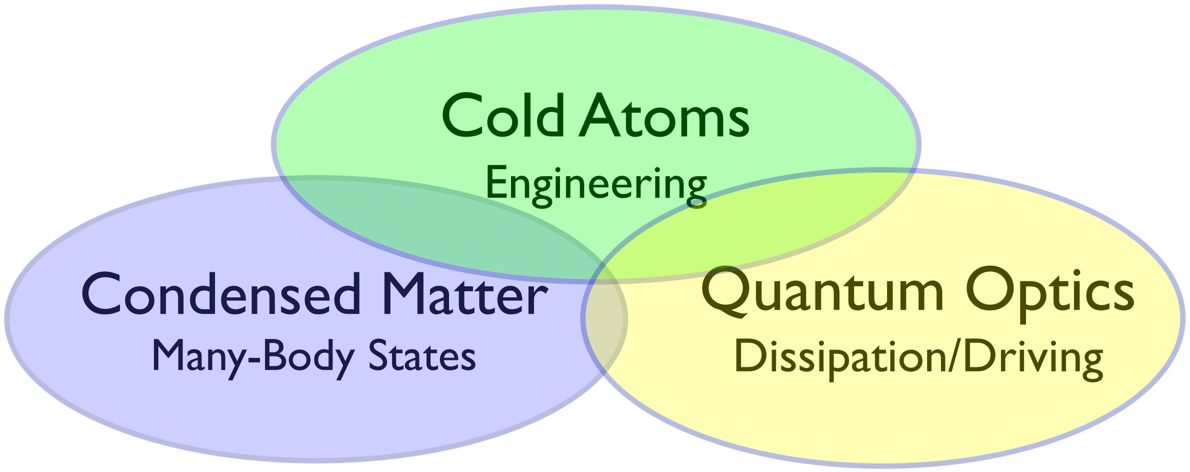
ASC Lectures, October 14/15 2009,
Arnold Sommerfeld Center, Munich

Quantum States and Phases in Dissipative Many-Body Systems with Cold Atoms

UNIVERSITY OF INNSBRUCK



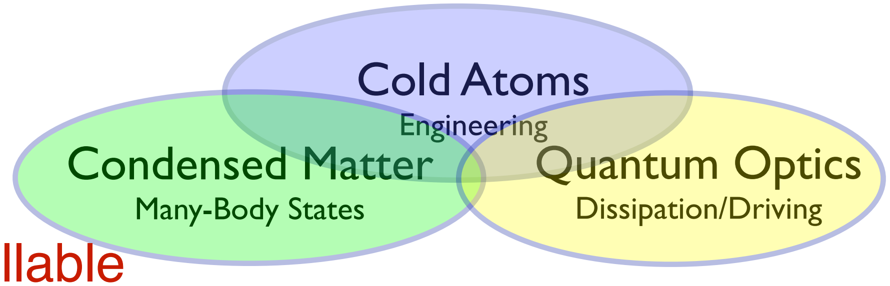
IQOQI
AUSTRIAN ACADEMY OF SCIENCES

SFB
Coherent Control of Quantum Systems

Sebastian Diehl

IQOQI Innsbruck

Lecture Overview



Main theme:
Dissipation can be turned into a favorable, controllable tool in cold atom many-body systems.

Part I: Quantum State Engineering in Driven Dissipative Many-Body Systems

- Proof of principle: Driven Dissipative BEC
- Application I: Nonequilibrium phase transition from competing unitary and dissipative dynamics
- Application II: Cooling into antiferromagnetic and d-wave states of fermions

- Collaboration: H. P. Büchler, A. Daley, A. Kantian, B. Kraus, A. Micheli, A. Tomadin, W. Yi, P. Zoller

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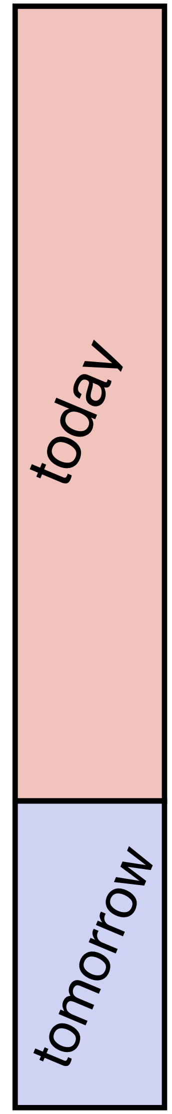
Part II: Dissipative Generation and Analysis of 3-Body Hardcore Models

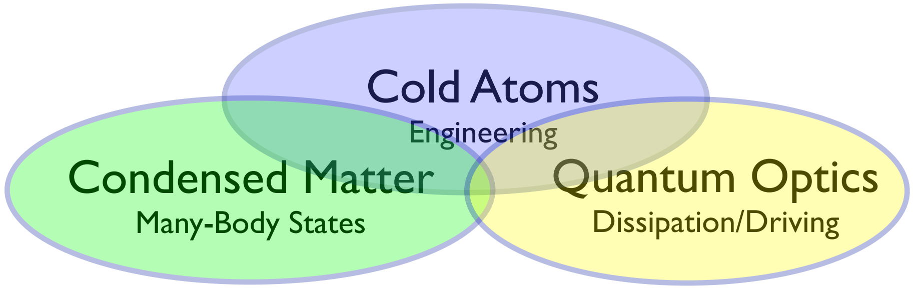
- Mechanism
- Experimental prospects, ground state preparation
- Application I: phase diagram for attractive 3-hardcore bosons
- Application II: atomic color superfluid for 3-component fermions

- Collaboration: M. Baranov, A. J. Daley, M. Dalmonte, A. Kantian, J. Taylor, P. Zoller

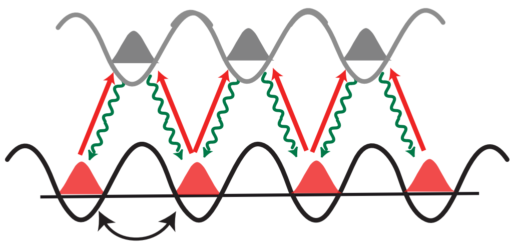


Outline Part I:

Quantum State Engineering in Driven Dissipative Many-Body Systems



- Introduction: Open Systems in Quantum Optics
- Driven Dissipative BEC:
  - Mechanism for pure DBEC: Many-Body Quantum Optics
  - Physical Implementation of DBEC: Reservoir Engineering, Bogoliubov bath
- Application I: Competition of unitary vs. dissipative dynamics
  - first look: weak interactions
  - strong interactions: nonequilibrium phase transition
- Application II: Targeting pure fermion states
  - An excited many-body state: η-condensate
  - Antiferromagnetic and d-wave fermion states



References:
SD, A. Micheli, A. Kantian, B. Kraus, H.P. Büchler, P. Zoller, Nature Physics 4, 878 (2008);
B. Kraus, SD, A. Micheli, A. Kantian, H.P. Büchler, P. Zoller, Phys. Rev. A 78, 042307 (2008)

F. Verstraete, M. Wolf, I. Cirac, Nature Physics 5, 633 (2009)

Quantum State Engineering in Many-Body Systems

- **thermodynamic equilibrium**
  - standard scenario of condensed matter & cold atom physics

