves (Goldstone for n > 0)

$$H = E_{\mathrm{GW}} + H_{\mathrm{SW}} + H_{\mathrm{int}}$$

Mean field: Gutzwiller Energy

higher order: interactions

microscopic

interactions

thermodynamic

interactions
condensation

long distance

interactions
condensation
spin waves

Hard-Core Constraint: Summary

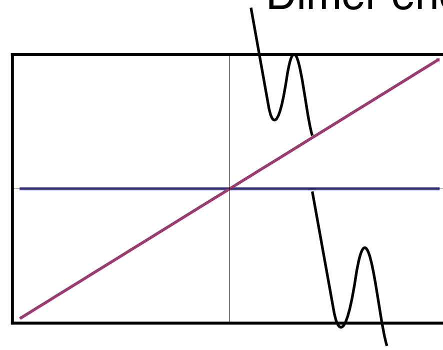
- Constrained Model can be mapped on coupled boson theory. This should be seen as a requantization of Gutzwiller mean field theory
- This theory automatically respects constraint: Decoupled physical and unphysical subspaces
- Effective Action path integral quantization favorable: symmetry principles are supplemented with a constraint principle

Vacuum Problem (n=0)

- Hamiltonian to third order is of Yukawa/Feshbach type:
  - quadratic part:

$$H_{\text{pot}} = \sum_i (U - 2\mu) n_{2,i} - \mu n_{1,i}$$

detuning from atom level



  - leading interaction:

$$H_{\text{kin}} = -J \sum_{\langle i,j\rangle} \left[t^\dagger_{1,i} t_{1,j} + \sqrt{2} (t^\dagger_{2,i} t_{1,i} t_{1,j} + t^\dagger_{1,i} t^\dagger_{1,j} t_{2,j}) \right]$$

(bilocal) dimer splitting into atoms

- Compare to standard Hubbard-Stratonovich decoupling:

usually: decouple interaction $U \rightarrow$ detuning $\sim 1/U$

here: interaction in quadratic part: detuning $\sim U$

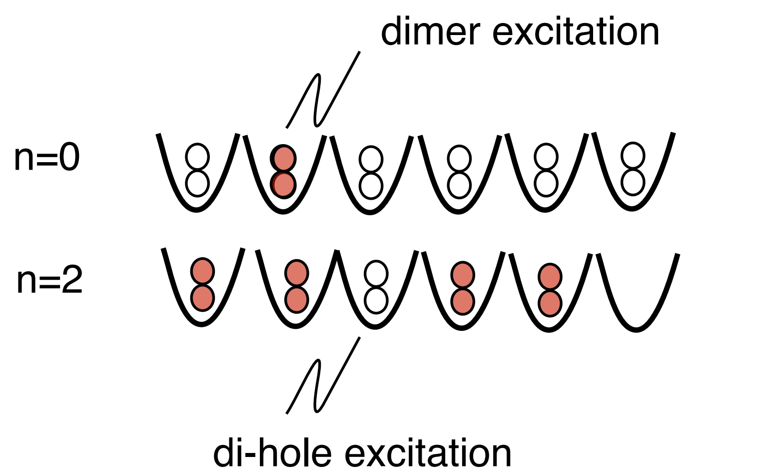
➡ realizes Feshbach model on the lattice

➡ we can expect resonant (strong coupling) phenomenology at weak coupling

$$H_{\text{kin}} = -J \sum_{\langle i,j\rangle} \left[t^\dagger_{1,i}(1 - n_{1,i} - n_{2,i})(1 - n_{1,j} - n_{2,j}) t_{1,j} + \sqrt{2} (t^\dagger_{2,i} t_{1,i} (1 - n_{1,j} - n_{2,j}) t_{1,j} + t^\dagger_{1,i} (1 - n_{1,i} - n_{2,i}) t^\dagger_{1,j} t_{2,j}) + 2 t^\dagger_{2,i} t_{2,j} t^\dagger_{1,j} t_{1,i} \right]$$

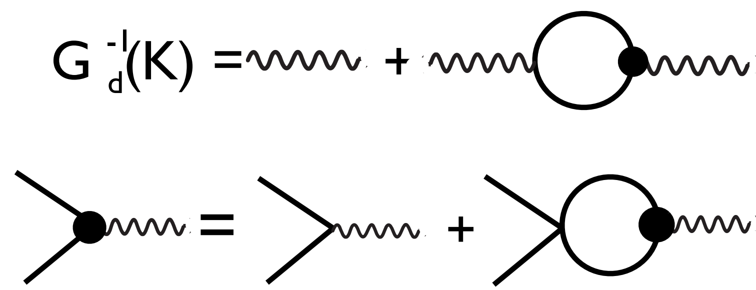
Additional Material

Vacuum Problems



- The physics at n=0 and n=2 are closely connected:
  - no spontaneous symmetry breaking
  - low lying excitations:
    - n=0: dimers on the physical vacuum
    - n=2: di-holes on the fully packed lattice

- Two-body problems can be solved exactly
  - Bound state formation: $G_d^{-1}(\omega = \mathbf{q} = 0) = 0$



$$\frac{1}{a_n|\tilde{U}| + b_n} = \int \frac{d^d q}{(2\pi)^d} \frac{1}{-\tilde{E}_b + 2/d \sum_\lambda (1 - \cos \mathbf{q}\mathbf{e}_\lambda)}$$

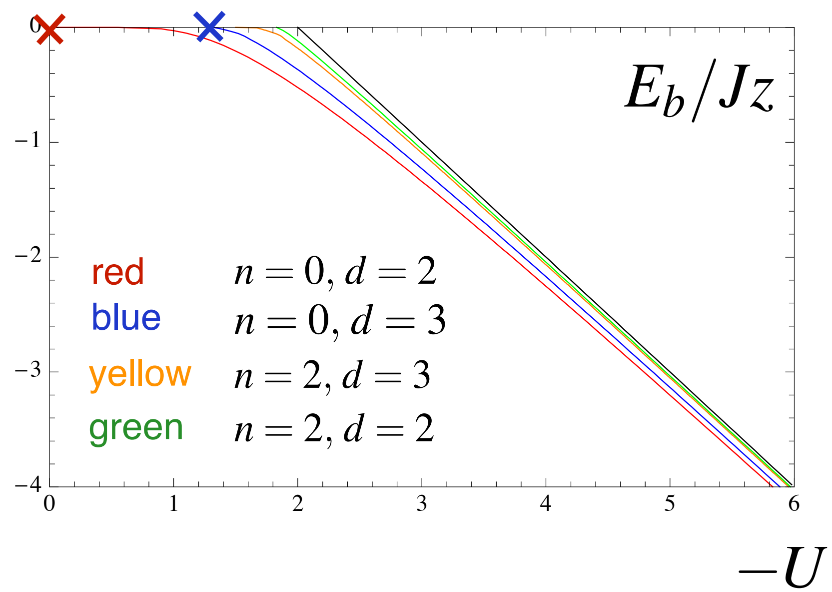
$$n = 0: \quad a_0 = 1,\ b_0 = 0$$

➡ reproduces Schrödinger Equation: benchmark

➡ Square root expansion of constraint fails

$$n = 2: \quad a_2 = 4,\ b_2 = -6 + 3\tilde{E}_b$$

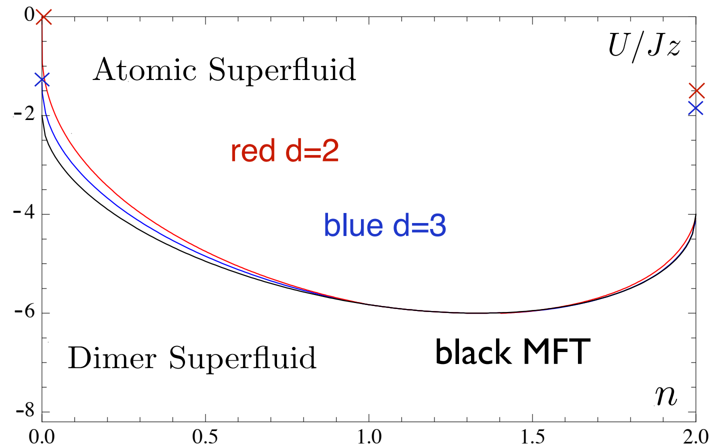
➡ di-hole-bound state formation at finite U in 2D



ASF - DSF Phase Border

Additional Material

shifts of the phase border



- Goal: Effects of quantum fluctuations on phase border
- Strategy:
  - Atomic mass matrix signals instability of ASF:

$$\det G_1^{-1}(\omega = \mathbf{k} = 0) = 0$$

  - ordering principle: small density expansion around

$$n \approx 0,\ n \approx 2$$

- Results:
  - dominant fluctuations: associated to bound state formation
  - two scales: bound state formation (G_2) and atom criticality (G_1)

(i) low density: coincidence of scales

➡ strong shifts, nonanalytic nonuniversal behavior

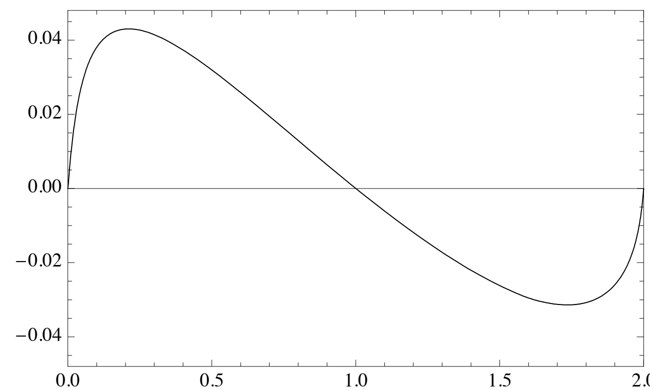
e.g. d=3: $$\frac{U_c}{Jz} \approx \frac{U_c(n=0)}{Jz} - \sqrt{\frac{\theta |U_c(n=0)|}{2Jz\sigma}}, \quad \sigma \approx 0.53$$

condensate angle

(ii) maximum density: mismatch of scales, di-hole bound state forms prior to atom criticality

➡ mean field like behavior

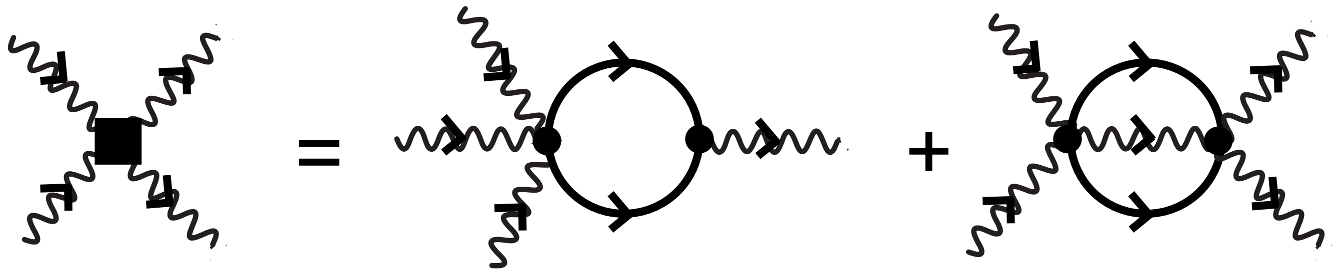
condensate depletion d=2



- Note: No particle-hole symmetry!

Effective Field Theory in Strong Coupling

- Perturbative limit U >> J: expect dimer hardcore model
- Perturbation theory second order J for interaction coefficient:



  - Strong quantum mechanical fluctuations: one and two-loop graph contribute equally
  - Constraint vertices describe forbidden decay possibilities for dimers
  - Resulting Hamiltonian (use constraint principle)

$$H_{\text{eff}} = \sum_{\langle i,j \rangle} \underbrace{t\, t^\dagger_{2,j}(1-\hat{n}_{1,j}-\hat{n}_{2,j})(1-\hat{n}_{1,i}-\hat{n}_{2,i})t_{2,i}}_{\text{constrained hopping}} + \underbrace{v\hat{n}_{2,i}\hat{n}_{2,j}}_{\text{effective nn-repulsion}} + \mu_{\text{eff}}\sum_i \hat{n}_{2,i}$$

$$\boxed{t = \frac{v}{2} = \frac{2J^2}{|U|}}$$

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UNIVERSITY OF INNSBRUCK

Quantum States and Phases in Dissipative Many-Body Systems with Cold Atoms

IQOQI
AUSTRIAN ACADEMY OF SCIENCES

Sebastian Diehl

Institute for theoretical physics, Innsbruck University,
and IQOQI Innsbruck

SFB
Coherent Control of Quantum Systems

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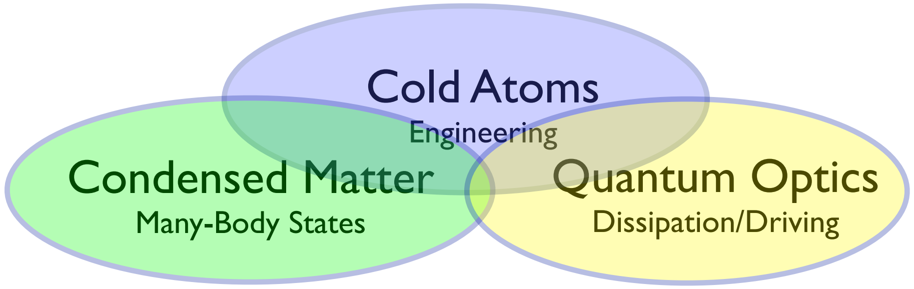
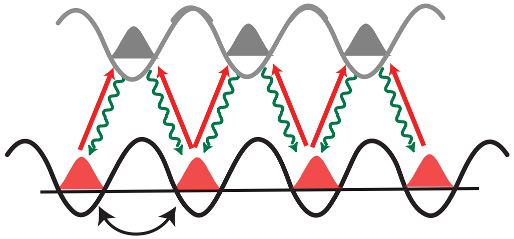
A. J. Daley (Innsbruck)	A. Tomadin (Innsbruck)
A. Kantian (Geneva)	W. Yi (Innsbruck)
B. Kraus (Innsbruck)	P. Zoller (Innsbruck)

Outline

Quantum State Engineering in Driven Dissipative Many-Body Systems



- Driven Dissipative BEC:
  - Mechanism for pure DBEC: Many-Body Quantum Optics
  - Physical Implementation of DBEC: Reservoir Engineering, Bogoliubov bath
- Application I: Competition of unitary vs. dissipative dynamics
  - first look: weak interactions
  - strong interactions: nonequilibrium phase transition
- Application II: Targeting pure fermion states
  - An excited many-body state: η-condensate
  - Antiferromagnetic and d-wave fermion states



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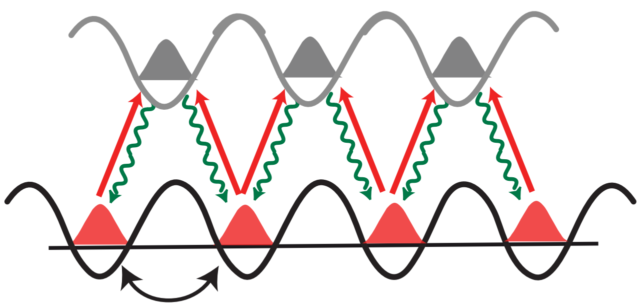
F. Verstraete, M. Wolf, I. Cirac, Nature Physics 5, 633 (2009)

Driven Dissipative BEC



Quantum State Engineering in Many-Body Systems

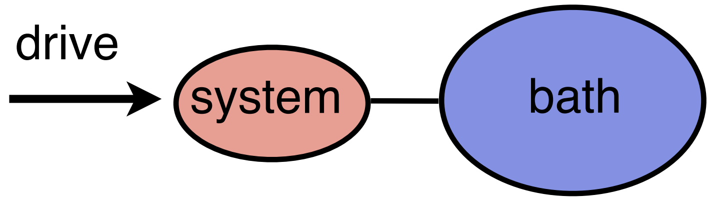
- **thermodynamic equilibrium**
  - standard scenario of condensed matter & cold atom physics

$$H|E_g\rangle = E_g|E_g\rangle \qquad \rho \sim e^{-H/k_BT} \xrightarrow{T\to 0} |E_g\rangle\langle E_g|$$

Hamiltonian (many body) | cooling to ground state

Hamiltonian Engineering: ✓interesting ground states
✓quantum phases

- **driven / dissipative dynamical equilibrium**
  - quantum optics



$$\frac{d\rho}{dt} = -i\,[H,\rho] + \mathcal{L}\rho \qquad \rho(t) \xrightarrow{t\to\infty} \rho_{ss} \quad \text{mixed state}$$

$$\overset{!?}{=} \quad |D\rangle\langle D| \quad \text{pure state (“dark state”)}$$

competing dynamics

master equation | steady state

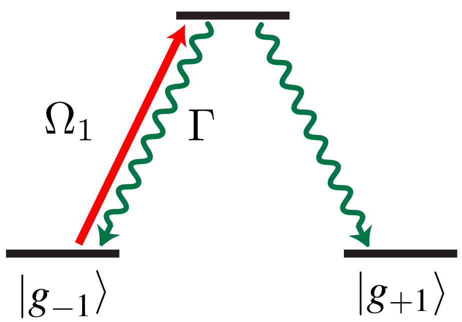
Liouvillian Engineering: ✓many body pure states / driven quantum phases
✓phase transtions from competing Hamiltonian and Liouvillian dynamics
✓useful and interesting fermion states

Dark States in Quantum Optics

- Goal: pure BEC as steady state solution, independent of initial density matrix:

$$\rho(t) \longrightarrow |BEC\rangle\langle BEC| \text{ for } t \to \infty$$

- Such situation is well-known quantum optics (three level system): optical pumping
  (Kastler, Aspect, Cohen-Tannoudji; Kasevich, Chu; ...)



$$\rho(t) \xrightarrow{t\to\infty} |g_+\rangle\langle g_+|$$

➡ Driven dissipative dynamics "purifies" the state

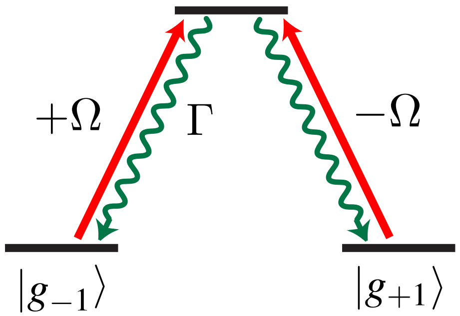
➡ $|g_+\rangle$ is a "dark state" decoupled from light

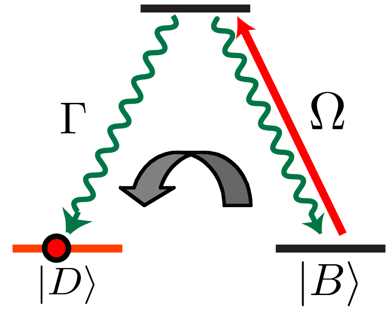
$$c_\alpha |g_+\rangle = 0$$

➡ Dark state is Eigenstate of jump operators with zero Eigenvalue
➡ Time evolution stops when system is in DS: pure steady state

An Analogy

- Λ-system: three electronic levels (VSCPT by Aspect, Cohen-Tannoudji; Kasevich, Chu)

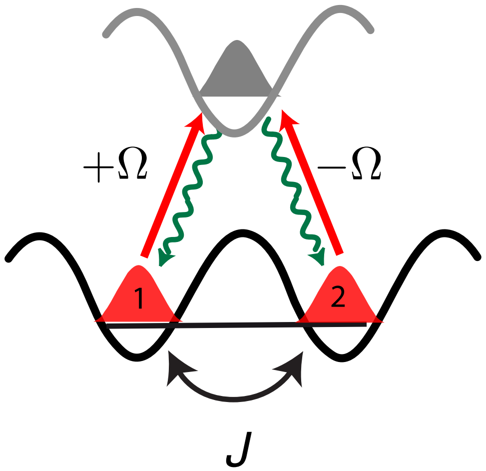




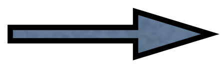
dark state | bright state

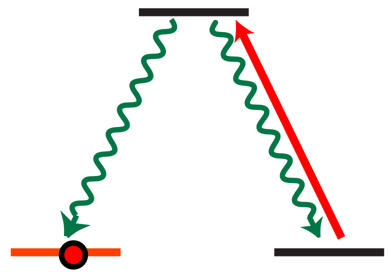
$|D\rangle \sim |g_{+1}\rangle + |g_{-1}\rangle$ $\quad$ $|B\rangle \sim |g_{+1}\rangle - |g_{-1}\rangle$