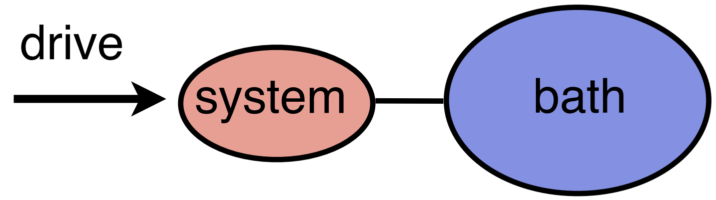
$$H|E_g\rangle = E_g|E_g\rangle \qquad \rho \sim e^{-H/k_BT} \xrightarrow{T\to 0} |E_g\rangle\langle E_g|$$

Hamiltonian (many body) cooling to ground state

Hamiltonian Engineering:

✓ interesting ground states
✓ quantum phases

- **driven / dissipative dynamical equilibrium**
  - quantum optics



$$\frac{d\rho}{dt} = -i[H,\rho] + \mathcal{L}\rho \qquad \rho(t) \xrightarrow{t\to\infty} \rho_{ss}$$

competing dynamics

mixed state

$$\stackrel{!?}{=} \quad |D\rangle\langle D|$$

pure state ("dark state")

master equation steady state

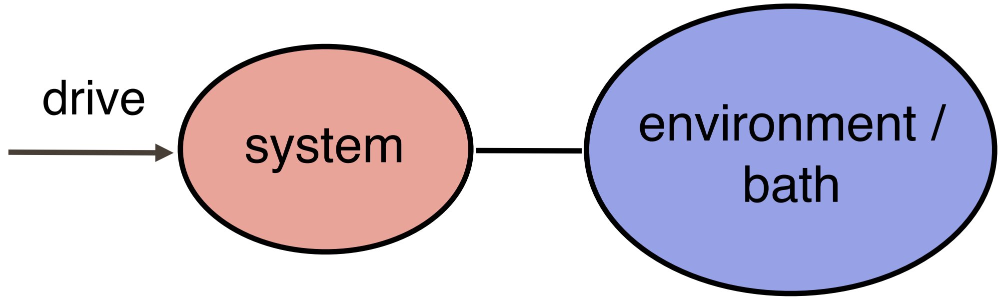
Liouvillian Engineering:

✓ many body pure states / driven quantum phases
✓ mixed states ~ "finite temperature"
✓ useful an interesting fermion states

Open Quantum Systems

Open Quantum Systems

$$H = H_S + H_B + H_{\rm int}$$

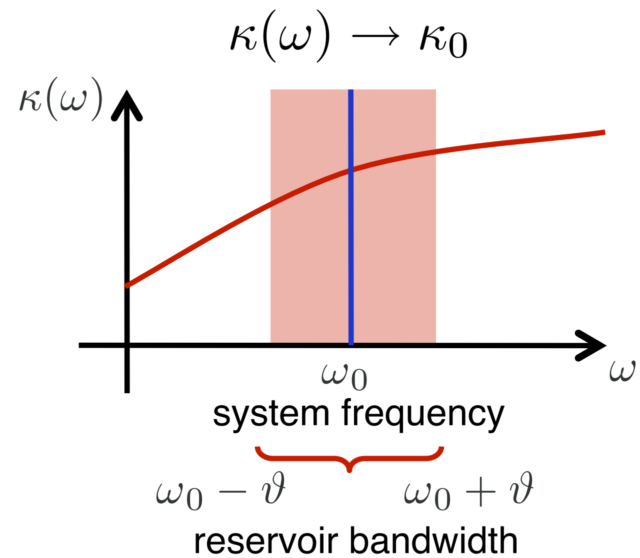


$$H_B = \int d\omega\, \omega b_\omega^\dagger b_\omega$$ continuum bath of harmonic oscillators

$$H_{\rm int} = i \int d\omega \kappa(\omega) \left[b_\omega^\dagger J - b_\omega J^\dagger \right]$$

quantum jump operators
polynomial in system operators

linear bath operator coupling to the system



Three approximations:

(1) Born approximation: $\kappa(\omega)/\omega_0 \ll 1$

(2) Markov approximation: $\kappa(\omega) \approx$ const. $\Rightarrow \kappa(t-t') \sim \delta(t-t')$

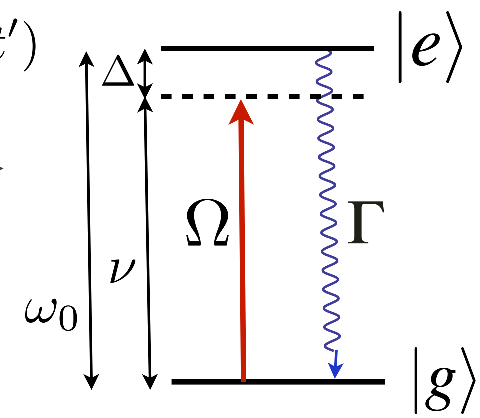
(3) Rotating wave approximation: $\dfrac{\omega_0 - \nu}{\omega_0 + \nu} \ll 1$

$$J_\alpha = |g\rangle\langle e| = \sigma^-$$

$$\omega_0 - \nu = \Delta$$

detuning

$$H = (|e\rangle, |g\rangle) \begin{pmatrix} \Delta & \Omega \\ \Omega & 0 \end{pmatrix} \begin{pmatrix} \langle e| \\ \langle g| \end{pmatrix}$$

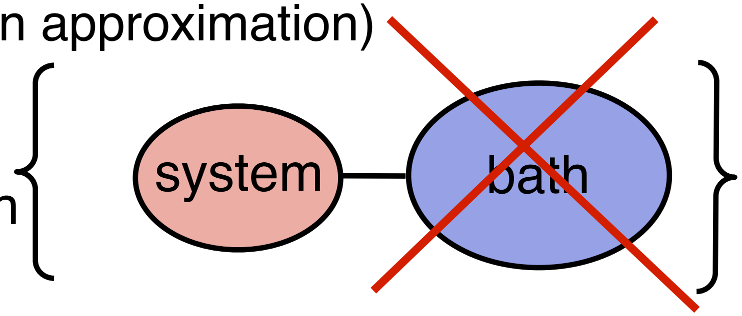


Open Quantum Systems

$$\partial_t \rho_{\text{tot}} = -i[H_S + H_B + H_{\text{int}}, \rho_{\text{tot}}]$$

➡ Eliminate bath degrees of freedom in second order time-dependent perturbation theory (Born approximation)

$\mathrm{Tr}_{\text{bath}}$



quantum jump operators

effective system dynamics from Master Equation (zero temperature bath)

$$\partial_t \rho = -i[H_S, \rho] + \underbrace{\kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger - \tfrac{1}{2}\{J_\alpha^\dagger J_\alpha, \rho\}}_{\mathcal{L}[\rho]}$$

$\mathcal{L}[\rho]$ Liouvillian operator in Lindblad form

- Structure: second order perturbation theory
- mnemonic: norm conservation $\partial_t \mathrm{tr}\rho = 0$
- but: $\partial_t \mathrm{tr}\rho^2 \neq 0$

pure state: $\mathrm{tr}\rho = \mathrm{tr}\rho^2 = 1$

$\Rightarrow \mathrm{tr}\rho^2$ -- “purity”

➡ Purity is not conserved

➡ go for $\partial_t \mathrm{tr}\rho^2 < 0$

Open Quantum Systems

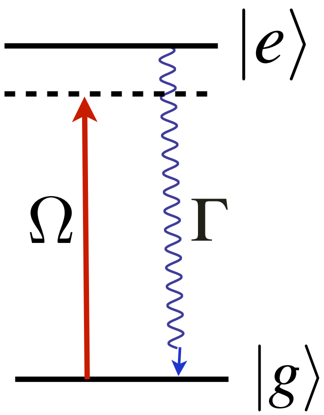
- Stochastic Interpretation: Quantum Jumps

$$\partial_t \rho = -i[H,\rho] + \kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger - \tfrac{1}{2}\{J_\alpha^\dagger J_\alpha, \rho\}$$

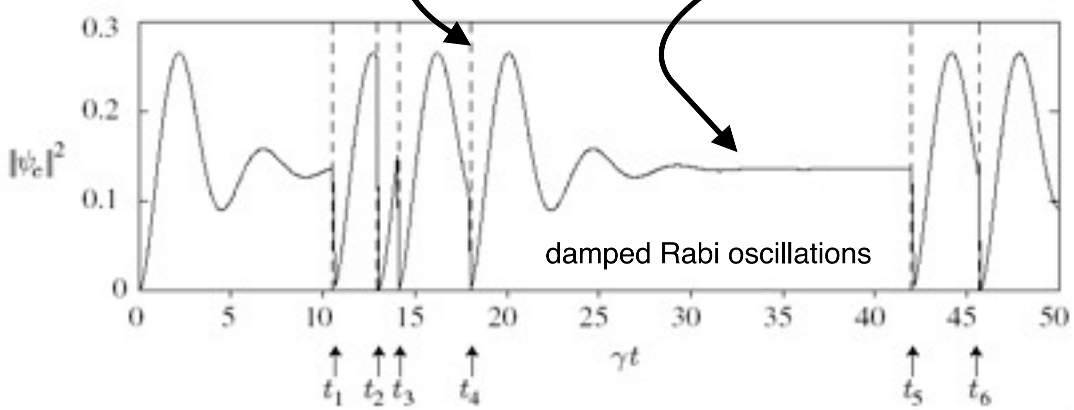
$$= -i[H_{\text{eff}},\rho]^* + \kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger \qquad H_{\text{eff}} = H - i\kappa/2 \sum_\alpha J_\alpha^\dagger J_\alpha$$

quantum jump operators

decay



$$J_\alpha = |g\rangle\langle e| = \sigma^-$$



time evolution of upper state population of driven dissipative two-level system (single run)

- Averaging over "quantum trajectories" generates all correlation functions

➡ Engineer the jump operators J_α

$$[A,B]^* := AB - B^\dagger A^\dagger$$

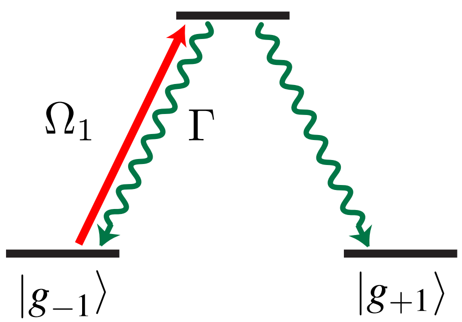
Driven Dissipative BEC

Dark States in Quantum Optics

- Goal: pure BEC as steady state solution, independent of initial density matrix:

$$\rho(t) \longrightarrow |BEC\rangle\langle BEC| \text{ for } t \to \infty$$

- Such situation is well-known quantum optics (three level system): optical pumping (Kastler, Aspect, Cohen-Tannoudji; Kasevich, Chu; ...)



$$\rho(t) \xrightarrow{t\to\infty} |g_+\rangle\langle g_+|$$

➡ Driven dissipative dynamics “purifies” the state

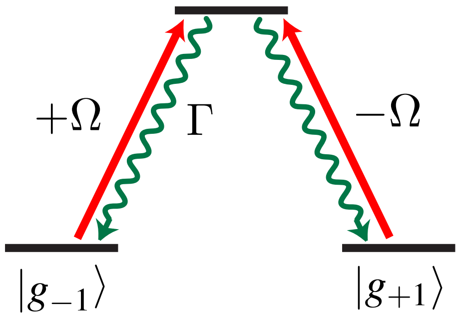
➡ $|g_+\rangle$ is a “dark state” decoupled from light

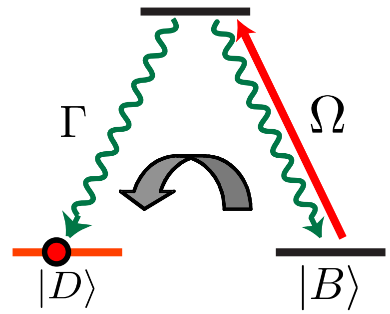
$$c_\alpha |g_+\rangle = 0$$

➡ Dark state is Eigenstate of jump operators with zero Eigenvalue
➡ Time evolution stops when system is in DS: pure steady state

An Analogy

- Λ-system: three electronic levels (VSCPT by Aspect, Cohen-Tannoudji; Kasevich, Chu)

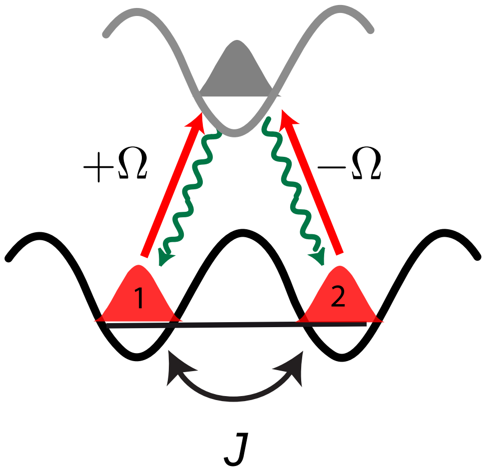




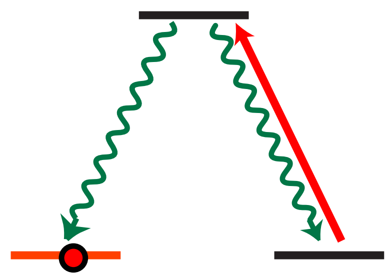
dark state | bright state

$|D\rangle \sim |g_{+1}\rangle + |g_{-1}\rangle$ $\quad$ $|B\rangle \sim |g_{+1}\rangle - |g_{-1}\rangle$

- 1 atom on 2 sites



~ dissipative Josephson junction



$(a_1^\dagger + a_2^\dagger)\,|\mathrm{vac}\rangle$ $\quad$ $(a_1^\dagger - a_2^\dagger)\,|\mathrm{vac}\rangle$

symmetric "in-phase" | anti-symmetric "out-of-phase"

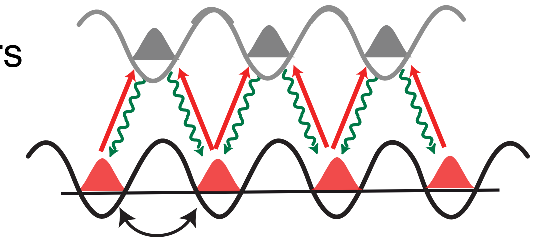
pumping into symmetric state

➡ "phase locking": like a BEC

Driven Dissipative lattice BEC

- Consider jump operator:

nearest neighbours



$$c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j)$$

(1) BEC state is a dark state: $|BEC\rangle = \frac{1}{N!}\Big(\sum_\ell a_\ell^\dagger\Big)^N |vac\rangle$

$$c_{ij}|BEC\rangle = 0\ \forall i \qquad (a_i - a_j)\sum_\ell a_\ell^\dagger = \sum_\ell a_\ell^\dagger (a_i - a_j) + \sum_\ell \delta_{i\ell} - \delta_{j\ell}$$

(2) BEC state is the only dark state:

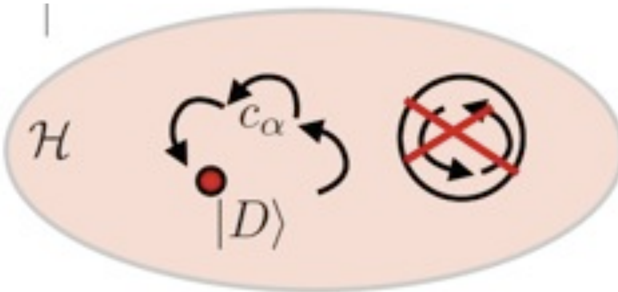
- $(a_i^\dagger + a_j^\dagger)$ has no eigenvalues
- $(a_i - a_j)$ has unique zero eigenvalue

$$(a_i - a_j)\ \forall i \longrightarrow (1 - e^{i\mathbf{q}\mathbf{e}_\lambda})a_{\mathbf{q}}\ \forall \mathbf{q}$$

Driven Dissipative lattice BEC

(3) Uniqueness: |BEC> is the only stationary state (sufficient condition)

If there exists a stationary state which is not a dark state, then there must exist a subspace of the full Hilbert space which is left invariant under the set $\{c_\alpha\}$



(4) Compatibility of unitary and dissipative dynamics

$|D\rangle$ be an eigenstate of H, $H|D\rangle = E|D\rangle$

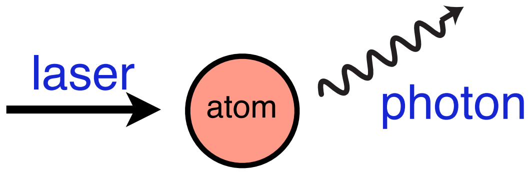
$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\langle D|$$

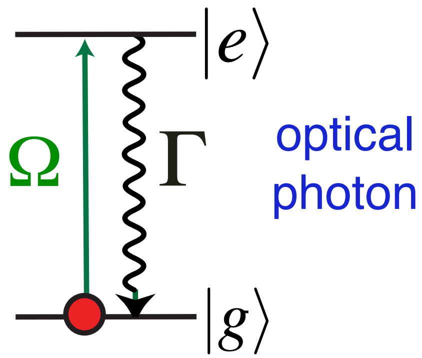
- Long range order in many-body system from quasi-local dissipative operations
- Uniqueness: Final state independent of initial density matrix
- Criteria are general: jump operators for AKLT states (spin model), eta-states (fermions), d-wave states (fermions, next lecture)

A. Griessner, A. Daley et al. PRL 2006; NJP 2007
(noninteracting atom)

Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission





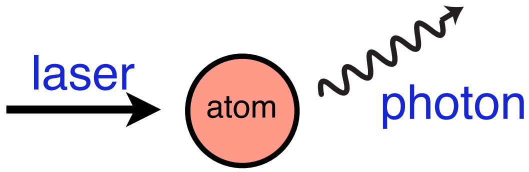
- reservoir: vacuum modes of the radiation field (T=0)
- $\omega \sim 2\pi \times 10^{14} Hz$

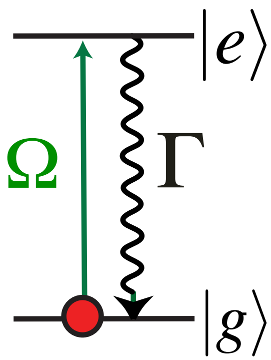
Quantum optics ideas/techniques

- much lower energy scales...

Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission





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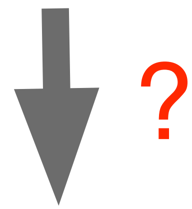
Quantum optics ideas/techniques



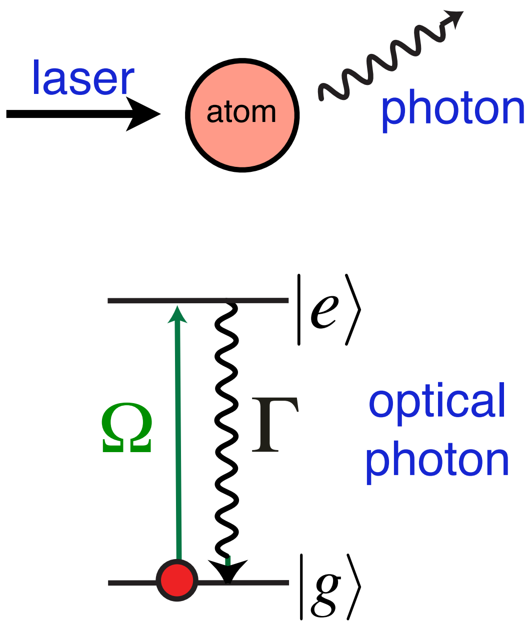
(many body) cold atom systems

- much lower energy scales...

A. Griessner, A. Daley et al. PRL 2006; NJP 2007
(noninteracting atom)

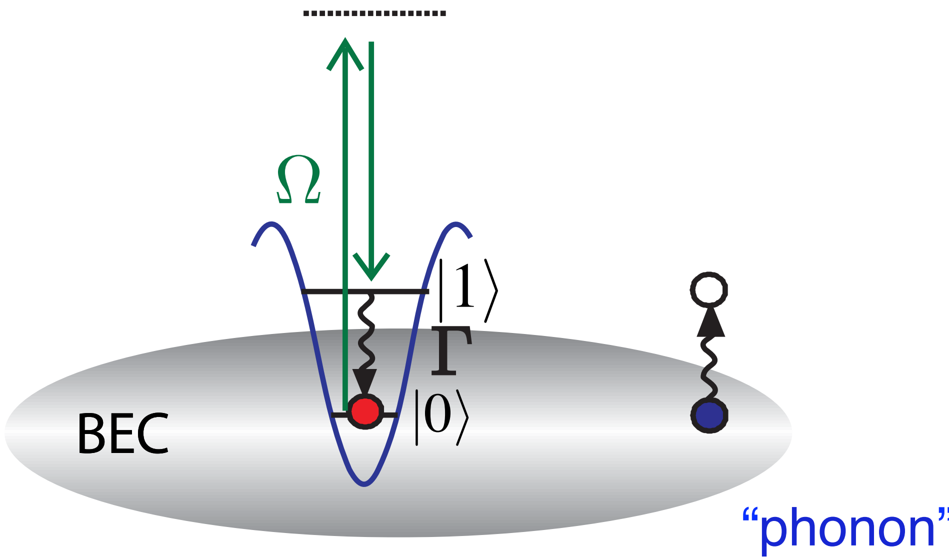
Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission



- reservoir: vacuum modes of the radiation field (T=0)
- $\omega \sim 2\pi \times 10^{14} Hz$