- 1 atom on 2 sites



~ dissipative Josephson junction





$(a_1^\dagger + a_2^\dagger)\,|\mathrm{vac}\rangle$ $\quad$ $(a_1^\dagger - a_2^\dagger)\,|\mathrm{vac}\rangle$

symmetric "in-phase" | anti-symmetric "out-of-phase"

pumping into symmetric state

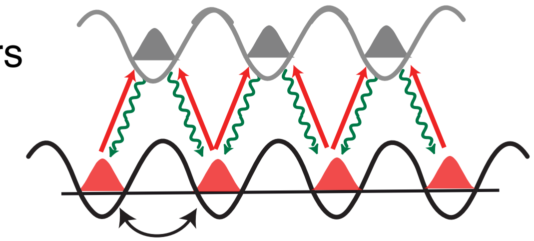
➡ "phase locking": like a BEC

Driven Dissipative lattice BEC

- Consider jump operator:

nearest neighbours

$$c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j)$$



(1) BEC state is a dark state: $|BEC\rangle = \frac{1}{N!}\left(\sum_\ell a_\ell^\dagger\right)^N |vac\rangle$

$$c_{ij}|BEC\rangle = 0 \;\forall i \qquad (a_i - a_j)\sum_\ell a_\ell^\dagger = \sum_\ell a_\ell^\dagger (a_i - a_j) + \sum_\ell \delta_{i\ell} - \delta_{j\ell}$$

(2) BEC state is the only dark state:

- $(a_i^\dagger + a_j^\dagger)$ has no eigenvalues
- $(a_i - a_j)$ has unique zero eigenvalue

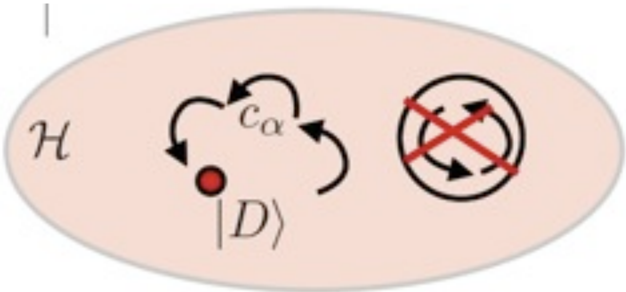
$$(a_i - a_j)\;\forall i \longrightarrow (1 - e^{i\mathbf{q}\mathbf{e}_\lambda})a_\mathbf{q}\;\forall \mathbf{q}$$

Driven Dissipative lattice BEC

(3) Uniqueness: |BEC> is the only stationary state (sufficient condition)

If there exists a stationary state which is not a dark state, then there must exist a subspace of the full Hilbert space which is left invariant under the set $\{c_\alpha\}$

(4) Compatibility of unitary and dissipative dynamics



$|D\rangle$ be an eigenstate of H, $H\,|D\rangle = E\,|D\rangle$

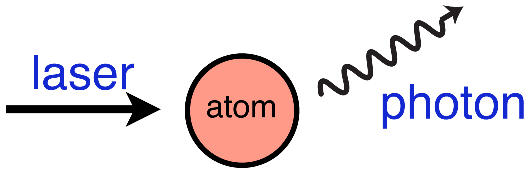
$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\,\langle D|$$

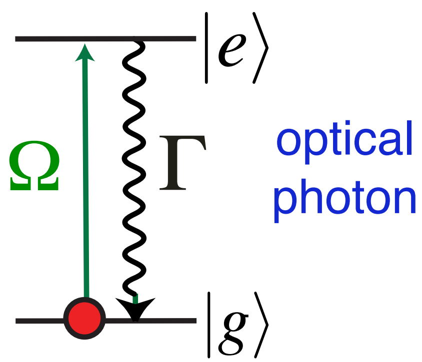
- Long range order in many-body system from quasi-local dissipative operations
- Uniqueness: Final state independent of initial density matrix
- Criteria are general: jump operators for AKLT states (spin model), eta-states (fermions), d-wave states (fermions, next lecture)

A. Griessner, A. Daley et al. PRL 2006; NJP 2007
(noninteracting atom)

Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission





- reservoir: vacuum modes of the radiation field (T=0)
- $\omega \sim 2\pi \times 10^{14} Hz$

Quantum optics ideas/techniques

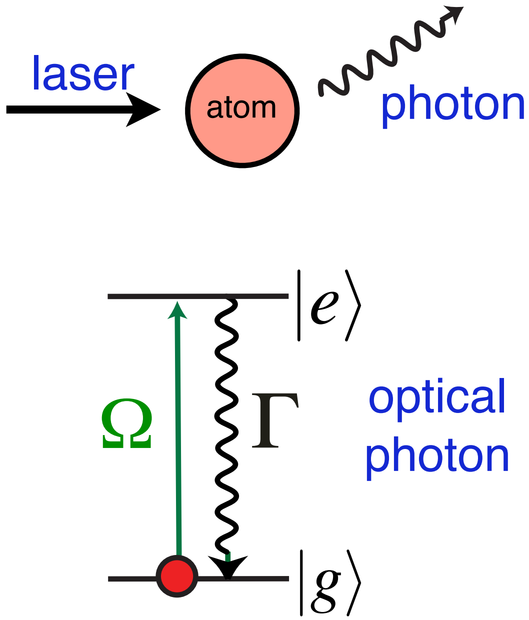
?

(many body) cold atom systems

- much lower energy scales...

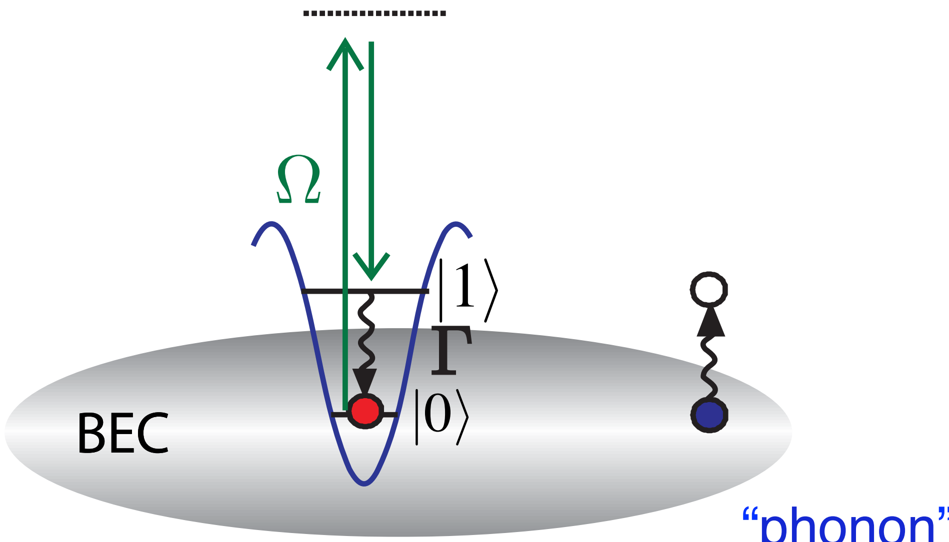
Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission



- reservoir: vacuum modes of the radiation field (T=0)
- $\omega \sim 2\pi \times 10^{14} Hz$

- trapped atom in a BEC reservoir



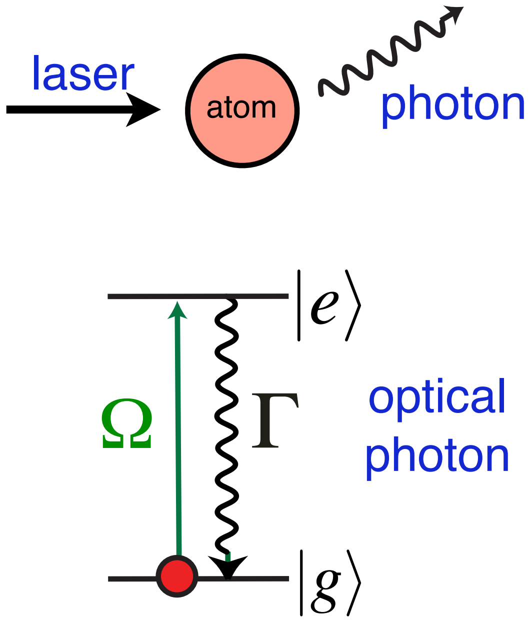
laser assisted atom + BEC collision

$\omega_{bd} \sim 2\pi \times kHz$

A. Griessner, A. Daley et al. PRL 2006; NJP 2007
(noninteracting atom)

Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission



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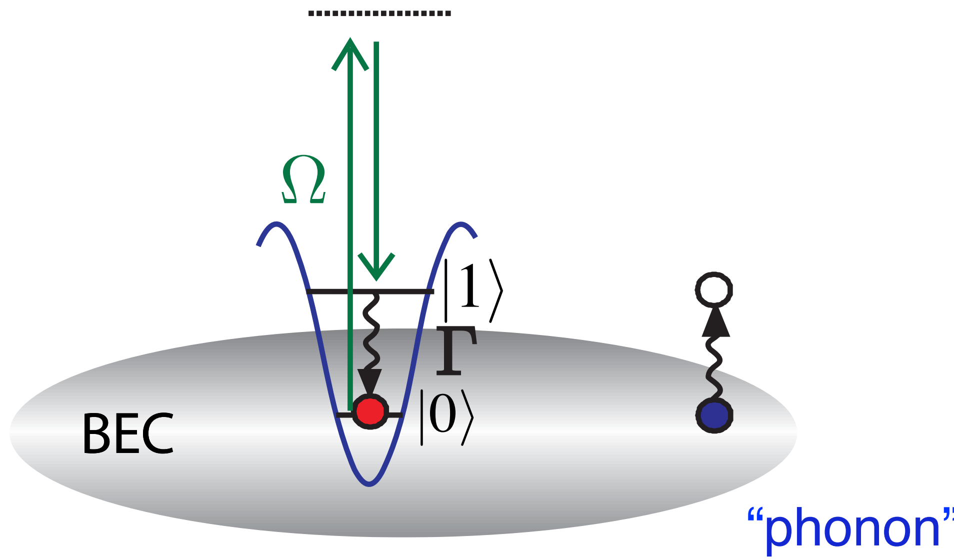
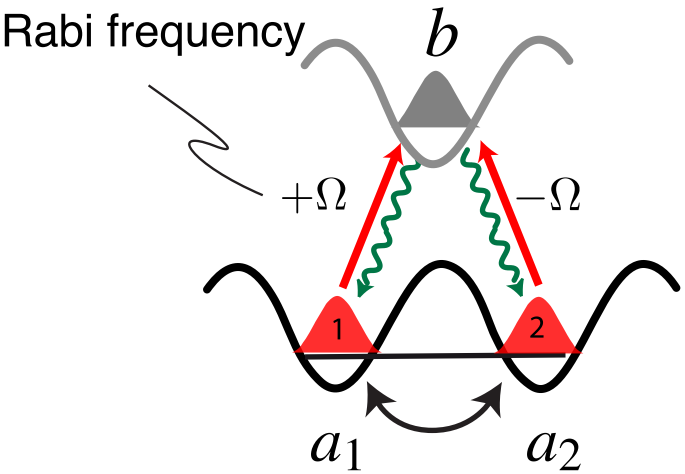


laser assisted atom + BEC collision

- reservoir: Bogoliubov excitations of the BEC (at temperature T)
- $\omega_{bd} \sim 2\pi \times kHz$

Physical Realization

Schematic



(1) Coherent excitation with opposite sign of Rabi frequency

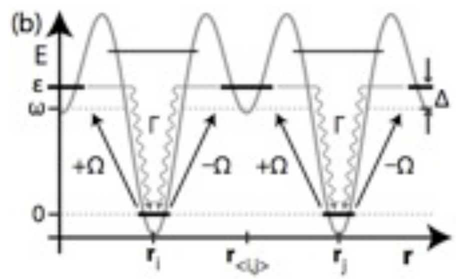
$$\Omega\, b^\dagger (a_1 - a_2) + h.c.$$

antisymmetric

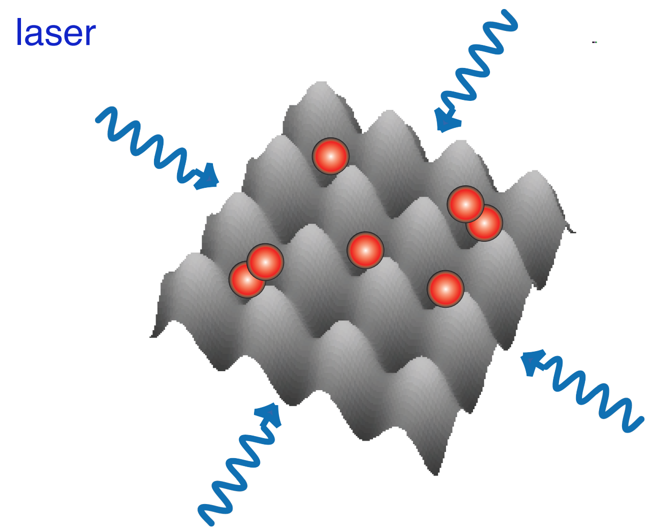
$$c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j)$$

In practice

- level structure: optical superlattice

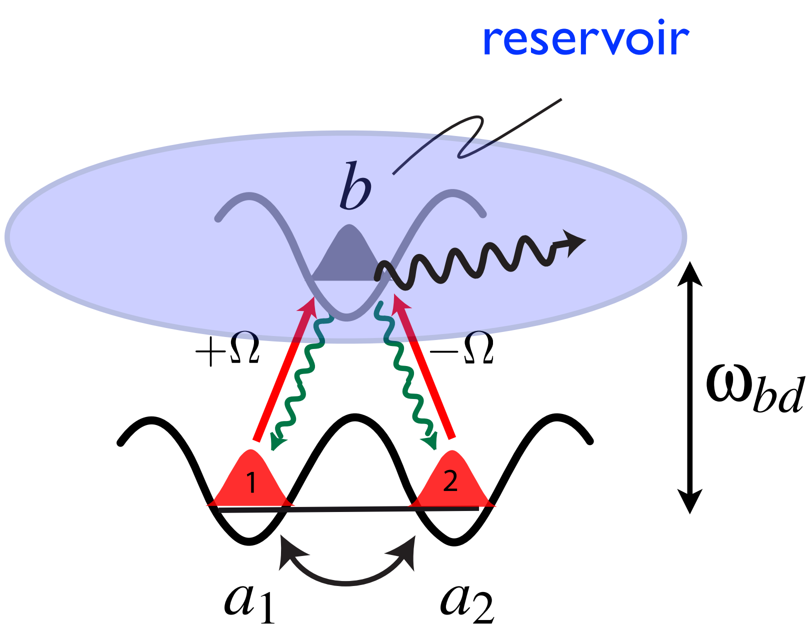


- coherent excitation: Raman laser



Physical Realization

Schematic



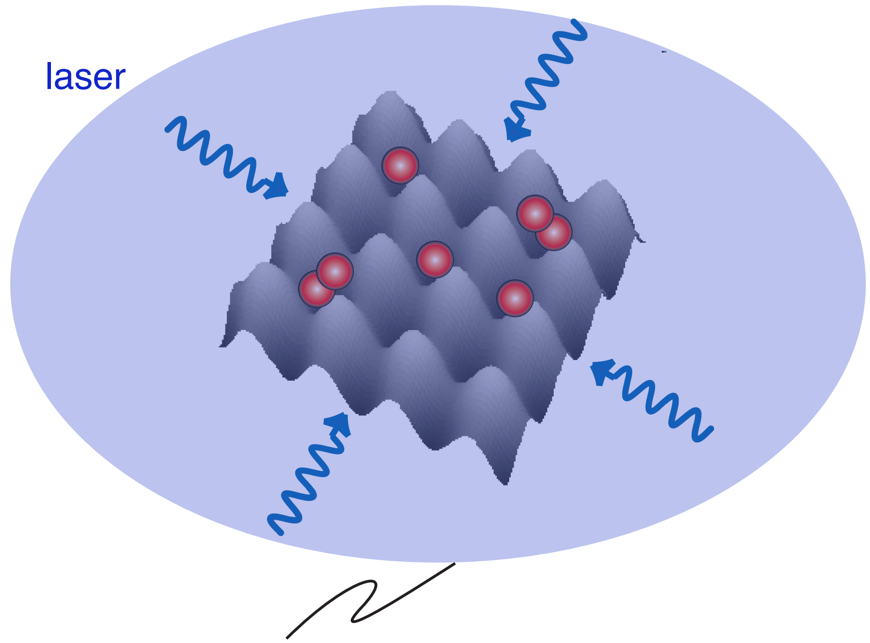
(2) Dissipative decay back:
coupling of upper level to reservoir

$$\kappa(a_1^\dagger + a_2^\dagger)b\sum_{\mathbf{k}}(r_{\mathbf{k}} + r_{\mathbf{k}}^\dagger)$$

symmetric

$$c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j)$$

In practice

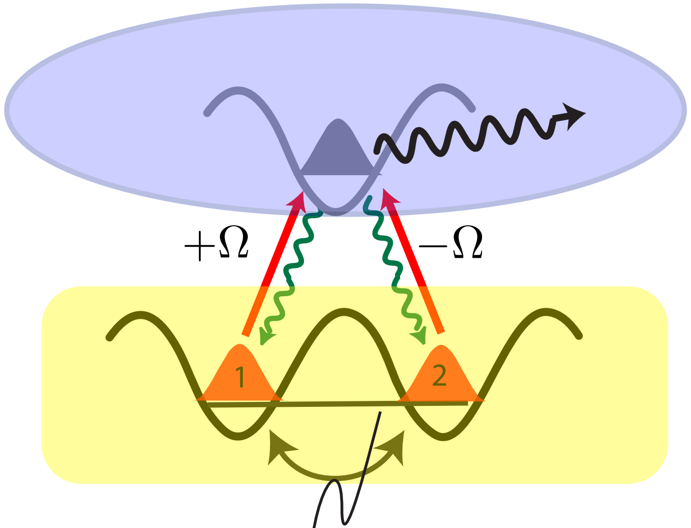


BEC = reservoir of
Bogoliubov excitations

➡ $T_{BEC} \ll \omega_{bd}$ effective
zero temperature reservoir

- coupling to system: interspecies interaction
- short coherence length in bath provides quasi-local dissipative processes, but not mandatory for our setup to work