- trapped atom in a BEC reservoir



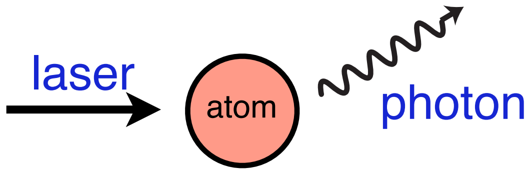
laser assisted atom + BEC collision

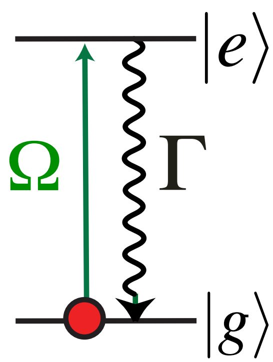
$\omega_{bd} \sim 2\pi \times kHz$

A. Griessner, A. Daley et al. PRL 2006; NJP 2007
(noninteracting atom)

Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission

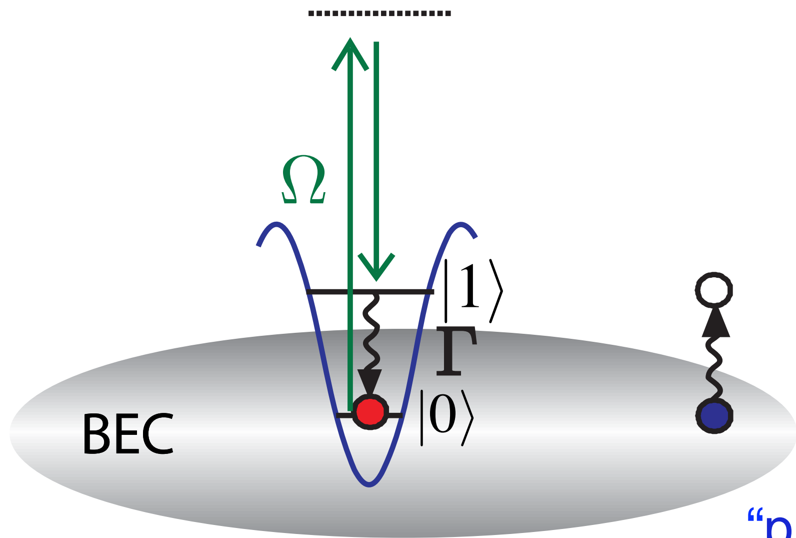




optical photon

- reservoir: vacuum modes of the radiation field (T=0)
- $\omega \sim 2\pi \times 10^{14} Hz$

- trapped atom in a BEC reservoir



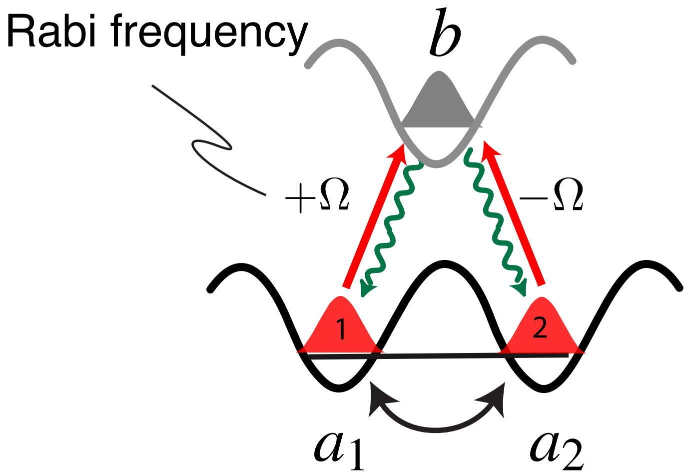
"phonon"

laser assisted atom + BEC collision

- reservoir: Bogoliubov excitations of the BEC (at temperature T)
- $\omega_{bd} \sim 2\pi \times kHz$

Physical Realization

Schematic



(1) Coherent excitation with opposite sign of Rabi frequency

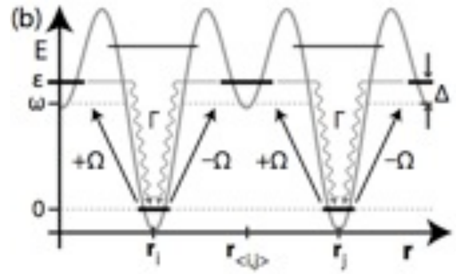
$$\Omega\, b^{\dagger}(a_1 - a_2) + h.c.$$

antisymmetric

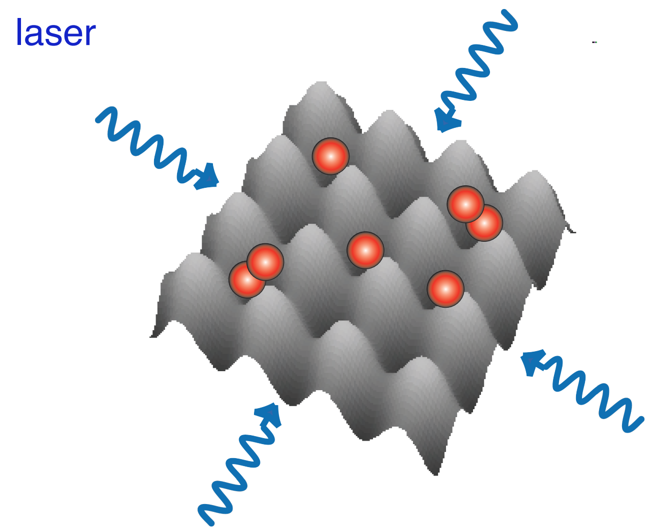
$$c_{ij} = (a_i^{\dagger} + a_j^{\dagger})(a_i - a_j)$$

In practice

- level structure: optical superlattice

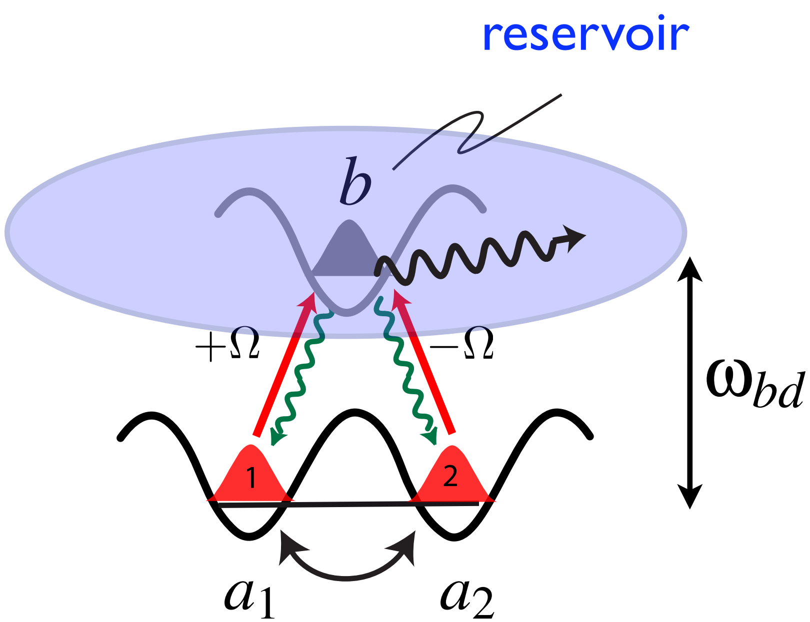


- coherent excitation: Raman laser



Physical Realization

Schematic



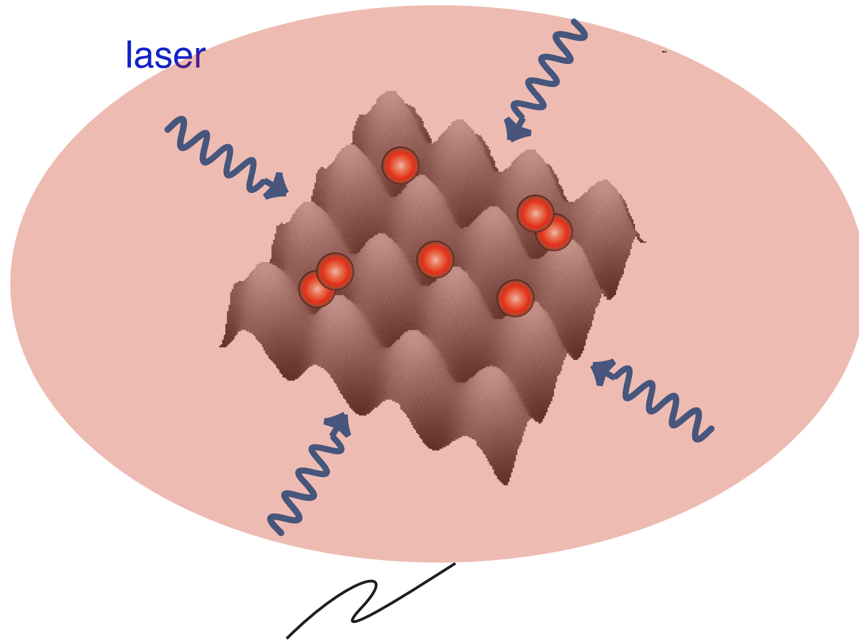
(2) Dissipative decay back:
coupling of upper level to reservoir