Physical Realization

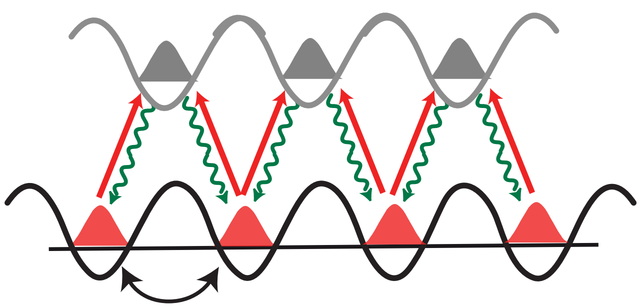


(3) adiabatic elimination of auxiliary level, trace out the bath

Effective single band jump operators

$$c_{12} = (a_1^\dagger + a_2^\dagger)(a_1 - a_2)$$

Many sites: Array of dissipative junctions



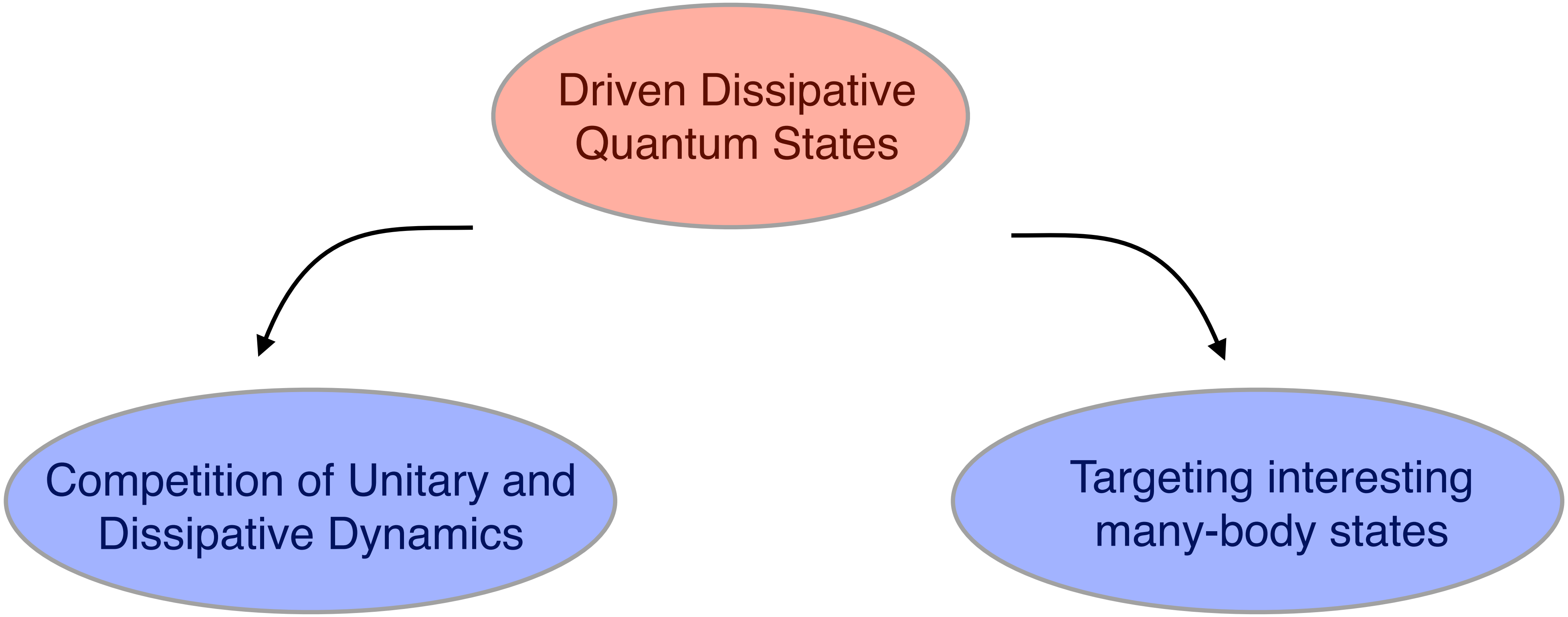
Comments:

- Long range phase coherence from quasi-local dissipative operations
- - Coherent drive: locks phases

  - Dissipation: randomizes

  - Conspiracy: purification
- The coherence of the driving laser is mapped on the matter system
- Setting is therefore robust

Applications: Preview



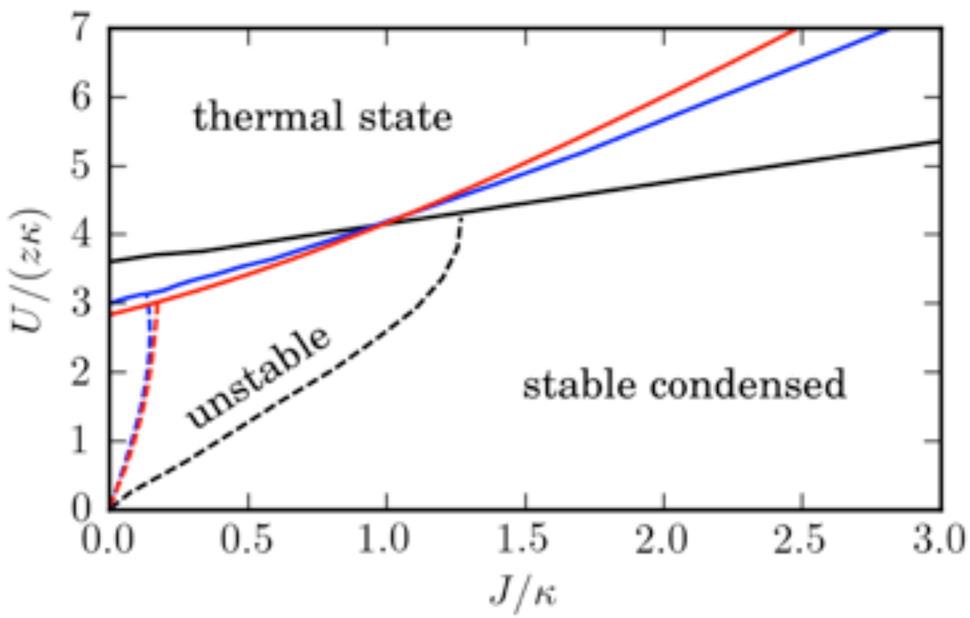
- Nonequilibrium Phase Transitions
- Dynamical Instabilities

- paired fermion states for quantum simulation

New class of nonequilibrium systems

Practical use for future cold atom experiments

Competition of Unitary vs. Dissipative Dynamics



Effects of finite interactions

$$H = -J \sum_{<i,j>} a_i^\dagger a_j + U \sum_i a_i^{\dagger 2} a_i^2$$

dissipative dynamics favors pure BEC state

interacting Hamiltonian dynamics not compatible

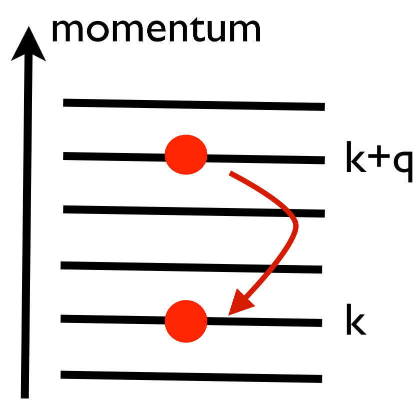
Competition

$$\frac{d\rho}{dt} = -i\,[H, \rho] + \mathcal{L}\rho$$

treating interactions in

- weak coupling
  - 3D: true (depleted) condensate, fixed phase: Bogoliubov theory
  - 1,2D: phase fluctuations destroy long range order: Luttinger theory
- Strong coupling, 3D
  - mixed state Gutzwiller Ansatz

Weak Coupling: Linearized jump operators

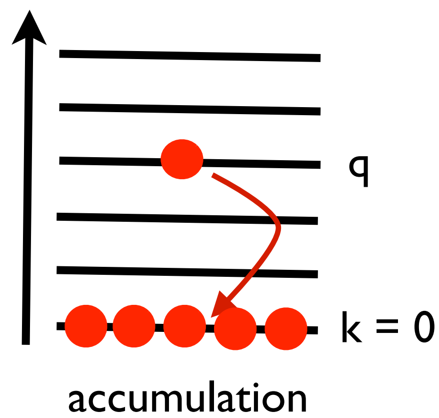


- momentum space jump operators are nonlocal nonlinear objects

$$c_{\mathbf{q},\lambda} = \frac{1}{M^{d/2}} \sum_{\mathbf{k}} (1 + \mathrm{e}^{\mathrm{i}\mathbf{k}\mathbf{e}_\lambda})(1 - \mathrm{e}^{-\mathrm{i}(\mathbf{k}+\mathbf{q})\mathbf{e}_\lambda}) a^\dagger_{\mathbf{k}} a_{\mathbf{k}+\mathbf{q}}$$

- In a linearized theory the reduce to (any dimension)

$$c_{\mathbf{q},\lambda} = f_{\mathbf{q},\lambda} a_{\mathbf{q}} \qquad f_{\mathbf{q},\lambda} = 2\sqrt{n}(1 - e^{-\mathrm{i}\mathbf{q}\mathbf{e}_\lambda})$$

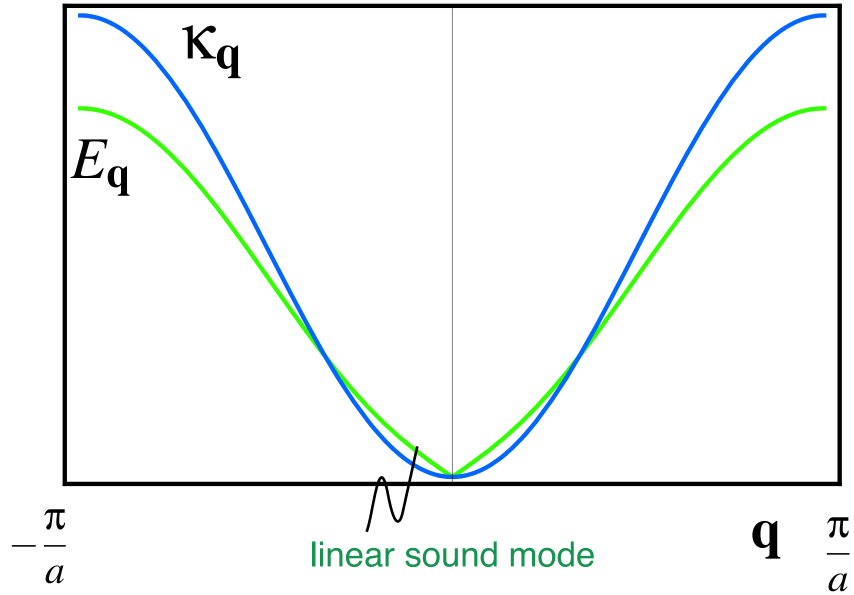


- Interpretation:
  - bosonic mode operators: depopulation of momentum q in favor of condensate
  - zero mode explicit: $f_{\mathbf{q}=0,\lambda} = 0$
  - lead to momentum dependent decay rate

$$\kappa_{\mathbf{q}} = \sum_\lambda \kappa |f_{\mathbf{q},\lambda}|^2 \sim \mathbf{q}^2$$

Many-Body Master Equation

- Interpretation: How close are we to the GS of the Hamiltonian?
  - Diagonalize H
  - consider equation for single mode



$$\partial_t \rho = \overbrace{-i\frac{E}{2}[d^\dagger d, \rho]}^{\text{Bogoliubov / hydrodynamic excitation}}$$

$$+2\kappa(\underbrace{u^2 d\rho d^\dagger}_{\text{“cooling”}} + \underbrace{v^2 d^\dagger \rho d}_{\text{“heating”}} - \underbrace{uv(d^\dagger \rho d^\dagger + d\rho d)}_{\text{squeezing}})$$ + anticommutator term

$v^2_{\mathbf{q}},\ u^2_{\mathbf{q}} = v^2_{\mathbf{q}} + 1$ generalized Bogoliubov coefficients

$N, \qquad N+1$ cf. thermal reservoir

➡ Intrinsic heating/cooling, though reservoir is at T = 0

Characterization of Steady State: Density Operator

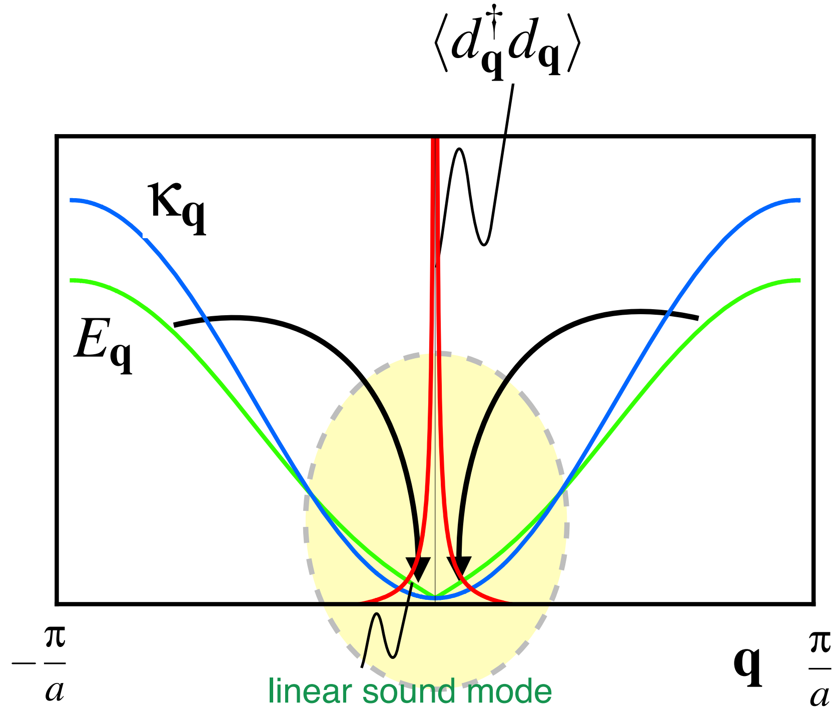
- linearized ME exactly solvable: Gaussian density operator for each mode expressible as

$$\rho_{\mathbf{k}} = \exp\left(-\beta_{\mathbf{k}} b^\dagger_{\mathbf{k}} b_{\mathbf{k}}\right)$$

with squeezed operators b (Bogoliubov transformation)

➡ mixed state with

$$\coth^2\left(\beta_{\mathbf{k}}/2\right) = \frac{\kappa_{\mathbf{k}}^2 + (\varepsilon_{\mathbf{k}} + Un)^2}{\kappa_{\mathbf{k}}^2 + E_{\mathbf{k}}^2}$$



- at low momenta, resemblance to thermal state:

$$\beta_{\mathbf{k}} \approx \frac{E_{\mathbf{k}}}{T_{\text{eff}}}, \quad T_{\text{eff}} = \frac{Un}{2}$$

‣ role of temperature played by interaction

Correlations in various dimension: 3D

- Steady state: condensate depletion:

$$n_D = n - n_0 = \frac{1}{2}\int \frac{d\mathbf{q}}{v_0} \frac{(Un)^2}{\kappa_q^2 + E_q^2}$$

  - small depletion justifies Bogoliubov theory
  - squeezing and mixing effects tied to interaction strength (unlike th. equilibrium)

- Approach to the steady state:

$$n_{0,eq} - n_0(t) \sim \sqrt{\frac{Un}{8J}} \frac{1}{2\kappa n} t^{-1}$$

  - power-law: Many-body effect due to mode continuum
  - sensitive probe to interactions: cf. for noninteracting system

$$n_{0,eq} - n_0(t) \sim t^{-3/2}$$

  - universal at late times

Correlations in various dimension: 1/2D

- Steady State: quasi-condensates in low “temperature” phase

$$\langle a_x^\dagger a_0 \rangle \sim \langle \exp i(\phi_x - \phi_0) \rangle \sim \begin{cases} \mathrm{e}^{-\frac{T_{\mathrm{eff}}}{8Jn} x}, & d = 1 \\ (x/x_0)^{-T_{\mathrm{eff}}/4T_{\mathrm{KT}}}, & d = 2 \end{cases}$$

$$T_{\mathrm{KT}} = \pi J n \gg T_{\mathrm{eff}} \qquad T_{\mathrm{eff}} = Un/2 \qquad x_0 = 2\kappa n (T_{\mathrm{eff}} J)^{-1/2}$$

Kosterlitz-Thouless temperature of 2D quasi-condensate

Dissipative coupling: only sets cutoff scale

- steady state well understood as thermal Luttinger liquid
- similar results for temporal correlations (from ME via quantum regression theorem)
- weak effect of dissipation on phase fluctuations:

$$E_{\mathbf{q}} \sim |\mathbf{q}|, \kappa_{\mathbf{q}} \sim \mathbf{q}^2$$

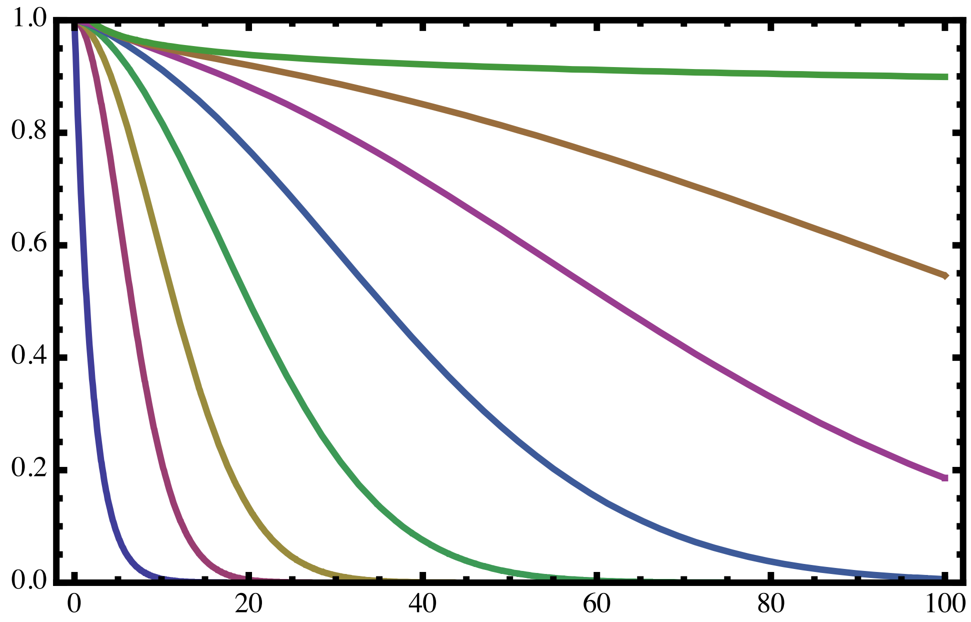
2D: Real Time Evolution

- Buildup of spatial correlations from disordered state

$$\Psi_t(x,0) \sim \begin{cases} e^{-|x|/\xi} & t=0 \\ (x/x_0)^{-\frac{T_{\mathrm{eff}}}{4T_{\mathrm{KT}}}}\, e^{-\frac{x^2}{4\xi\sqrt{\pi\kappa n t}}} & t \longrightarrow \infty \end{cases}$$

broadening of Gaussian governed by time-dependent length scale

$$x_t = 2(\pi\xi^2\kappa n t)^{1/4}$$



Strong Coupling: Nonequilibrium Phase Transition

- Analogy to Mott insulator / Superfluid quantum phase transition: Competition

• enhancement of superfluidity:	Hopping J	driven dissipation κ
• suppression of superfluidity:	interaction U	interaction U

➡ Expect phase transition as function of J/U κ/U

- Differences:

➡ Competition of two unitary evolutions vs. competition of unitary and dissipative evolution

✓ phase transition (temperature T)

✓ quantum phase transition (g)

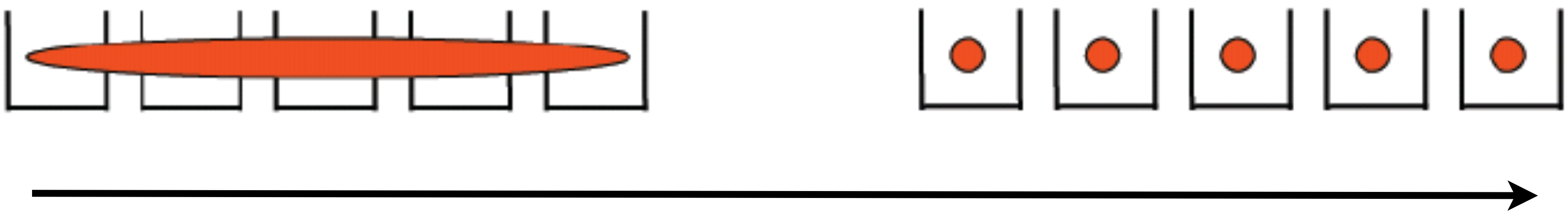
Reminder: Mott Insulator-Superfluid Phase Transition

$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j - \mu \sum_i \hat{n}_i + \tfrac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

- Hopping J favors delocalization in real space:
- Condensate (local in momentum space!)
- Fixed condensate phase: Breaking of phase rotation symmetry

- Interaction U favors localization in real space for integer particle numbers:
- Mott state with quantized particle no.
- no expectation value: phase symmetry intact (unbroken)

$$\langle b_i \rangle \sim e^{\mathrm{i}\varphi}$$



➡ Competition gives rise to a quantum phase transition as a function of

$$U/J$$

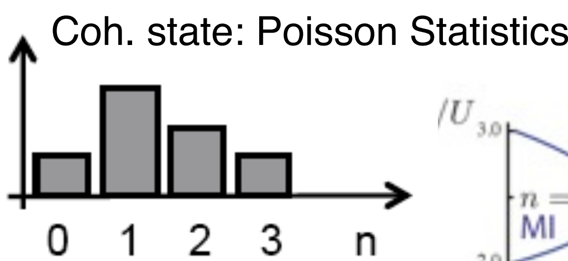
Reminder: Gutzwiller Ansatz

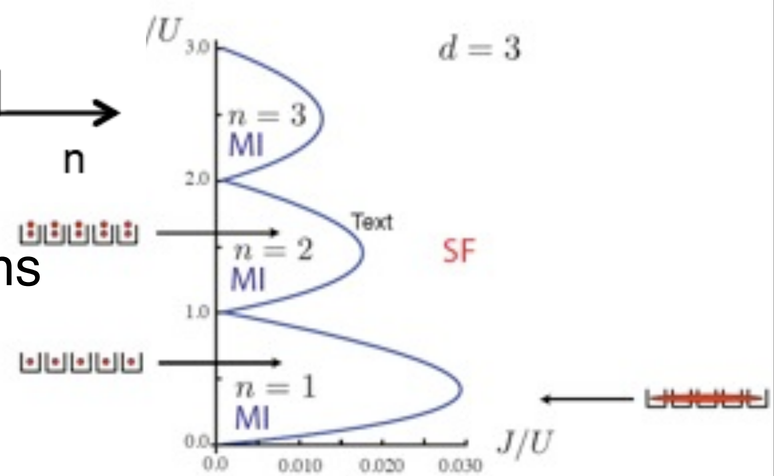
- Interpolation scheme encompassing the full range J/U.
  - Main ingredient: product wave function ansatz

$$|\psi\rangle = \prod_i |\psi\rangle_i, \quad |\psi\rangle_i = \sum_n f_n^{(i)} |n\rangle_i, \quad {}_i\langle\psi|\psi\rangle_i = 1\,\forall i$$

complex amplitudes — wave function normalization

  - Limiting cases (homogeous, drop site index, amplitudes chosen real):
    - Mott state with particle number m: $f_n = \delta_{n,m}$
    - coherent state: $f_n = \sqrt{N/n!}e^{-N/2}$





- Validity: approximation neglects all spatial correlations
  - becomes exact in infinite dimensions
  - reasonable in d=2,3 (T=0)

Mixed State Gutzwiller Approach

- Product ansatz for the density operator (instead of wave function)

$$\rho(t) = \prod_i \rho_i(t), \quad \rho_i(t) = \sum_{nm} |n\rangle_i \langle m| \rho_{nm}^{(i)}(t)$$

Interpretation:

✓ off-diagonal: SF

✓ diagonal: atom statistics

- Project on on-site density operator:

$$\rho_k = \mathrm{Tr}_{\neq k}\, \rho$$

➡ Nonlinear Mean Field Master Equation for reduced density operator (homogenous, drop index)

$$B^r = \{\hat{n}, b, b^\dagger, \mathbf{1}\}$$

$$\dot{\rho} = -\mathrm{i}\left[-Jz(\langle b\rangle b^\dagger + \langle b^\dagger\rangle b) + \tfrac{1}{2}U b^{\dagger 2} b^2, \rho\right]$$

$$+\kappa z \sum_{r,r'} \Gamma^{r,r'} \left\{2B^r \rho B^{\dagger\, r'} - B^{\dagger\, r'} B^r \rho - \rho B^{\dagger\, r'} B^r\right\}$$

with correlation matrix

$$\Gamma^{r,r'} = \begin{bmatrix} \langle \hat{n}^2\rangle & \langle b^\dagger \hat{n}\rangle & -\langle b\hat{n}\rangle & -\langle \hat{n}\rangle \\ \langle \hat{n} b\rangle & \langle \hat{n}\rangle & -\langle b^2\rangle & \langle b\rangle \\ -\langle \hat{n} b^\dagger\rangle & -\langle b^{\dagger 2}\rangle & \langle \hat{n}\rangle + 1 & \langle b^\dagger\rangle \\ -\langle \hat{n}\rangle & -\langle b^\dagger\rangle & \langle b\rangle & \langle \mathbf{1}\rangle \end{bmatrix}$$

Properties of ME:

✓ trace conserving

✓ mean particle number conserving

- Nonlinearity emerging in approximation to linear qm equation: similar GP equation

Condensed Steady State

- Vanishing interaction: Liouvillian and hopping are compatible operators. The steady state is a pure coherent state (i.e. condensate).

$$\hat{b}_\ell \to \psi_\ell \in \mathbb{C} \qquad |\psi_\ell|^2/n = 1 \qquad \langle \hat{b}_\ell \hat{b}^\dagger_{\ell'} \rangle \to \psi_\ell \psi^*_{\ell'}$$

decoupling of the correlation functions

- Qualitative effect of small interactions: dissipative Gross-Pitaevskii equation

$$\partial_t \psi_\ell = -i(-J \sum_{\langle \ell' | \ell \rangle} \psi_{\ell'} + U|\psi_\ell|^2 \psi_\ell) - 2\kappa \sum_{\langle \ell' | \ell \rangle} (\psi_\ell - \psi_{\ell'} + \psi^*_{\ell'} \psi^2_\ell - |\psi_{\ell'}|^2 \psi_{\ell'})$$

homogeneous system $\partial_t \psi = i(Jz + U|\psi|^2)\psi$

- Choice of the chemical potential to enforce vanishing of the unitary term: steady state condition:

$$\psi(t) = \sqrt{n} e^{-i\mu t} \Rightarrow \mu = -Jz + Un$$

- From now on, we work with equation including chemical potential

Thermal Steady State

- Strong interaction destroy the phase coherence:

transformation to a rotating frame of reference with unitary

$$V \equiv e^{iU\hat{n}(\hat{n}-1)t}$$

annihilation operator in the rotating frame

$$V\hat{b}V^{-1} = e^{-iU\hat{n}t}\hat{b} = \sum_n e^{inUt}|n\rangle\langle n|\hat{b}$$

dephasing & average out

- The master equation for a diagonal (mixed!) state reduces to

$$\partial_t \rho_\ell = \kappa[(\bar{n}+1)(2b_\ell \rho b_\ell^\dagger - \{b_\ell^\dagger b_\ell, \rho_\ell\}) + \bar{n}(2b_\ell^\dagger \rho_\ell b_\ell - \{b_\ell b_\ell^\dagger, \rho_\ell\})]$$

* factorization of correlation function + vanishing order parameter = no kinetic term
* diagonal state in Fock space = no interaction contribution
* the system acts as its own reservoir

- Thermal state solution, determined only by the average density

$$[\rho_\ell]_{n,n'} = \delta_{n,n'} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}$$

- Note: The thermal solution is always a dynamical fixed point of the mean field master equation. However, below a critical U it is unstable (cf. Mexican hat potential)

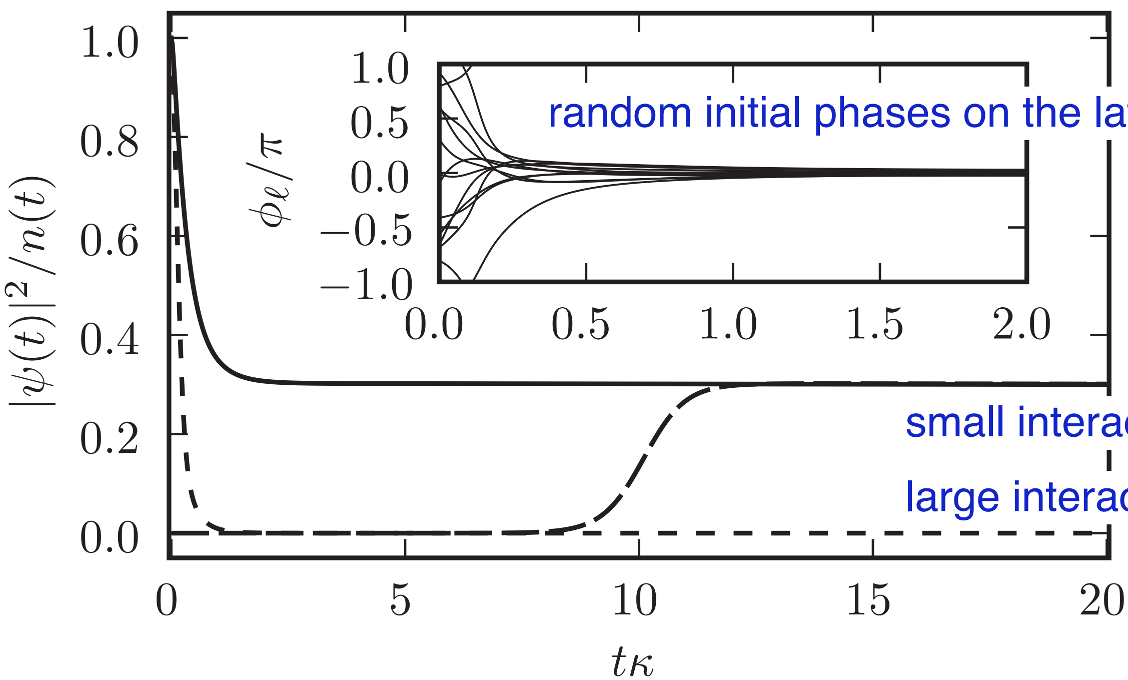
Numerical Solution of the Equation of Motion

- Forward time-evolution of the nonlinear Liouville equation

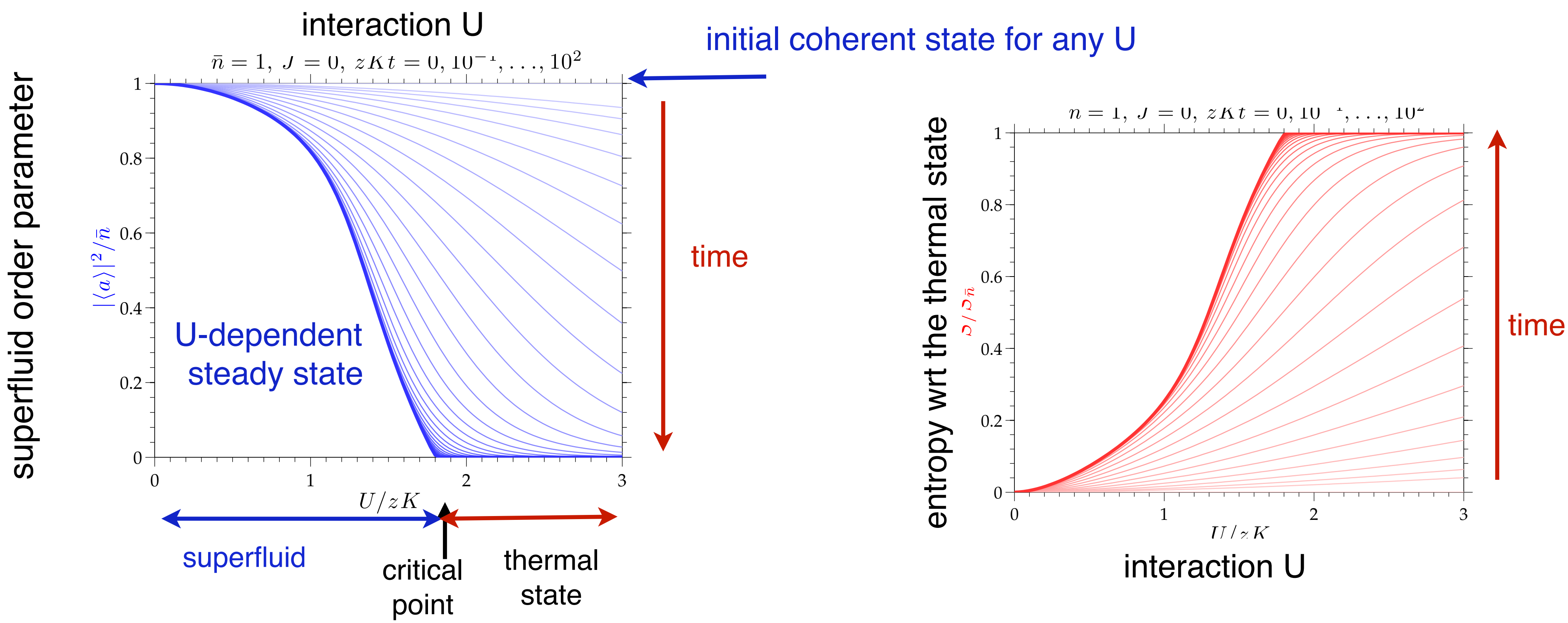
$$\rho(t)$$

$$\rho(t+{\color{red}dt}) \simeq \rho(t) + {\color{red}dt} \times \mathcal{L}[\rho(t); \boxed{\psi(t), \langle \hat{n}(t)\rangle, \langle \hat{b}^2(t)\rangle, \ldots}\,]$$

- Independence of the final state of the system from the initial state



Dependence of the Steady State on the Interaction

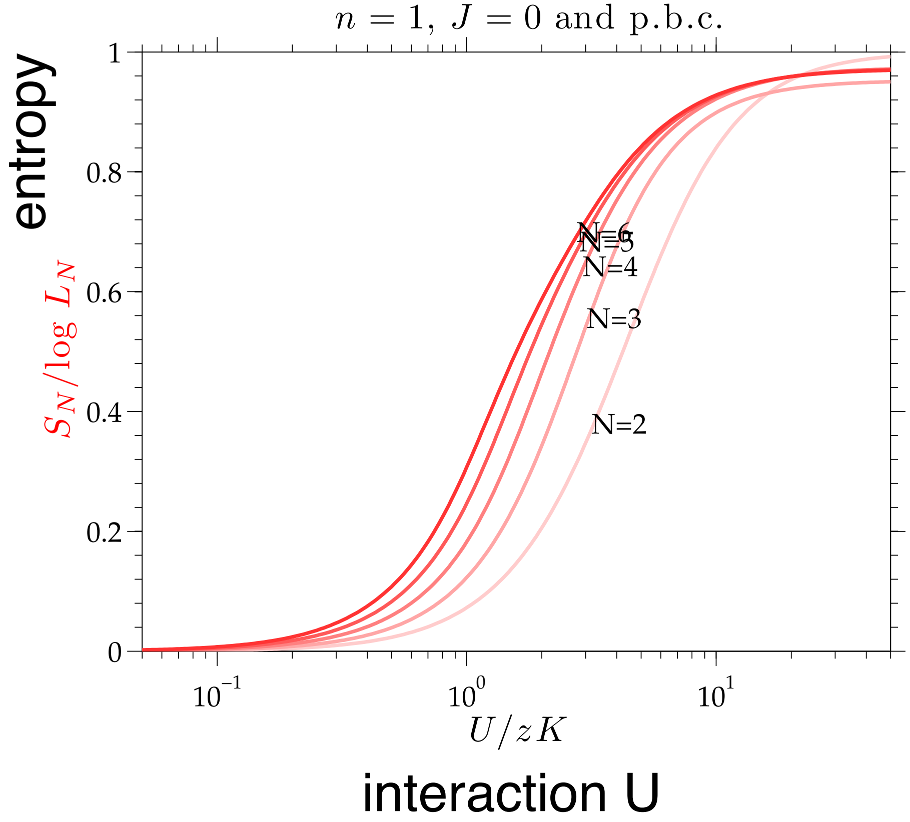


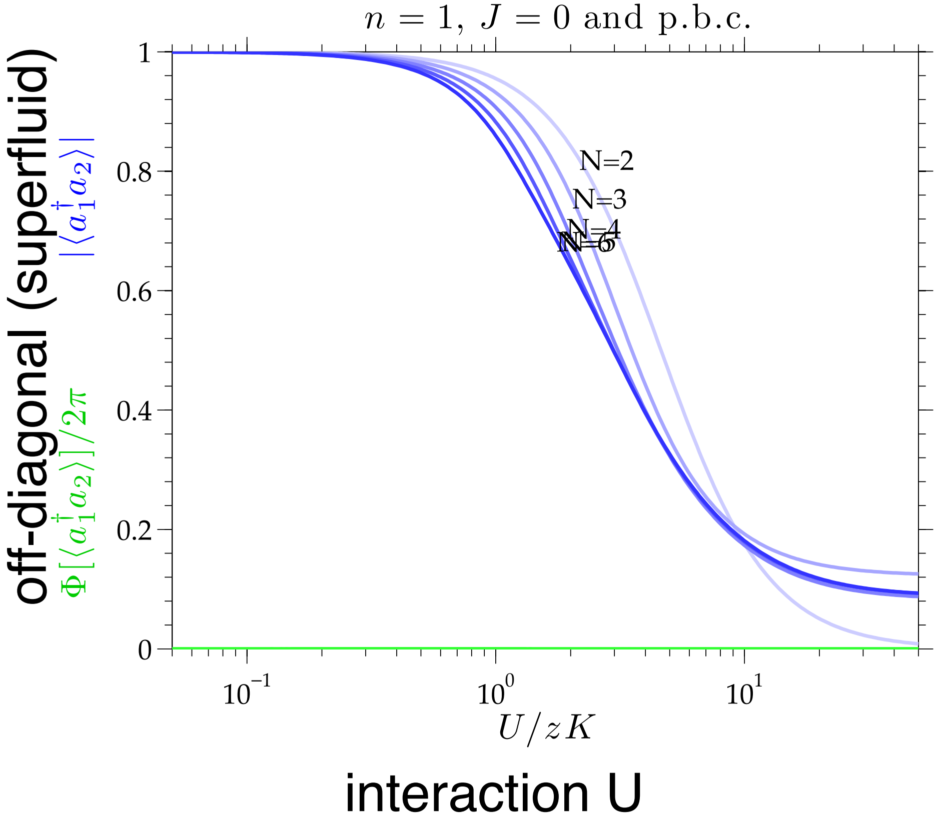
Nonequilibrium phase transition between pure and mixed state,

driven by a competition between unitary and dissipative dynamics

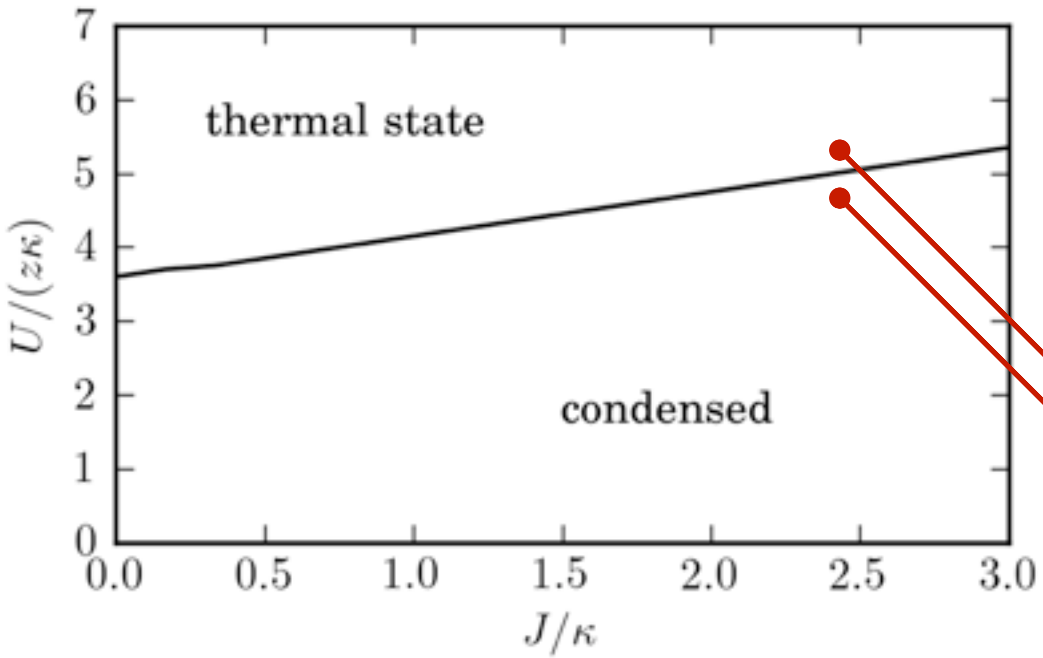
- Shares features of:
  - Quantum phase transition: interaction driven
  - Classical phase transition: ordered phase terminates in a thermal state
- Development in time of the non-analyticity at the critical point
- No signature of commensurability effects (Mott) due to strong mixing of U