$$\kappa(a_1^\dagger + a_2^\dagger) b \sum_{\mathbf{k}} (r_{\mathbf{k}} + r_{\mathbf{k}}^\dagger)$$

symmetric

$$c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j)$$

In practice

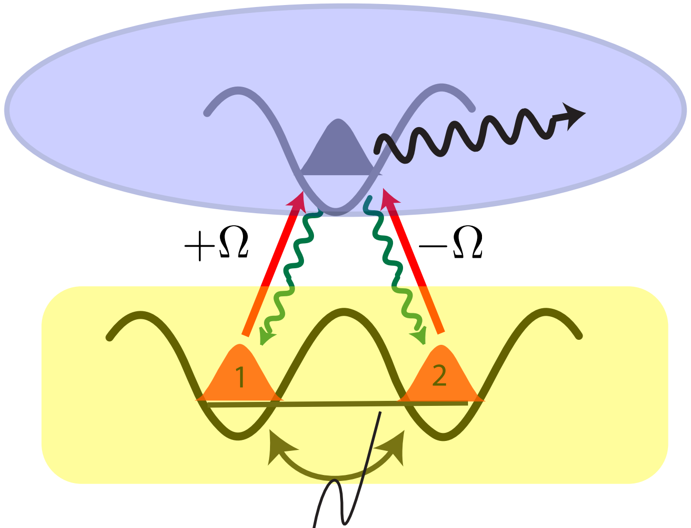


BEC = reservoir of
Bogoliubov excitations

➡ $T_{BEC} \ll \omega_{bd}$ effective
zero temperature reservoir

- coupling to system: interspecies interaction
- short coherence length in bath provides quasi-local dissipative processes, but not mandatory for our setup to work

Physical Realization

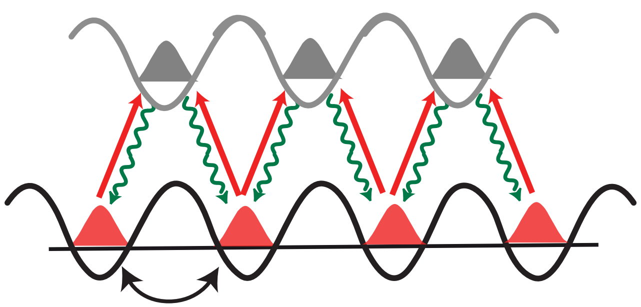


(3) adiabatic elimination of auxiliary level, trace out the bath

Effective single band jump operators

$$c_{12} = (a_1^\dagger + a_2^\dagger)(a_1 - a_2)$$

Many sites: Array of dissipative junctions



Comments:

- Long range phase coherence from quasi-local dissipative operations
- - Coherent drive: locks phases

  - Dissipation: randomizes

  - Conspiracy: purification
- The coherence of the driving laser is mapped on the matter system
- Setting is therefore robust

Competition of unitary vs. dissipative dynamics

Effects of finite interactions

$$H = -J \sum_{<i,j>} a_i^\dagger a_j + U \sum_i a_i^{\dagger 2} a_i^2$$

dissipative dynamics favors pure BEC state

interacting Hamiltonian dynamics not compatible

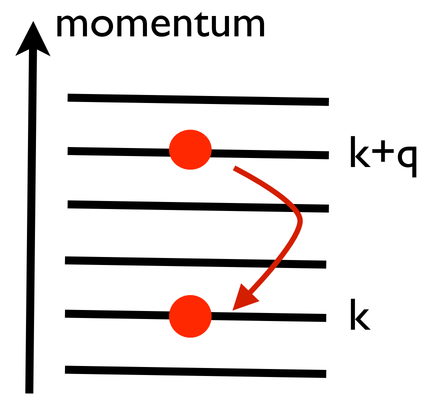
Competition

$$\frac{d\rho}{dt} = -i\,[H,\rho] + \mathcal{L}\rho$$

treating interactions in

- weak coupling
  - 3D: true (depleted) condensate, fixed phase: Bogoliubov theory
  - 1,2D: phase fluctuations destroy long range order: Luttinger theory
- Strong coupling, 3D
  - mixed state Gutzwiller Ansatz

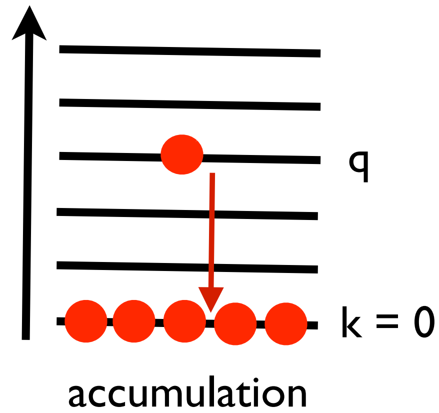
Weak Coupling: Linearized jump operators



- momentum space jump operators are nonlocal nonlinear objects

$$c_{\mathbf{q},\lambda} = \frac{1}{M^{d/2}} \sum_{\mathbf{k}} (1 + \mathrm{e}^{\mathrm{i}\mathbf{k}\mathbf{e}_\lambda})(1 - \mathrm{e}^{-\mathrm{i}(\mathbf{k}+\mathbf{q})\mathbf{e}_\lambda}) a^\dagger_{\mathbf{k}} a_{\mathbf{k}+\mathbf{q}}$$

- In a linearized theory the reduce to (any dimension)



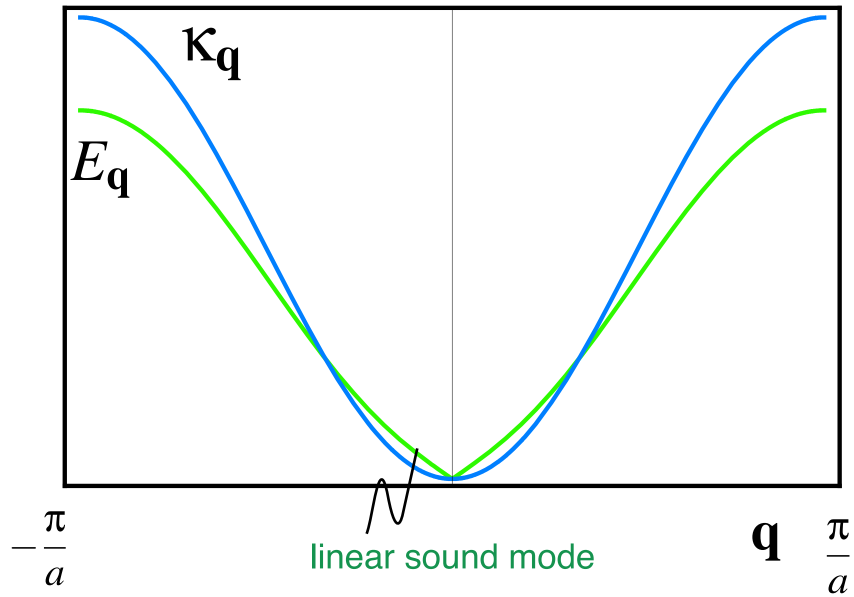
$$c_{\mathbf{q},\lambda} = f_{\mathbf{q},\lambda} a_{\mathbf{q}} \qquad f_{\mathbf{q},\lambda} = 2\sqrt{n}(1 - e^{-\mathrm{i}\mathbf{q}\mathbf{e}_\lambda})$$

- Interpretation:
  - bosonic mode operators: depopulation of momentum q in favor of condensate
  - zero mode explicit: $f_{\mathbf{q}=0,\lambda} = 0$
  - lead to momentum dependent decay rate

$$\kappa_{\mathbf{q}} = \sum_\lambda \kappa |f_{\mathbf{q},\lambda}|^2 \sim \mathbf{q}^2$$

Many-Body Master Equation

- Interpretation: How close are we to the GS of the Hamiltonian?
  - Diagonalize H
  - consider equation for single mode



$$\partial_t \rho = \overbrace{\boxed{-i\frac{E}{2}[d^\dagger d, \rho]}}^{\text{Bogoliubov / hydrodynamic excitation}}$$

$$+2\kappa(\underbrace{\boxed{u^2 d\rho d^\dagger}}_{\text{“cooling”}} + \underbrace{\boxed{v^2 d^\dagger \rho d}}_{\text{“heating”}} - \underbrace{\boxed{uv(d^\dagger \rho d^\dagger + d\rho d}}_{\text{squeezing}})$$ + anticommutator term

$v_{\mathbf{q}}^2,\ u_{\mathbf{q}}^2 = v_{\mathbf{q}}^2 + 1$ generalized Bogoliubov coefficients

$N, \qquad N+1$ cf. thermal reservoir

➡ Intrinsic heating/cooling, though reservoir is at T = 0

Characterization of Steady State: Density Operator

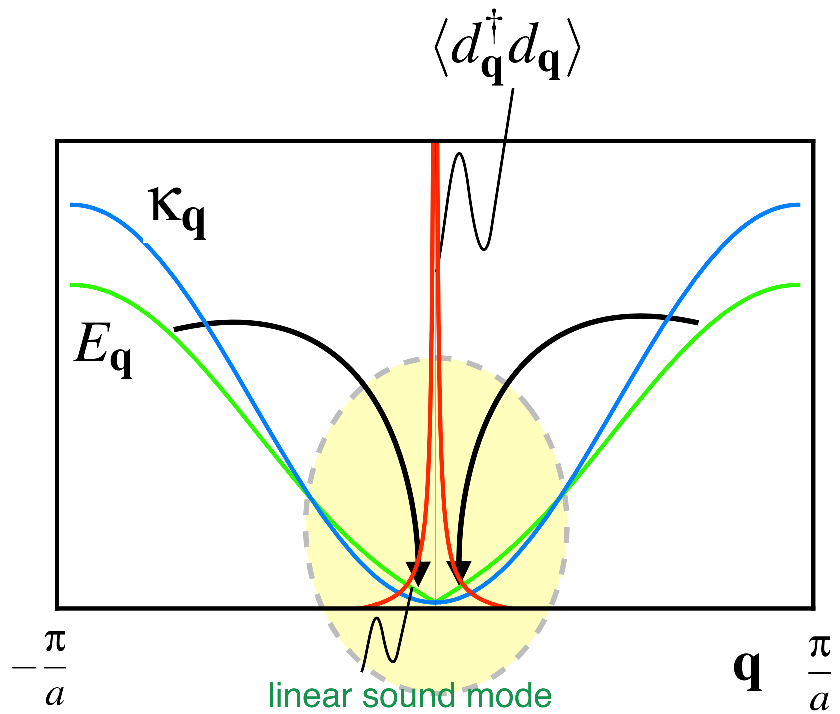
- linearized ME exactly solvable: Gaussian density operator for each mode expressible as

$$\rho_{\mathbf{k}} = \exp\left(-\beta_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}}\right)$$

with squeezed operators b (Bogoliubov transformation)

➡ mixed state with

$$\coth^2(\beta_{\mathbf{k}}/2) = \frac{\kappa_{\mathbf{k}}^2 + (\epsilon_{\mathbf{k}} + Un)^2}{\kappa_{\mathbf{k}}^2 + E_{\mathbf{k}}^2}$$



- at low momenta, resemblance to thermal state:

$$\beta_{\mathbf{k}} \approx \frac{E_{\mathbf{k}}}{T_{\mathrm{eff}}}, \quad T_{\mathrm{eff}} = \frac{Un}{2}$$

▸ role of temperature played by interaction

Correlations in various dimension: 3D

- Steady state: condensate depletion:

$$n_{\mathrm{D}} = n - n_0 = \frac{1}{2}\int \frac{d\mathbf{q}}{v_0} \frac{(Un)^2}{\kappa_q^2 + E_q^2}$$

  - small depletion justifies Bogoliubov theory
  - squeezing and mixing effects tied to interaction strength (unlike th. equilibrium)

- Approach to the steady state:

$$n_{0,eq} - n_0(t) \sim \sqrt{\frac{Un}{8J}} \frac{1}{2\kappa n} \, t^{-1}$$

  - power-law: Many-body effect due to mode continuum
  - sensitive probe to interactions: cf. for noninteracting system

$$n_{0,eq} - n_0(t) \sim t^{-3/2}$$

  - universal at late times

Correlations in various dimension: 1/2D

- Steady State: quasi-condensates in low “temperature” phase

$$\langle a_x^\dagger a_0 \rangle \sim \langle \exp i(\phi_x - \phi_0) \rangle \sim \begin{cases} e^{-\frac{T_{\mathrm{eff}}}{8Jn} x}, & d = 1 \\ (x/x_0)^{-T_{\mathrm{eff}}/4T_{\mathrm{KT}}}, & d = 2 \end{cases}$$

$$T_{\mathrm{KT}} = \pi J n \gg T_{\mathrm{eff}} \qquad T_{\mathrm{eff}} = Un/2 \qquad x_0 = 2\kappa n (T_{\mathrm{eff}} J)^{-1/2}$$