Exact calculations for N=6 sites





Nonequilibrium Phase Diagram



- critical point U for vanishing J
- increasing J stabilizes the condensate

thermal state stable
thermal state unstable towards condensed

- Linear instability analysis around the thermal state to determine the boundary

$$\partial_t \Delta\rho_\ell(t) = -i[\hat{h}_\ell^{(0)}, \Delta\rho_\ell(t)] + \mathcal{L}_\ell^{(0)}[\Delta\rho_\ell(t)] - i[\Delta\hat{h}_\ell, \rho_\ell^{(\mathrm{th})}] + \Delta\mathcal{L}_\ell[\rho_\ell^{(\mathrm{th})}]$$

variation in time

$$\partial_t \delta\rho_{n,n-1} = M[\rho^{(\mathrm{th})}; \delta\rho_{n-1,n-2}, \delta\rho_{n+1,n}]$$

spectrum of the linear form determines the stability

$$\begin{aligned} \delta\langle\hat{b}_{\ell'}\rangle &= \delta\langle\hat{b}_\ell\rangle e^{-i\phi_0(\ell'-\ell)} \\ \delta\langle\hat{b}_{\ell'}\hat{n}_{\ell'}\rangle &= \delta\langle\hat{b}_\ell\hat{n}_\ell\rangle e^{-i\phi_0(\ell'-\ell)} \end{aligned}$$

Analytical Approach in the Limit of Low Density

- Study the equations of motion of the correlation functions

$$
\begin{aligned}
\partial_t \langle (b_\ell^\dagger)^n b_\ell^m \rangle &= \mathrm{Tr}[(b_\ell^\dagger)^n b_\ell^m \, \partial_t \rho_\ell(t)] \\
&= -i\mathrm{Tr}[(b_\ell^\dagger)^n b_\ell^m \, [\mathcal{H}_\ell, \rho_\ell(t)]] + \mathrm{Tr}[(b_\ell^\dagger)^n b_\ell^m \mathcal{L}[\rho_\ell(t)]]
\end{aligned}
$$

(nonlocal) coupling to other correlation functions: infinite hierarchy

- Introduce a power counting: $\boxed{b_\ell \sim \sqrt{n}, b_\ell^\dagger \sim \sqrt{n}}$
  and keep only the leading order for $n \to 0$

- (Infinite) hierarchy exhibits a closed nonlinear subset for the three correlation functions

$$
\begin{aligned}
\partial_t \psi &= i\mu\psi + (-iU + 4\kappa)\langle b^\dagger b^2 \rangle - 4\kappa\psi^* \langle b^2 \rangle \ , \\
\partial_t \langle b^\dagger b^2 \rangle &= 8n\kappa\psi + (-iU + i\mu - 8\kappa)\langle b^\dagger b^2 \rangle \ , \\
\partial_t \langle b^2 \rangle &= (-iU + 2i\mu - 8\kappa)\langle b^2 \rangle + 8\kappa\psi^2 \ .
\end{aligned}
$$

(homogeneous system)

- Fix the chemical potential to make the linear term vanish in the steady state

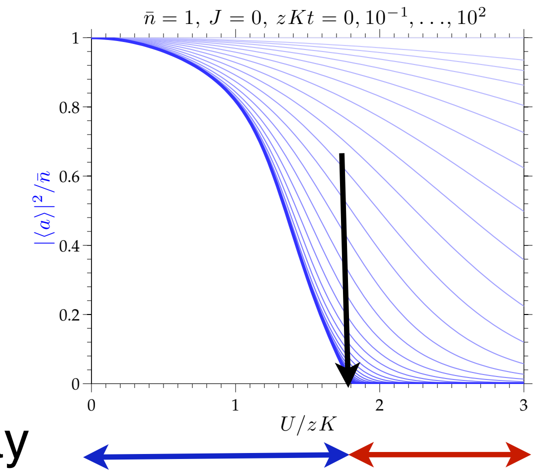
Critical Exponent of the Phase Transition

- Critical exponents can be extracted from approaching the phase transition in time

- In linear response, expect form of the order parameter evolution

$$|\psi(t)| \sim \frac{e^{-m^2 t}}{t^{\alpha}}$$

real part of lowest eigenvalue: “mass”



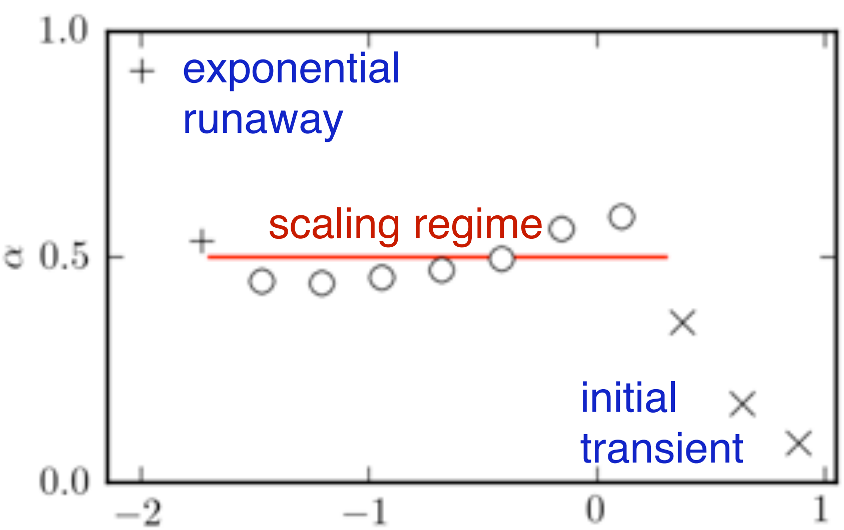
$m^2 < 0$ $m^2 > 0$

unstable
disorderd state

- At criticality: zero eigenvalue and thus dominant polynomial decay

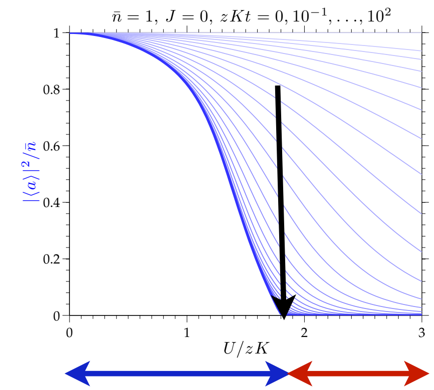
$$\alpha = \lim_{t \to \infty} \frac{d \log \psi(t)}{d \log(1/t)}$$

- Numerical Result:



$$\alpha \approx 1/2$$

Critical Exponent of the Phase Transition



$m^2 < 0$ $m^2 > 0$

unstable
disorderd state

- Analytically for low density:
  - At criticality, order parameter evolution is

$$\partial_t \psi \quad = \quad -4\kappa \psi^* \langle b^2 \rangle$$

- $\langle b^2 \rangle$ evolves fast (exponentially) and can be obtained in adiabatic approximation

$$\langle b^2 \rangle \approx \frac{8\kappa\psi^2}{(8\kappa + iU - 2inU)} \propto \psi^2 \qquad \Rightarrow \partial_t \psi \quad \propto \quad \psi^* \psi^2$$

- Thus, Landau-Ginzburg type cubic but dissipative nonlinearity

$$\boxed{|\psi(t)| \sim t^{-1/2}, \quad \alpha = 1/2}$$

- This is the mean field value as expected. But it governs the time evolution

Analytical Computation of the Steady State

- Introduction of “connected” correlation functions

$\delta\hat{b} \equiv \hat{b} - \psi_\infty$ unknown constant to be determined self-consistently

$\langle \delta\hat{b} \rangle = 0$ equilibrium property of the steady state

- Equations of motion become *linear* in the correlation functions:
  - Solution for the correlation functions, with μ and ψ_∞ parameters
  - Choice of μ from condition that drive for $\langle \delta\hat{b} \rangle, \langle \delta\hat{b}^\dagger \rangle$ vanish (cf. Goldstone’s theorem)
  - Solution for ψ_∞ from the (nonlinear) identity $|\psi_\infty|^2 + \langle \delta\hat{b}^\dagger \delta\hat{b} \rangle = n$

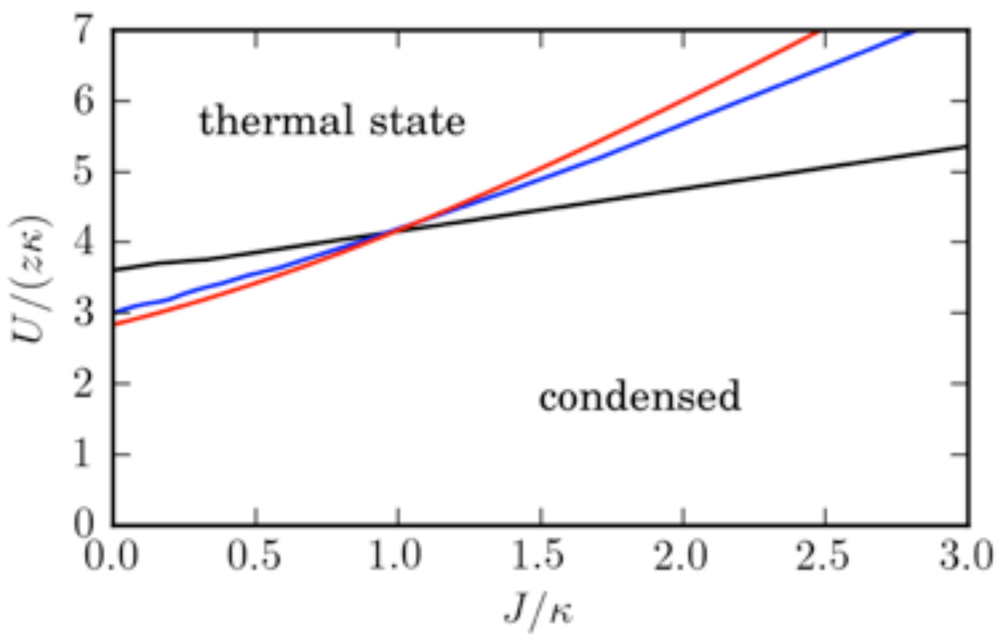
Analytical Results for the Steady State

- Explicit expression for the condensate fraction $\quad [j = J/(4\kappa), \quad u = U/(4\kappa z)]$

$$\frac{|\psi_\infty|^2}{n} = 1 - \frac{2u^2\left(1+(j+u)^2\right)}{1+u^2+j(8u+6j\left(1+2u^2\right)+24j^2u+8j^3)}$$

- Depletion for vanishing hopping $\quad \frac{|\psi_\infty|^2}{n} = 1 - \frac{U^2}{32\kappa^2}$

- Border $J(U)$ of the phase transition $\psi_\infty(J,U)=0$



$n = 0.1$ analytical

$n = 0.1$ numerical (linear instability)

$n = 1$

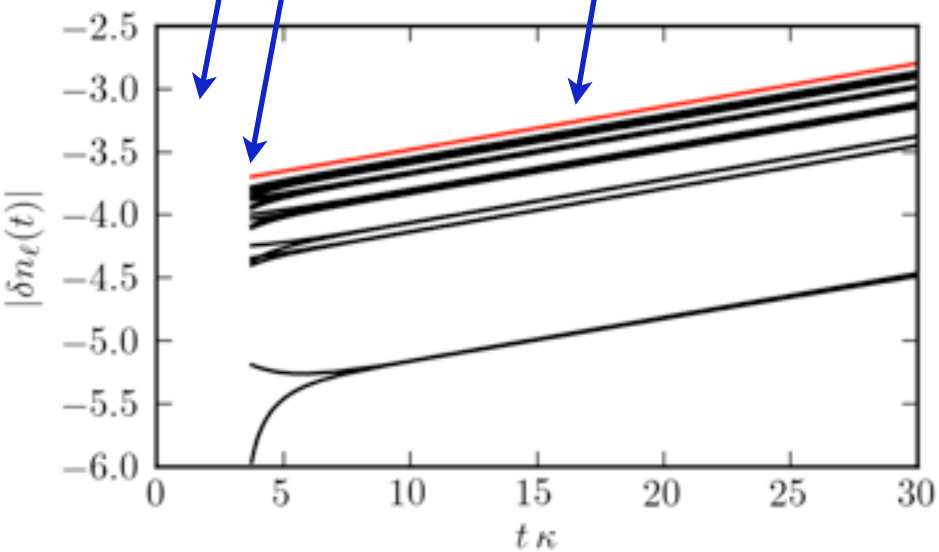
Dynamical Instability in the Condensate Phase

- Numerical experiment to probe the stability: subject the inhomogeneous system to a “kick” (instantaneous perturbation of the density matrix)

initial preparation in the homogeneous steady state

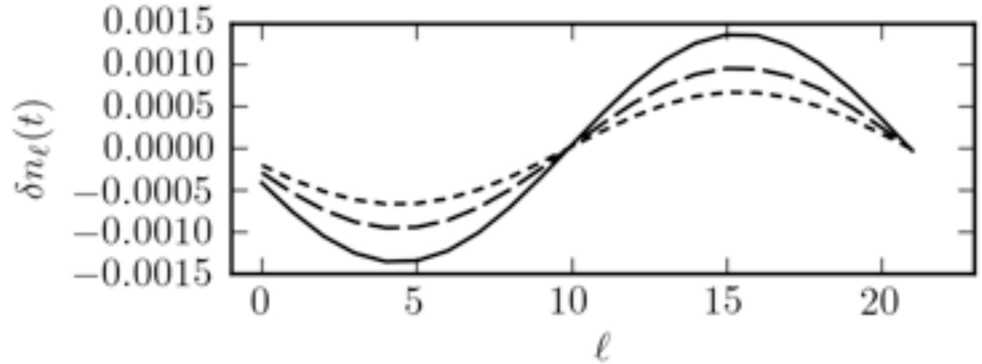
kick

exponential increase of the fluctuation on all sites with uniform rate



$$|n_\ell(t) - n(0)| \sim e^{\gamma t} \sin q\ell$$

long-wavelength density wave

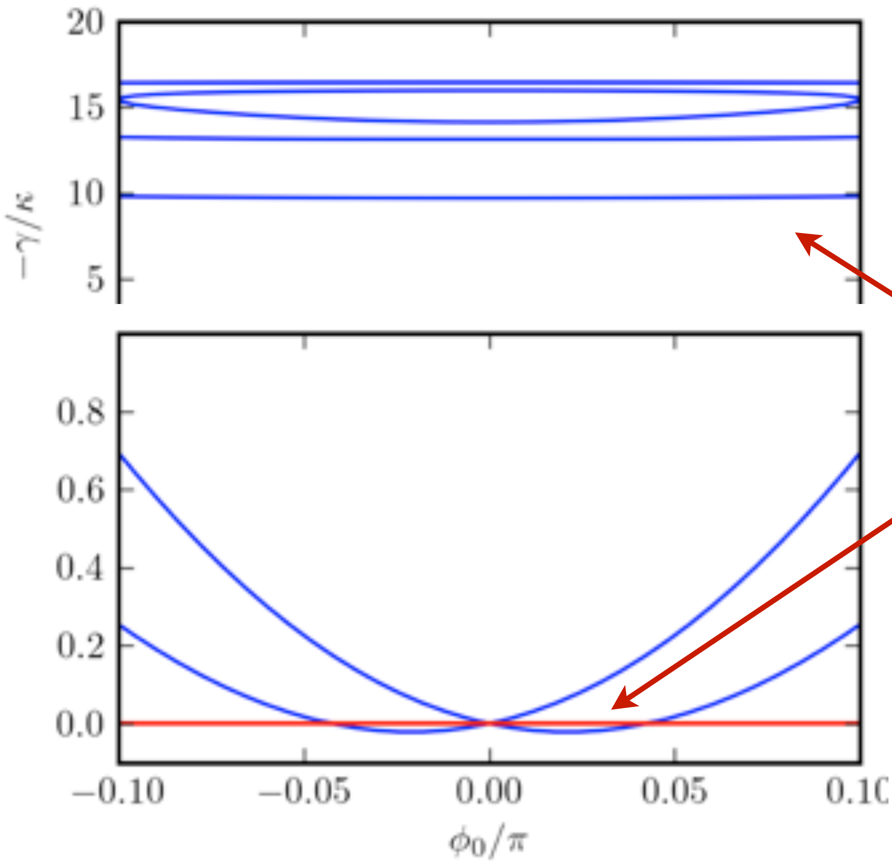


- Very slow effect: linearization of the master equation around the initial state, computation of the rate of the instability.
- This was a computation on 22 sites, linearization makes larger systems accessible

Dynamical Instability in the Condensate Phase

- Result of the linearized equations of motion $\partial_t \delta\rho_\ell(t) = L[\rho(0)]\delta\rho_\ell(t)$

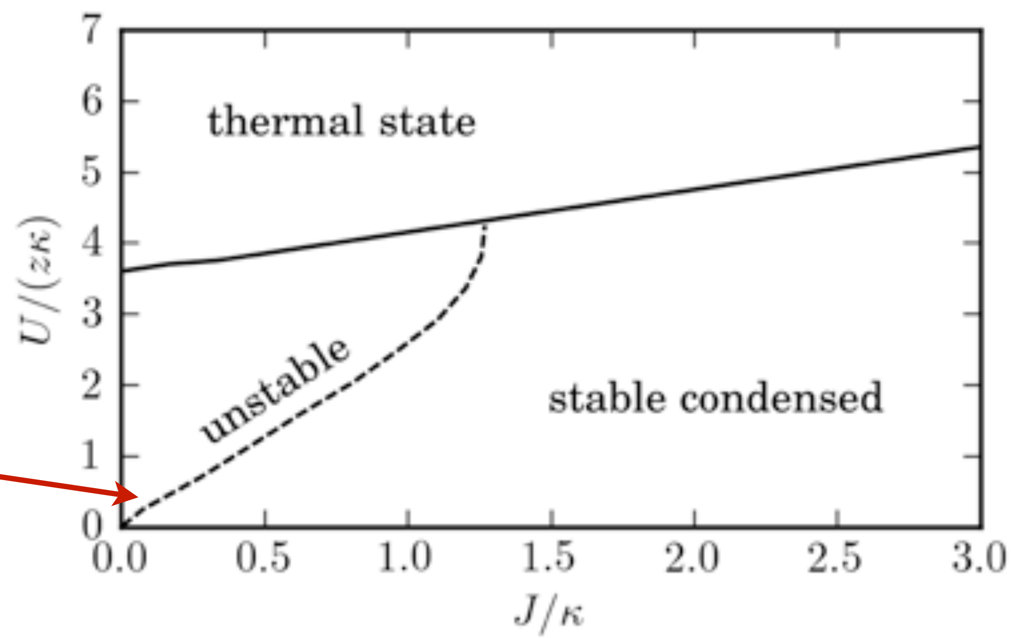
with the hypothesis on the spatial dependence of the perturbation $\delta\rho_\ell = e^{i\phi_0\ell}\delta\rho_0$



Imaginary part of the spectrum of the linearized equation

many stable branches, fluctuation decay

one branch with unstable low momentum modes



- The instability arises for any small interaction in the absence of hopping!

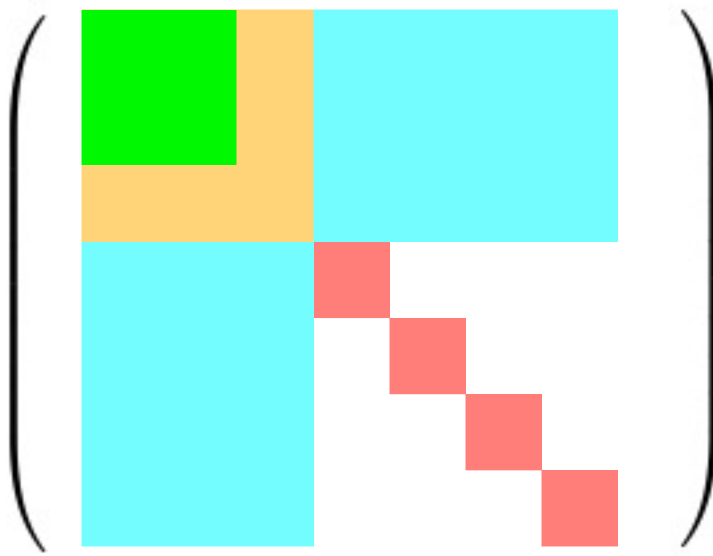
Analytical Treatment of the Condensate Instability

- Linearize the equation of motion for the "connected" correlation functions around the steady state of the system

$$\begin{aligned}\langle(\delta b_\ell^\dagger)^m \delta b_\ell^n\rangle &= \mathrm{Tr}[(\delta b_\ell^\dagger)^m \delta b_\ell^n (\rho_\infty + \Delta\rho_\ell(t))] \\ &\equiv \langle(\delta b_\ell^\dagger)^m \delta b_\ell^n\rangle_\infty + \Delta\langle(\delta b_\ell^\dagger)^m \delta b_\ell^n\rangle\end{aligned}$$

- The zero-order term vanishes because we perturb around the steady state.
- The time-fluctuation $\Delta\langle\delta b_\ell\rangle$ of the linear terms does not vanish!
- From the lattice to continuous Fourier variables $f_{\ell+1} - 2f_\ell + f_{\ell-1} \rightarrow -q^2 f_q$

$$\partial_t \Delta \begin{pmatrix} \langle\delta b\rangle_q \\ \langle\delta b^\dagger\rangle_q \\ \langle\delta n\rangle_q \\ \langle(\delta b^\dagger)^2\rangle_q \\ \langle\delta b^2\rangle_q \\ \langle\delta b^\dagger \delta b^2\rangle_q \\ \langle(\delta b^\dagger)^2 \delta b\rangle_q \end{pmatrix} =$$



$$\Delta \begin{pmatrix} \langle\delta b\rangle_q \\ \langle\delta b^\dagger\rangle_q \\ \langle\delta n\rangle_q \\ \langle(\delta b^\dagger)^2\rangle_q \\ \langle\delta b^2\rangle_q \\ \langle\delta b^\dagger \delta b^2\rangle_q \\ \langle(\delta b^\dagger)^2 \delta b\rangle_q \end{pmatrix}$$

Long-wavelength Dynamical Instability

- Diagonalization of the 7x7 matrix

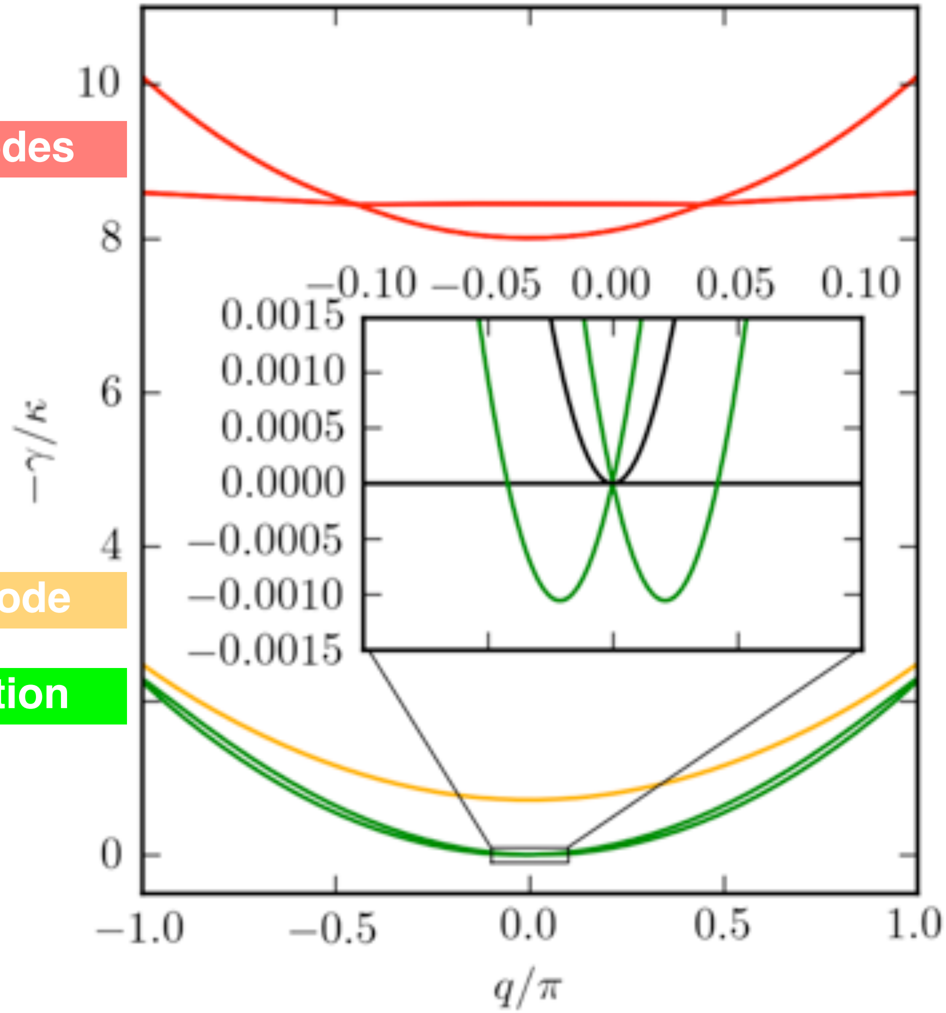
fast-decaying modes

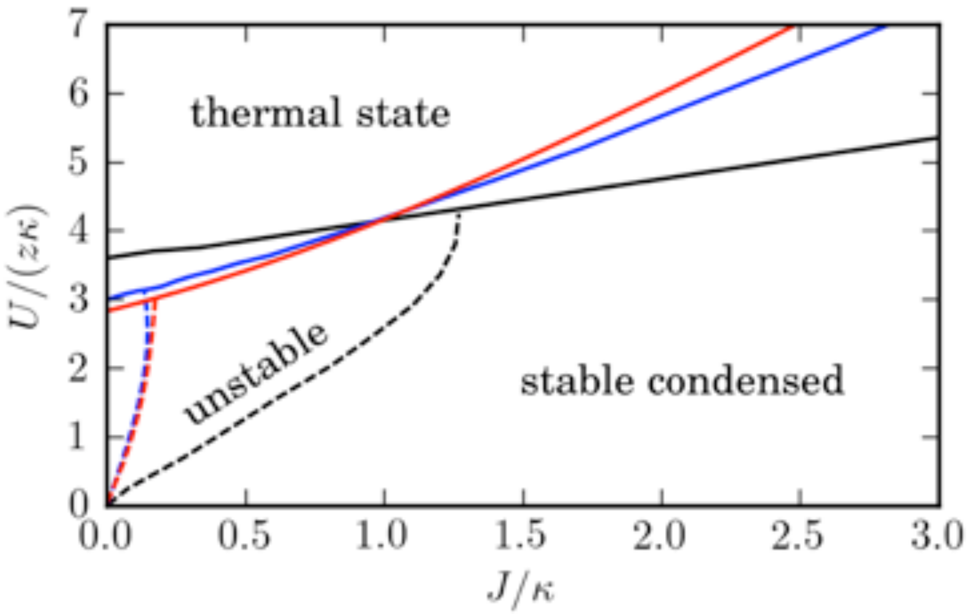
✱ effect at very low-momenta: large lattices

✱ accurate description of the low-lying modes

density mode

condensate fluctuation





$n = 0.1$ analytical

$n = 0.1$

$n = 1$ numerical (linear instability)

Reduction to the Low-Lying Modes

- Adiabatic elimination of the fast-decaying modes (two times)

$$\begin{pmatrix} \partial_t \Psi_1 \\ 0 \equiv \partial_t \Psi_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

1. solve for the fast mode Ψ_2

2. substitute and obtain an equation for the slow mode only

- Reduce to the modes of the condensate only [note that $\langle \delta b^\dagger \rangle_q = \langle \delta b^\dagger_{-q} \rangle$]

$$\partial_t \begin{pmatrix} \Delta\psi_q \\ \Delta\psi^*_{-q} \end{pmatrix} = \begin{pmatrix} Un + \epsilon_{\mathbf{q}} - i\kappa_{\mathbf{q}} & Un + 9un\kappa_{\mathbf{q}} \\ -Un - 9un\kappa_{\mathbf{q}} & -Un - \epsilon_{\mathbf{q}} - i\kappa_{\mathbf{q}} \end{pmatrix} \begin{pmatrix} \Delta\psi_q \\ \Delta\psi^*_{-q} \end{pmatrix}$$

$$\epsilon_q \equiv Jq^2, \quad \kappa_q = 2(2n+1)\kappa q^2$$

* contribution to the off-diagonal terms, that is absent in the dissipative GPE

* contribution due to the correlations that are high-order in the Fock space and fast-decaying

Origin of the Instability

- Diagonalization of the matrix of the low-lying modes: eigenvalues

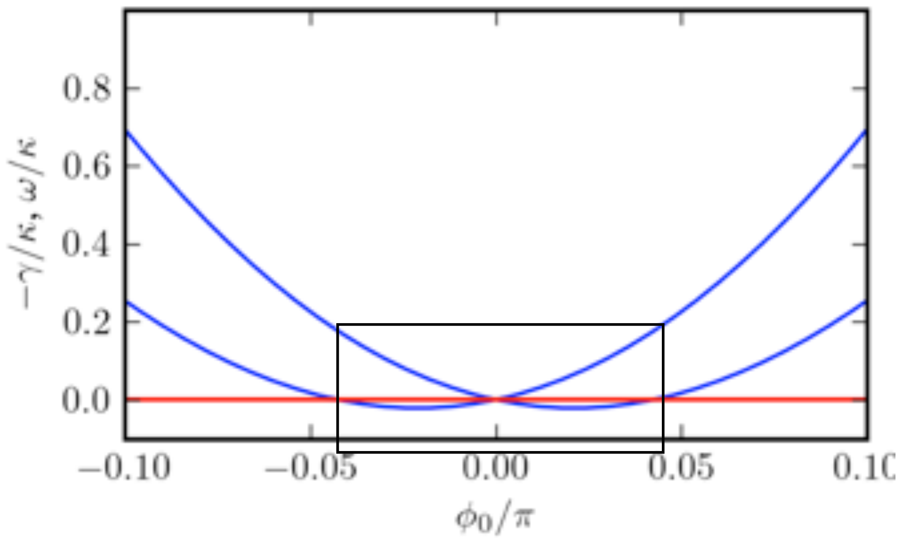
$$\gamma_{\mathbf{q}} = \kappa_{\mathbf{q}} + ic|\mathbf{q}|, \quad c = \sqrt{2Un(J - 9Un/(2z))}$$

- for $J > 9Un/(2z)$ the speed of sound is real and the dissipation rate $\mathrm{Re}\gamma_{\mathbf{q}} = \kappa_{\mathbf{q}} = \kappa \mathbf{q}^2$ quadratic

- Below a critical value

$$J = 9Un/(2z)$$

the speed of sound becomes imaginary. The nonanalytic linear momentum dependence always dominates the quadratic term at sufficiently small momenta, cf. picture



Validity of Inhomogeneous Gutzwiller Approximation

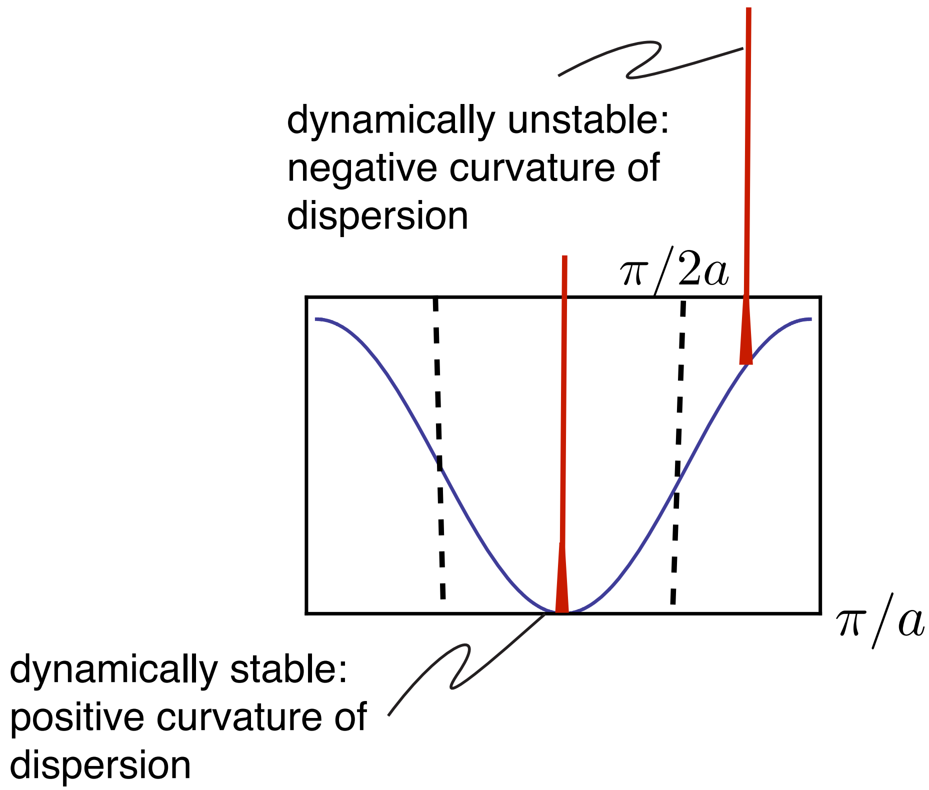
- The instability arises at weak coupling already, where the system is well described by the inhomogeneous Gutzwiller mean-field theory.

  - The instability is due to a renormalization of the single particle (complex) excitation spectrum, and thus encoded in the evolution of $(\Delta\psi_i(t), \Delta\psi_i^*(t))$
  - The exact equation of motion is a nonlinear equation, with nonlocal spatial correlations
  - The Gutzwiller approximation factorizes the correlations functions in real space, but treats onsite correlations exactly
  - The factorization is real space is justified at weak coupling (large condensate): The dominant scattering processes are those for (*-q, q)* off the macroscopically occupied condensate
  - In contrary, treating the onsite correlations properly is mandatory for the effect: Further (onsite) factorization of correlation functions (GP approximation) is insufficient

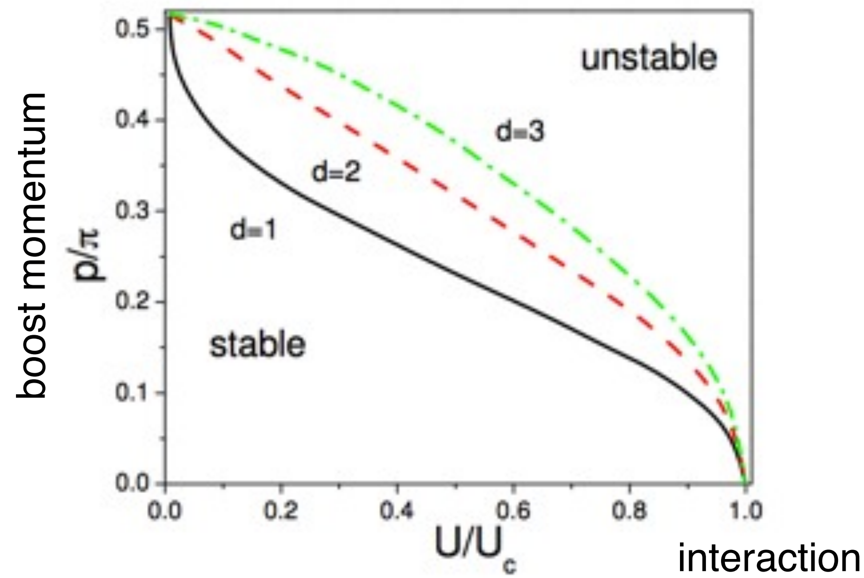
- Picture: Onsite (temporal, quantum) correlations prepare the ground for long wavelength spatial (classical) fluctuations becoming unstable

Comparison to Other Dynamical Instabilities

- Dynamical instabilities can arise out of equilibrium
- A prominent example: Boosting a zero temperature lattice condensate (Niu ’02 et al., Altman et al. ’04)



Stability phase diagram

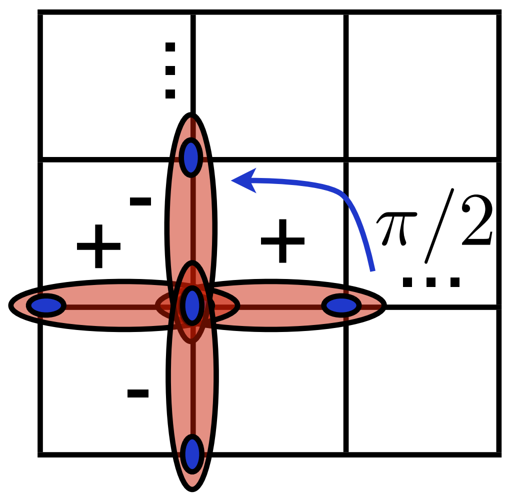


from Altman et al. ’04

- Differences to our scenario:
  - This is a classical effect, obtained by externally tuning system parameters
  - There is a finite “critical point” (but: similarities at the BH critical point)

Dissipative Driving of Fermions

- Excited states: η Condensate
- Cooling into Antiferromagnetic and d-Wave States



Cooling to Excited States: η-Condensate

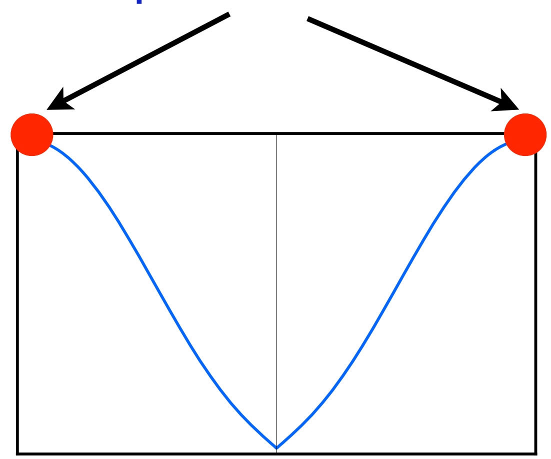
- η-state: exact excited (i.e. metastable) eigenstate of the two-species Fermi Hubbard Hamiltonian in d dimensions [Yang '89]

$$H = -J \sum_{\langle i,j\rangle,\sigma} f^\dagger_{i\sigma} f_{j\sigma} + U \sum_i f^\dagger_{i\uparrow} f^\dagger_{i\downarrow} f_{i\downarrow} f_{i\uparrow}$$

- local "doublon" $\eta^\dagger_i = f^\dagger_{i\uparrow} f^\dagger_{i\downarrow}$

- checkerboard superposition η-particle

$$\eta^\dagger = \frac{1}{M^{d/2}} \sum_i \phi_i \eta^\dagger_i \qquad \phi_i = \pm 1$$



- N-η-condensate:

$$H(\eta^\dagger)^N|0\rangle = NU(\eta^\dagger)^N|0\rangle$$

exact eigenstate,
off-diagonal long range order

Cooling to Excited States: η-Condensate

- Small scale simulations (open BC) demonstrate η condensation for jumps

$$c_{ij}^{(1)} = (\eta_i^\dagger - \eta_j^\dagger)(\eta_i + \eta_j)$$

$$c_{ij}^{(2)} = n_{i\uparrow} f_{i\downarrow}^\dagger f_{j\downarrow} + n_{j\uparrow} f_{j\downarrow}^\dagger f_{i\downarrow}$$

- Interpretation: Quantum Jump picture

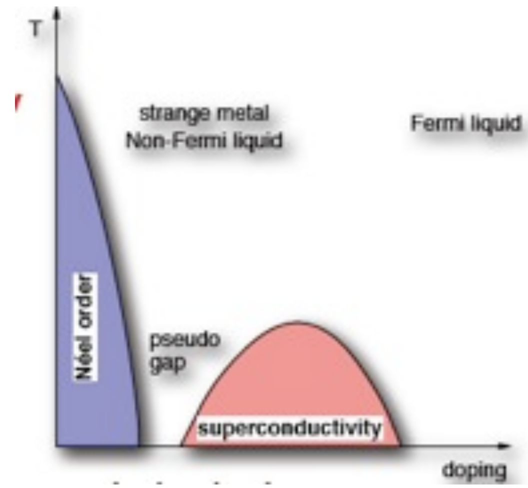
  - H generates spin-up and down configurations on each pair of sites (for any initial density matrix)
  - $c_{ij}^{(2)}$ associates into local doublons
  - $c_{ij}^{(1)}$ creates checkerboard superposition: η condensate

➡ May be conceptually interesting

➡ However, these jump operators are two-body: difficult to engineer

Motivation: Cooling Fermion Systems

Experimental phase diagram for cuprates



- High temperature superconductivity

- discovered in 1986 (Müller, Bednorz): cuprates show superconductivity at unconventionally high temperature
- riddle: attraction from repulsion
  - microscopically, strong Coulomb onsite repulsion
  - still, observe pairing of fermions with d-wave symmetry

- Minimal model: 2d Fermi-Hubbard model

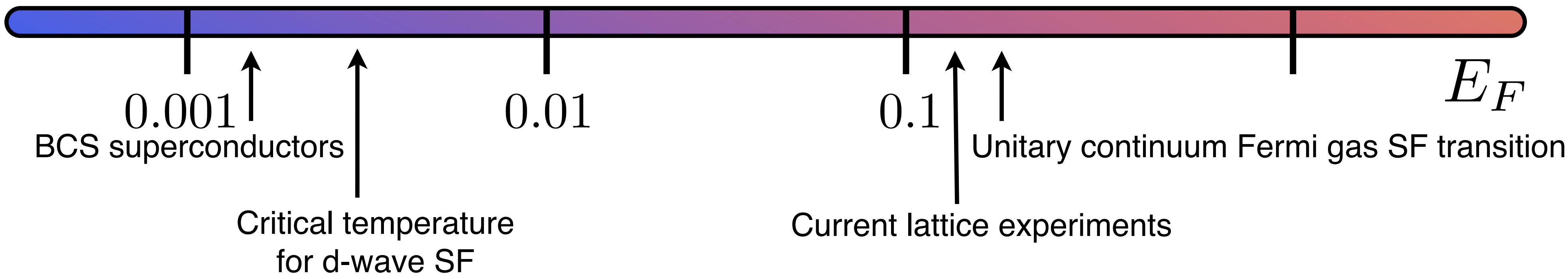
$$H_{\mathrm{FH}} = -J \sum_{\langle i,j \rangle, \sigma} c^{\dagger}_{i,\sigma} c_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} \qquad U \approx 10J$$

- realistic for cuprate high-temperature superconductors?
- hard to solve: strongly interacting fermion theory
  - no controlled analytical approach available
  - numerically (classical computer) intractable

➡ Quantum simulation of the Fermi-Hubbard model in optical lattices?

Quantum Simulation of Fermion Hubbard model

- Clean realization of fermion Hubbard model possible
  - Detection of Fermi surface in 40K (M. Köhl et al. PRL 94, 080403 (2005))
  - Fermionic Mott Insulators (R. Jördens et al. Nature 455, 204 (2008); U. Schneider et al., Science 322, 1520 (2008))
- Cooling problematic: small d-wave gap sets tough requirements



➡ Still need to be 10-100x cooler

- Existing proposal: Adiabatic quantum simulation (S. Trebst et al. PRL 96, 250402 (2006))
  - Start from a pure initial state of noninteracting model
  - Adiabatically transform to unknown ground state of interacting model
  - Concrete scheme: find path protected by large gaps:
    - prepare RVB ground state on isolated 2x2 plaquettes
    - couple these plaquettes to arrive at many-body ground state

Dissipative Quantum State Engineering Approach

- Roadmap:

(1) Precool the system (lowest Bloch band)

(2) Dissipatively prepare pure (zero entropy) state close to the expected ground state:
- energetically close
- symmetry-wise close
- spin-wise close

(3) Adapted adiabatic passage to the Hubbard ground state
- switch dissipation off
- switch Hamiltonian on

Precooling

Dissipative Cooling $H = 0, \quad \mathcal{L} \neq 0$

Adapted adiabatic passage

$H = H_{FH}, \quad \mathcal{L} = 0$

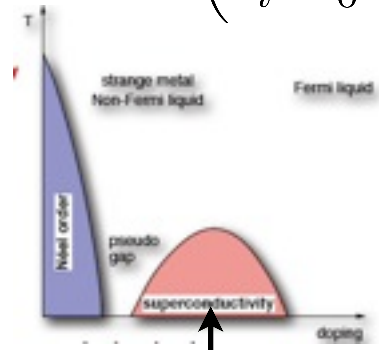
The State to Be Prepared

Pauli matrix $\sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$|\mathrm{D}\rangle = (D^\dagger)^N |\mathrm{vac}\rangle, \;\; D^\dagger = \sum_i (c^\dagger_{i+\mathbf{e}_x} - c^\dagger_{i+\mathbf{e}_y})\sigma^{(y)} c^\dagger_i$$

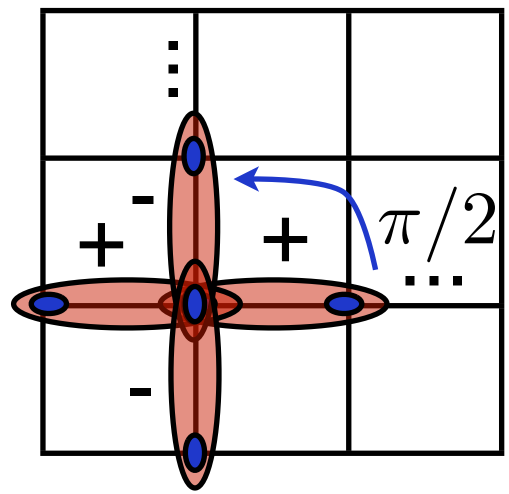
mean field (product) state

$c_i = \begin{pmatrix} c_{i,\uparrow} \\ c_{i,\downarrow} \end{pmatrix}$ two-component spinor



d-wave SC

- What does the state have in common with the expected Hubbard ground state



(1) Quantum numbers

- pairing in the singlet channel
- phase coherence: delocalization of singlet pairs
- transformation under spatial rotations: “d-wave”

➡ The state shares the symmetries of Hubbard GS

➡ No phase transition will be crossed in preparation process

- in the talk, we mainly consider 1-dimensional analog for simplicity:

$$|\mathrm{D}_1\rangle = (D^\dagger)^N |\mathrm{vac}\rangle, \;\; D^\dagger = \sum_i c^\dagger_{i+1}\sigma^{(y)} c^\dagger_i$$

The State to Be Prepared

Pauli matrix $\sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$|\mathrm{D}\rangle = (D^\dagger)^N |\mathrm{vac}\rangle, \;\; D^\dagger = \sum_i (c^\dagger_{i+\mathbf{e}_x} - c^\dagger_{i+\mathbf{e}_y}) \sigma^{(y)} c^\dagger_i$$

mean field (product) state

$c_i = \begin{pmatrix} c_{i,\uparrow} \\ c_{i,\downarrow} \end{pmatrix}$ two-component spinor

- What does the state have in common with the expected Hubbard ground state

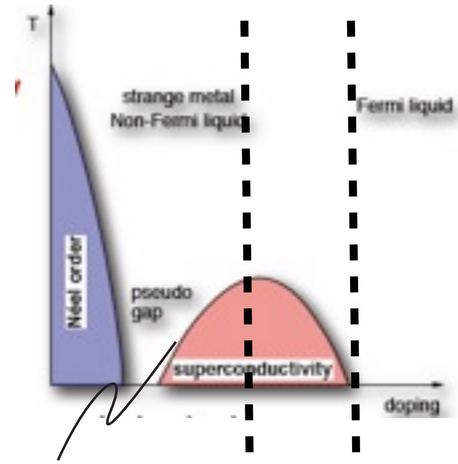
  (2) Energetically close? Not known, but:

  - off-site pairing $c^\dagger_{i+1}\sigma^{(y)}c^\dagger_i$ avoids excessive double occupancy

    cf onsite pairing: $c^\dagger_i \sigma^{(y)} c^\dagger_i$

  - the pairs are quasi-local, i.e. have a short coherence length in accord with observation in cuprates

doping not too close to AF



superfluidity decreases due to strong correlations

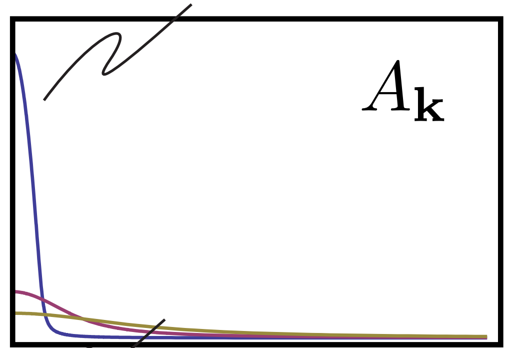
(A. Paramekanti, N. Trivedi, M. Randeria, PRB 70, 054504 (2004))

➡ State can be expected to be convenient starting point not too close to half filling

Relation to the BCS Wavefunction

- usually, fixed phase (coherent state) wave function

“BCS limit”



“BEC/molecular limit”

Fixed particle number wavefunction

$$|\psi\rangle \propto \prod_{\mathbf{k}} (1 + A_{\mathbf{k}} c^\dagger_{-\mathbf{k},\uparrow} c^\dagger_{\mathbf{k},\downarrow})|\text{vac}\rangle$$

$$= \exp\Big(\sum_{\mathbf{k}} A_{\mathbf{k}} c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow}\Big)|\text{vac}\rangle = \sum_N \frac{1}{N!}\Big(\overbrace{\sum_{\mathbf{k}} A_{\mathbf{k}} c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow}}\Big)^N |\text{vac}\rangle$$

BCS gap

BCS amplitude

$$A_{\mathbf{k}} = \frac{v_{\mathbf{k}}}{u_{\mathbf{k}}} = \frac{\Delta_{\mathbf{k}}}{E_{\mathbf{k}} + \xi_{\mathbf{k}}}$$

chemical potential

$$E_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + \Delta^2_{\mathbf{k}}}$$

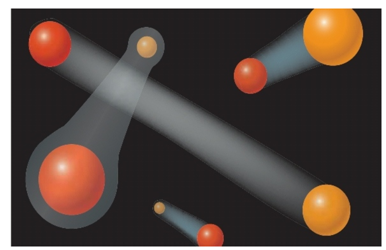
$$\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$$

dispersion

- distinct limits:

$\mu/\epsilon_F \to 1$
- localized in momentum space
- delocalized in position space

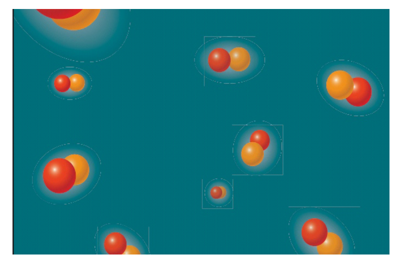
“BCS limit”



$\mu/\epsilon_F \to -\infty$
- delocalized in momentum space
- localized in position space

$A_{\mathbf{k}} \to \dfrac{\Delta_{\mathbf{k}}}{-\mu}$

“BEC / molecular limit”



- Relation to our state: $\sum_i c^\dagger_{i+1}\sigma^{(y)}c^\dagger_i = 2\sum_{\mathbf{k}} \cos \mathbf{k}\, c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{\mathbf{k},\downarrow}$ $\quad A_{\mathbf{k}} = 2\cos\mathbf{k}$

➡ State shares the symmetries, but can be energetically very different

Setting

- Goal: Construct jump operators with unique mean field dark states:

$$\mathcal{L}[\rho] = \sum_{\ell} j_\ell \rho j_\ell^\dagger - \tfrac{1}{2}\{j_\ell^\dagger j_\ell, \rho\}$$

$$j_\ell |\eta\rangle = 0 \ \forall \ \ell \qquad\qquad |\eta\rangle = \prod_a C_a^\dagger |\mathrm{vac}\rangle$$

dark state — mean field (product) state

solve: $\Rightarrow \boxed{[j_\ell, C_a^\dagger] = 0 \quad \forall \ell, a.}$ (sufficient for normal ordered jump operators)

- Requirements for implementation:

$$j_\ell = \sum_{\langle j|i\rangle \sigma,\sigma'} c^\dagger_{j,\sigma'} H_{\sigma,\sigma'} c_{i,\sigma}$$

- non-hermitian
- particle number conserving $[j_\ell, \sum_{i,\sigma} \hat{n}_{i,\sigma}] = 0 \, \forall \, \ell$
- quasi-local: j close central site i
- single-particle operation

this is what the eta operators suffered from!

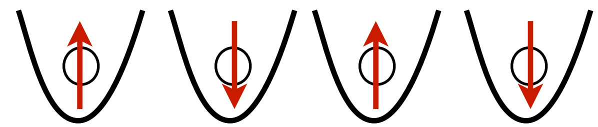
Antiferromagnetic Jump Operators

- Construct jump operators for antiferromagnetism as a preparation

- Antiferromagnetic "Neel state" is a product of AF "unit cell" operators

$$|\mathrm{AF}\pm\rangle = \prod_{i\in A} \hat{S}^{\pm}_{i-}|\mathrm{vac}\rangle = (-)^{M/2}\prod_{i\in B} \hat{S}^{\mp}_{i+}|\mathrm{vac}\rangle, \qquad C_a^\dagger = \hat{S}^{\pm}_{i\pm} = c^\dagger_{i\pm 1}\sigma^{\pm} c_i^\dagger$$

doubly degenerate   bipartite lattice with sublattices A,B



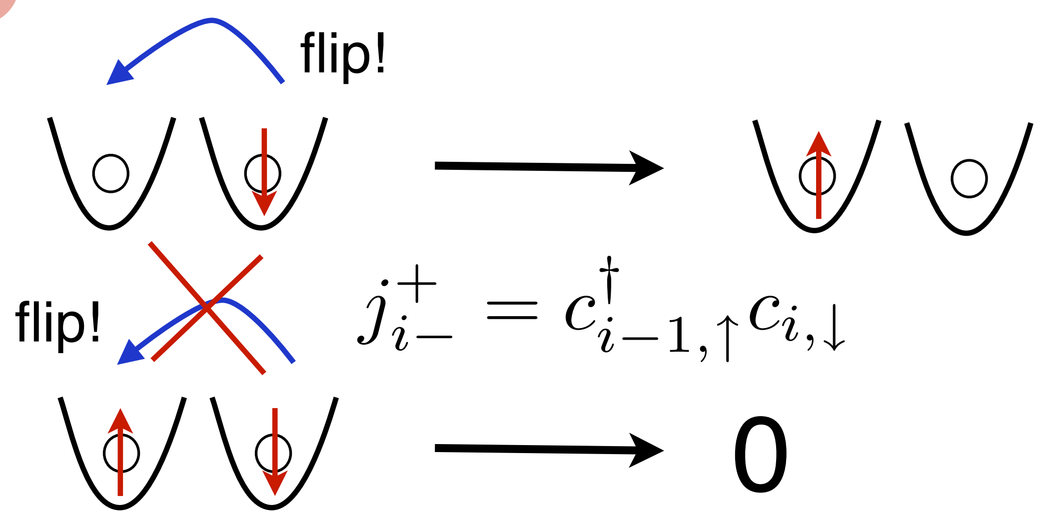
- Set of jump operators (one dimension):

$$\boxed{j_\ell = \{j^{\pm}_{i\pm},\ j^{z}_{i\pm}\}}$$

Pauli matrices

$$j^{\pm}_{i\pm} = c^\dagger_{i\pm 1}\sigma^{\pm}c_i, \quad j^{z}_{i\pm} = c^\dagger_{i\pm 1}(1\pm\sigma^z)c_i$$

- Action of jump operators
  - $j^{\pm}_{i\pm}$: Pauli blocking
  - j^{z}_{i}: spin transport



$$j^{+}_{i-} = c^\dagger_{i-1,\uparrow}c_{i,\downarrow}$$

$$S^{+}_{i-} = c^\dagger_{i-1,\uparrow}c^\dagger_{i,\downarrow}$$

d-Wave Jump Operators

- Rewrite the d-wave state in terms of AF unit cell operators:

shift invariance

$$|\mathrm{D}\rangle = (D^\dagger)^N|\mathrm{vac}\rangle, \;\; D^\dagger = \sum_i c^\dagger_{i+1}\sigma^{(y)}c^\dagger_i = \sum_i \hat{J}^\pm_i \qquad \hat{J}^\pm_i = \hat{S}^\pm_{i+} + \hat{S}^\pm_{i-}$$

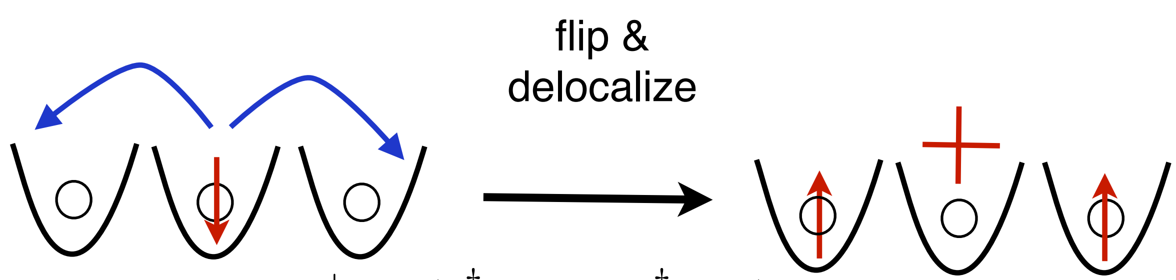
homogeneous product

but delocalized pairs

- Second equality: interpret the state as a symmetrically delocalized AF
- Set of jump operators:

$$\boxed{j_\ell = \{J^\pm_i, \;\; J^z_i\}} \qquad J^\pm_i \;\; = \;\; j^\pm_{i+} + j^\pm_{i-}, \qquad J^z_i = j^z_{i+} + j^z_{i-}$$

- Action of jump operators
  - $J^\pm_i$: Pauli blocking
  - J^z_i : spin transport
  - both: phase coherence via delocalization



$$J^+_i = (c^\dagger_{i-1,\uparrow} + c^\dagger_{i+1,\uparrow})c_{i,\downarrow}$$

➡ Combine fermionic Pauli blocking with delocalization as for bosons

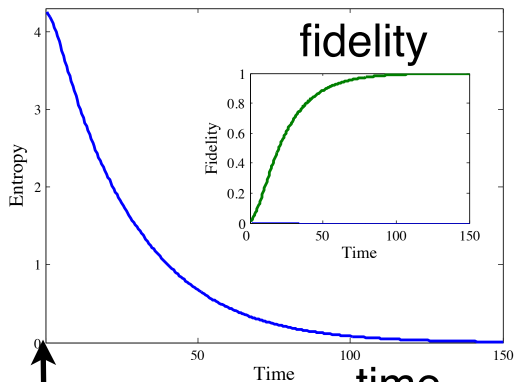
➡ Pauli blocking is the reason for single particle nature of operators

Uniqueness

- Recall: Unique dark state <-> state reached independent of initial condition
- Evidence for uniqueness from small scale numerical simulations

Antiferromagnetism

Entropy evolution

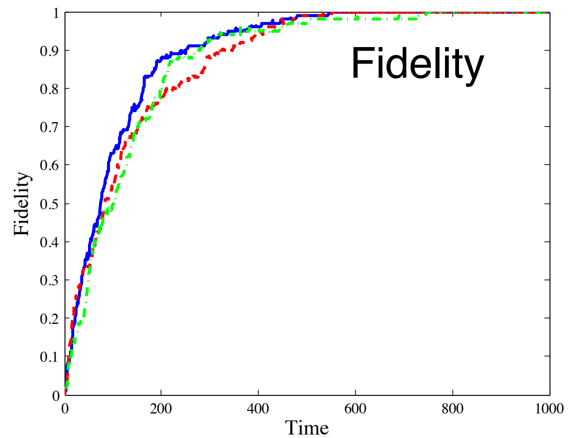


time

perfect mixture

6 sites,
6 particles

d-wave



time

3x3 sites, 4 particles, different random initial states

Entropy

$$S = \mathrm{tr}\rho(t)\log\rho(t)$$

Fidelity

$$\mathrm{tr}[\rho(t)|AF\pm\rangle\langle AF\pm|]$$

Uniqueness

- Understanding can be gained from symmetry considerations
  - Uniqueness of dark state equivalent to uniqueness of ground state (GS) of

$$H_\Delta = \sum_{i,\alpha=\pm,z} \Delta_\alpha J_i^{\alpha\,\dagger} J_i^\alpha \qquad \left[\mathcal{L}[\rho] = \sum_{\alpha,i} \kappa_\alpha J_i^\alpha \rho J_i^{\alpha\dagger} - \tfrac{1}{2}\{\underbrace{\kappa_\alpha J_i^{\alpha\dagger} J_i^\alpha}_{\text{-> effective Hamiltonian}}, \rho\}\right]$$

- H is semi-positive
- an exact GS is the above d-wave (E=0)
- unique iff no symmetry T such that

$$THT^{-1} = H, \quad T|\mathrm{D}\rangle \neq E|\mathrm{D}\rangle$$

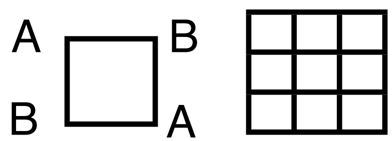
- Symmetries:
  - Translations
  - global phase rotations U(1)
  - global spin rotations SU(2) for $\Delta_z = \Delta_\pm/2$,

  } d-wave is an eigenstate to these

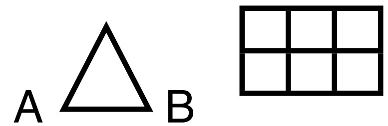
  - additional discrete symmetry on bipartite lattice for $\Delta_z = 0$ spoils uniqueness

$$T_d: \quad c_{i,\uparrow} \to -c_{i,\uparrow}; \quad c_{i,\downarrow} \to c_{i,\downarrow} \text{ for } i \in A,$$
$$c_{i,\uparrow} \to c_{i,\uparrow}; \quad c_{i,\downarrow} \to c_{i,\downarrow} \text{ for } i \in B$$

bipartite (periodic BC)



not bipartite (PBC)



SU(2) symmetry;
the jump operators
are SU(2) vectors

$$[S^\alpha, J_i^\beta] = \mathrm{i}\epsilon_{\alpha\beta\gamma} J_i^\gamma \quad \forall i$$

➡ Avoid symmetries

➡ All three operators needed for uniqueness

Comments on the effective Hamiltonian

- Amusing parallel: Above Hamiltonian is a parent Hamiltonian for the d-wave state

$$H_\Delta = \sum_{i,\alpha} \Delta_\alpha J_i^{\alpha\,\dagger} J_i^\alpha = \sum_i h_i$$

  - H is semi-positive
  - an exact unique GS is the above d-wave state(E=0)
  - GS is GS for each h_i separately: projectors on GS

➡ completely analogous to e.g. AKLT model

➡ there, ground state is valence bond solid with exponentially decaying correlations

➡ different: state has long range order due to strong delocalization

➡ study excitations

  - mean field decoupling

$$\Delta_+ \sum_i J_i^{+\,\dagger} J_i^+ = \Delta_+ \sum_i c^\dagger_{i,\downarrow}(c_{i+1,\uparrow} + c_{i-1,\uparrow})(c^\dagger_{i+1,\uparrow} + c^\dagger_{i-1,\uparrow})c_{i,\downarrow} = \Delta_+ \sum_i c^\dagger_{i,\downarrow}(c^\dagger_{i+1,\uparrow} + c^\dagger_{i-1,\uparrow})c_{i,\downarrow}(c_{i+1,\uparrow} + c_{i-1,$$

$$\approx \sum_{\mathbf{q}} \underbrace{\Delta^+ \cos \mathbf{q}}_{\text{single fermion gap}}\; c_{\mathbf{q},\downarrow} c_{-\mathbf{q},\uparrow} + h.c.$$

order parameter-like structure:
macroscopically populated
-> replace by c-number mean field
(-> loose particle number cons.)

  - "diagonal" contributions $\sim c^\dagger_{\mathbf{q}} c_{\mathbf{q}}$ from normal ordering J_i^z

➡ single fermion excitations are gapped: important for adabatic passage

Arbitrary phase coherent pairing states

- Any pairing product state can be characterized by 3 quantum numbers

$$O^{\dagger\,N}_{k,n,\mu}|\mathrm{vac}\rangle, \quad O^{\dagger}_{k,n,\mu} = \sum_i \exp ikx_i \; c^{\dagger}_{i+n}\sigma^{\mu}c^{\dagger}_i \qquad \sigma^{\mu} = (\mathbf{1}, \sigma^{\alpha})$$

pairing momentum

pairing distance

- Examples:

$k = 0,\; n = 0,\; \mu = 2$ s-wave BCS

$k = \pi,\; n = 0,\; \mu = 2$ eta-state

$k = 0,\; n = 1,\; \mu = 2$ d-wave like state

- Jump operators constructed for all k, mu, and n >0 (displayed just for completeness...)

$$\begin{aligned}
\mu &= 0 : (c^{\dagger}_{i+n} - e^{ik}c^{\dagger}_{i-n})(\mathbf{1} \pm \sigma^z)c^{\dagger}_i,\; (c^{\dagger}_{i+n} - e^{ik}c^{\dagger}_{i-n})\sigma^y c^{\dagger}_i \\
\mu &= 1 : (c^{\dagger}_{i+n} - e^{ik}c^{\dagger}_{i-n})\sigma^{\pm}c^{\dagger}_i,\; (c^{\dagger}_{i+n} + e^{ik}c^{\dagger}_{i-n})\mathbf{1}c^{\dagger}_i \\
\mu &= 2 : (c^{\dagger}_{i+n} + e^{ik}c^{\dagger}_{i-n})\sigma^{\pm}c^{\dagger}_i,\; (c^{\dagger}_{i+n} + e^{ik}c^{\dagger}_{i-n})\sigma^z c^{\dagger}_i \\
\mu &= 3 : (c^{\dagger}_{i+n} - e^{ik}c^{\dagger}_{i-n})(\mathbf{1} \pm \sigma^z)c^{\dagger}_i,\; (c^{\dagger}_{i+n} - e^{ik}c^{\dagger}_{i-n})\sigma^x c^{\dagger}_i
\end{aligned}$$

➡ arbitrary n > 0 pairing states can be targeted

➡ d-wave not distinguished, but off-site pairing special

➡ symmetries of the state inherited by the parent Hamiltonian

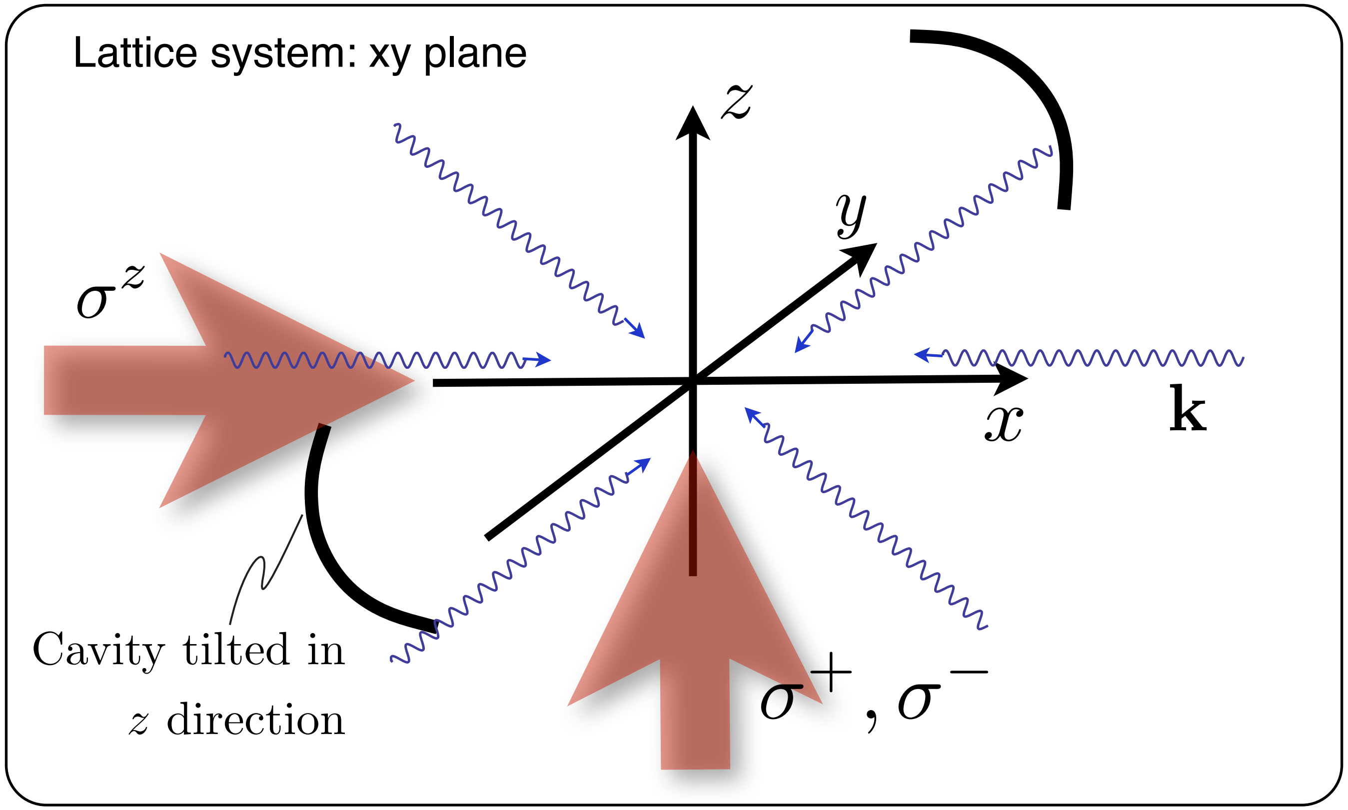
Implementation of d-wave jump operators

- Decisive property: single-particle nature of the jump operators
- Implement Fourier transformed operators:

$$\mathcal{L}[\rho] = \sum_{\alpha,i} J_i^\alpha \rho J_i^{\alpha\dagger} - \tfrac{1}{2}\{J_i^{\alpha\dagger} J_i^\alpha, \rho\} = \sum_{\alpha,\mathbf{k}} J_\mathbf{k}^\alpha \rho J_\mathbf{k}^{\alpha\dagger} - \tfrac{1}{2}\{J_\mathbf{k}^{\alpha\dagger} J_\mathbf{k}^\alpha, \rho\}$$

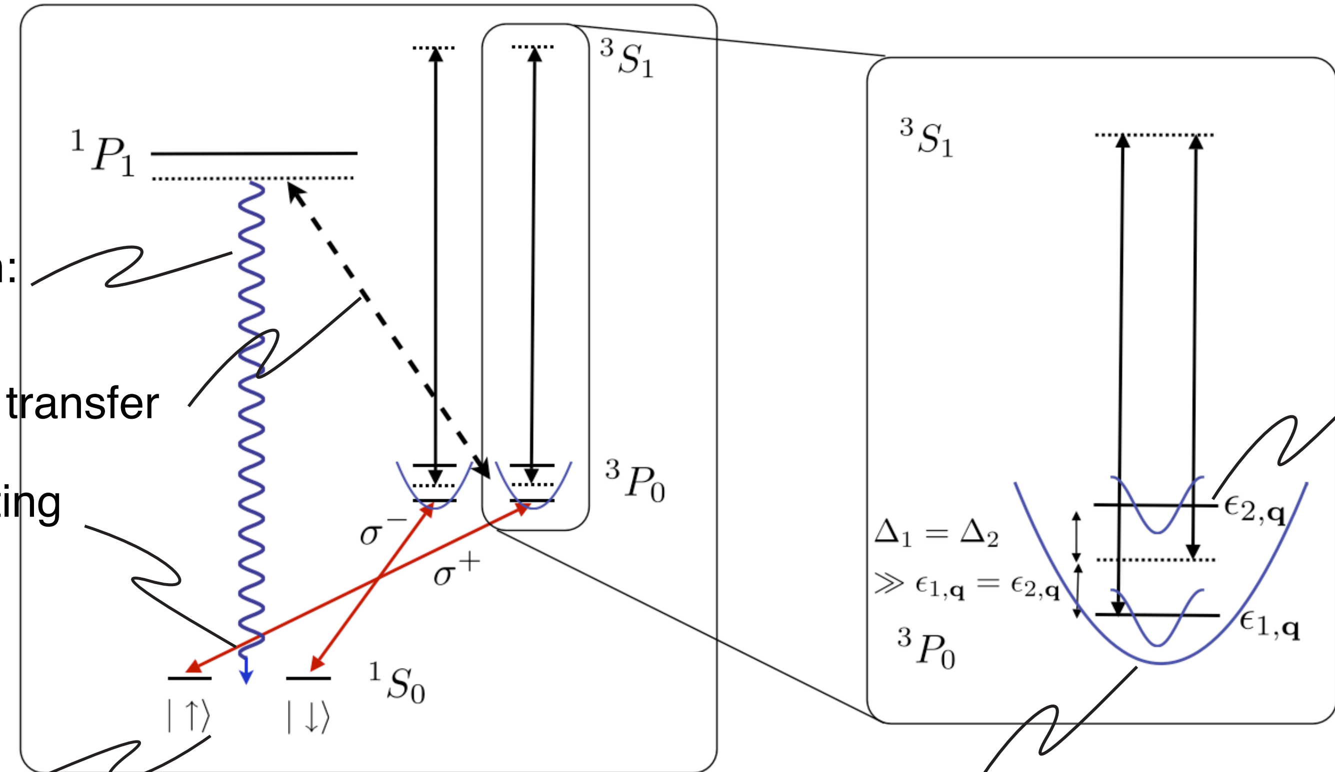
$$J_\mathbf{k}^\pm = \sum_\mathbf{q} \cos \mathbf{q}\, a_\mathbf{q}^\dagger \sigma^\pm a_{\mathbf{q}-\mathbf{k}}, \qquad J_\mathbf{k}^z = \sum_\mathbf{q} \cos \mathbf{q}\, a_\mathbf{q}^\dagger \sigma^z a_{\mathbf{q}-\mathbf{k}}$$

- Basic physical ingredients:
  - Dissipation: Emission in cavity
  - Use Earth Alkaline atoms in state dependent superlattice
- Engineering requirements:
  - Spin imprinting: Light Polarization
  - Momentum transfer: Laser angle (incoherent beams)
  - cos q dependence: Quantum Interference



Implementation of d-wave jump operators

- Level scheme: Earth Alkaline atoms



spont. emission:
cavity mode

momentum transfer

spin imprinting

physical spin

atom confinement
via optical lattice

Bloch bands

Quantum Interference:

$$\frac{1}{\Delta + \epsilon_{\mathbf{q}}} + \frac{1}{-\Delta + \epsilon_{\mathbf{q}}} = \frac{2\epsilon_{\mathbf{q}}}{-\Delta^2 + \epsilon_{\mathbf{q}}^2} \approx -2\frac{\epsilon_{\mathbf{q}}}{\Delta^2}$$

cos q: onsite
processes
interfere
destructively

$$J^{\pm}_{\mathbf{k}} = \sum_{\mathbf{q}} \cos \mathbf{q}\, a^\dagger_{\mathbf{q}} \sigma^{\pm} a_{\mathbf{q}-\mathbf{k}}$$

Adapted Adiabatic Passage

$$H \overset{?}{=} 0, \quad \mathcal{L} \neq 0$$

Adapted adiabatic passage $\downarrow$

$$H = H_{FH}, \quad \mathcal{L} = 0$$

- Assume we have prepared zero entropy d-wave
- Want to connect to Hubbard ground state
- Adiabatic passage (purely Hamiltonian dynamics):

$$H = \lambda(t) H_\Delta + (1 - \lambda(t)) H_{\mathrm{FH}},$$
$$\lambda(t_{\mathrm{in}}) = 1, \ \lambda(t_{\mathrm{fin}}) = 0$$

ramping slowly: remain in ground state

➡ Adapted adiabatic passage: Two ingredients

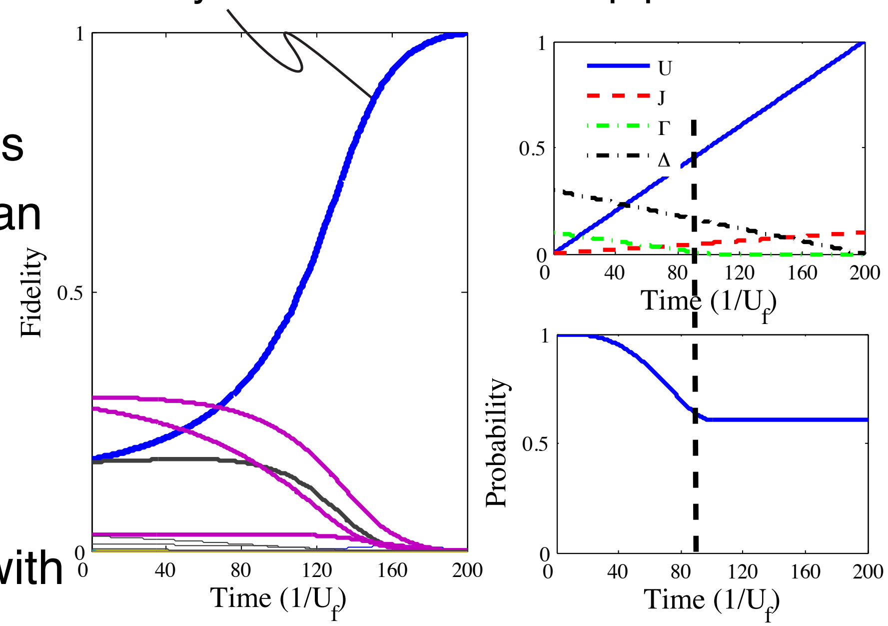
- gap protection from auxiliary Hamiltonian
  - parent Hamiltonian has d-wave eigenstate and is gapped: add detuning to the effective Hamiltonian

$$\gamma \to \gamma + \mathrm{i}\Delta$$

- probabilistic ground state preparation
  - dissipative and Hubbard dynamics compete
  - focus on time before first jump: state prepared with probability

fidelity to Hubbard GS

ramp parameters



preparation probability

Summary Driven Dissipation

By merging techniques from quantum optics and many-body systems:

Driven dissipation can be used as controllable tool in cold atom systems.

- Pure states with long range correlations from quasilocal dissipation
  - Many-body dark state, independent of initial density matrix
  - Laser coherence mapped on matter system
  - System steady state has zero entropy
- Nonequilibrium phase transition driven via competition of unitary and dissipative dynamics
  - driven by interactions (like quantum phase transition)
  - terminates into thermal state (like classical phase transition)
  - novel dynamical instability
- Strong potential applications for fermionic quantum simulation
  - cool into zero entropy d-wave state as intial state for Fermi-Hubbard model
  - single particle operations due to Pauli blocking
  - realistic setting using earth alkaline atoms in a cavity