Kosterlitz-Thouless temperature
of 2D quasi-condensate

Dissipative coupling:
only sets cutoff scale

- steady state well understood as thermal Luttinger liquid
- similar results for temporal correlations (from ME via quantum regression theorem)
- weak effect of dissipation on phase fluctuations:

$$E_{\mathbf{q}} \sim |\mathbf{q}|, \kappa_{\mathbf{q}} \sim \mathbf{q}^2$$

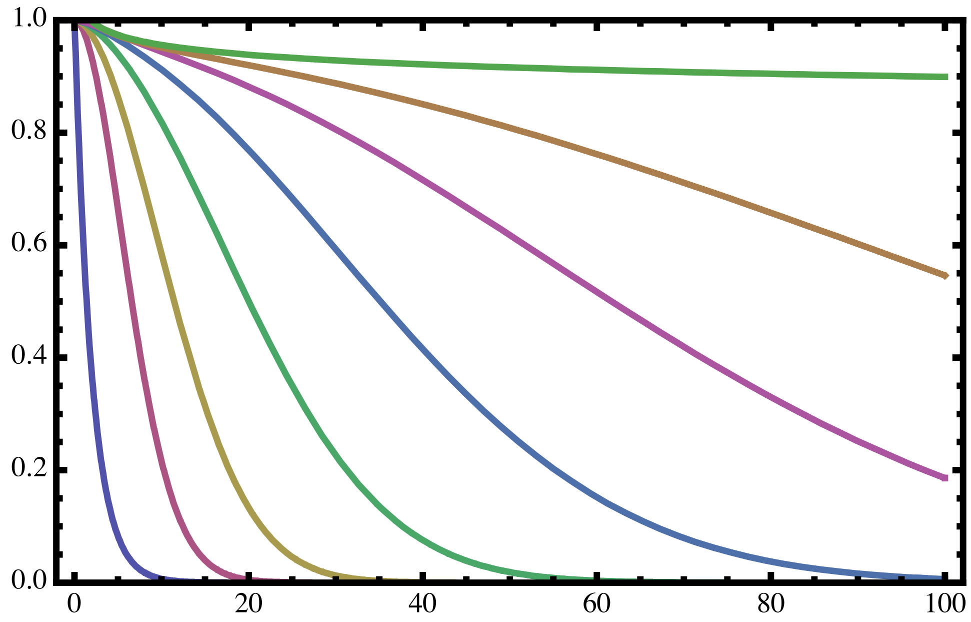
2D: Real Time Evolution

- Buildup of spatial correlations from disordered state

$$\Psi_t(x,0) \sim \begin{cases} e^{-|x|/\xi} & t = 0 \\ (x/x_0)^{-\frac{T_{\text{eff}}}{4T_{\text{KT}}}}\, e^{-\frac{x^2}{4\xi\sqrt{\pi\kappa n t}}} & t \to \infty \end{cases}$$

broadening of Gaussian governed by time-dependent length scale

$$x_t = 2(\pi\xi^2\kappa n t)^{1/4}$$



with A. Tomadin

Strong Coupling: Nonequilibrium Phase Transition

- Analogy to Mott insulator / Superfluid quantum phase transition :

• enhancement of superfluidity:	Hopping J	driven dissipation κ
• suppression of superfluidity:	interaction U	interaction U
➡ Expect phase transition as function of	J/U	κ/U

- Differences:

➡ Competition of two unitary evolutions vs. competition of unitary and dissipative evolution

✓phase transition (temperature T)

✓quantum phase transition (g)

Reminder: Mott Insulator-Superfluid Phase Transition

$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j - \mu \sum_i \hat{n}_i + \tfrac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

- Hopping J favors delocalization in real space:
- Condensate (local in momentum space!)
- Fixed condensate phase: Breaking of phase rotation symmetry

$$\langle b_i \rangle \sim e^{\mathrm{i}\varphi}$$

- Interaction U favors localization in real space for integer particle numbers:
- Mott state with quantized particle no.
- no expectation value: phase symmetry intact (unbroken)

➡ Competition gives rise to a quantum phase transition as a function of

$$U/J$$

Reminder: Gutzwiller Ansatz

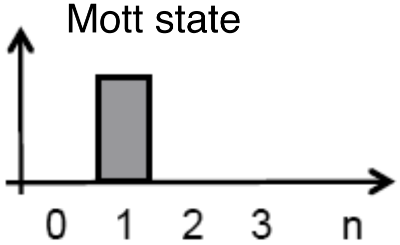
- Interpolation scheme encompassing the full range J/U.
  - Main ingredient: product wave function ansatz

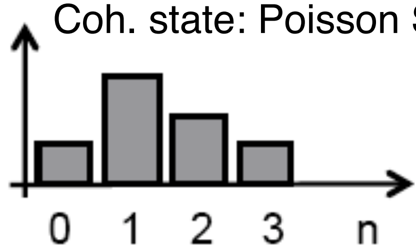
$$|\psi\rangle = \prod_i |\psi\rangle_i , \quad |\psi\rangle_i = \sum_n f_n^{(i)} |n\rangle_i , \quad {}_i\langle\psi|\psi\rangle_i = 1 \, \forall i$$

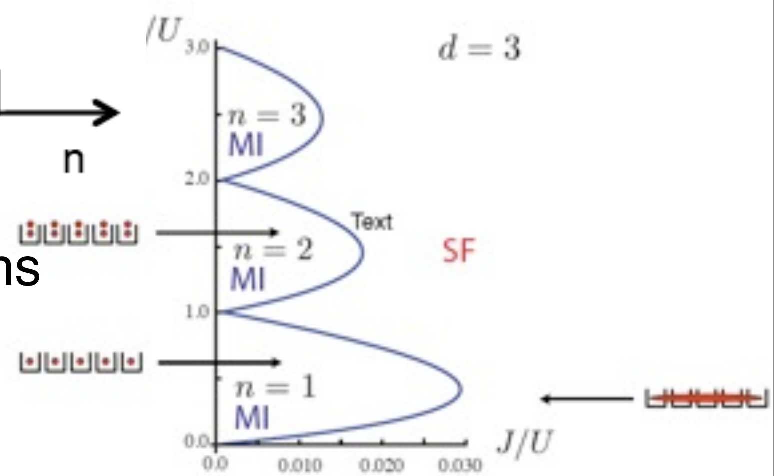
complex amplitudes

wave function normalization

  - Limiting cases (homogeous, drop site index, amplitudes chosen real):
    - Mott state with particle number m: $f_n = \delta_{n,m}$
    - coherent state: $f_n = \sqrt{N/n!} e^{-N/2}$







- Validity: approximation neglects all spatial correlations
  - becomes exact in infinite dimensions
  - reasonable in d=2,3 (T=0)

Mixed State Gutzwiller Approach

- Product ansatz for the density operator (instead of wave function)

$$\rho(t) = \prod_i \rho_i(t), \quad \rho_i(t) = \sum_{nm} |n\rangle_i \langle m| \rho_{nm}^{(i)}(t)$$

Interpretation:

✓ off-diagonal: SF

✓ diagonal: atom statistics

- Project on on-site density operator:

$$\rho_k = \mathrm{Tr}_{\neq k}\, \rho$$

➡ Nonlinear Mean Field Master Equation for reduced density operator (drop index)

$$\dot{\rho} = -\mathrm{i}\left[-ZJ(\langle b\rangle b^\dagger + \langle b^\dagger\rangle b) + \tfrac{1}{2}U b^{\dagger 2} b^2, \rho\right]$$

$$B^r = \{\hat{n}, b, b^\dagger, \mathbf{1}\}$$

$$+Z\kappa \textstyle\sum_{r,r'} \Gamma^{r,r'} \left\{2B^r \rho B^{\dagger\, r'} - B^{\dagger\, r'} B^r \rho - \rho B^{\dagger\, r'} B^r\right\}$$

with correlation matrix

$$\Gamma^{r,r'} = \begin{bmatrix} \langle \hat{n}^2\rangle & \langle b^\dagger \hat{n}\rangle & -\langle b\hat{n}\rangle & -\langle \hat{n}\rangle \\ \langle \hat{n} b\rangle & \langle \hat{n}\rangle & -\langle b^2\rangle & \langle b\rangle \\ -\langle \hat{n} b^\dagger\rangle & -\langle b^{\dagger 2}\rangle & \langle \hat{n}\rangle + 1 & \langle b^\dagger\rangle \\ -\langle \hat{n}\rangle & -\langle b^\dagger\rangle & \langle b\rangle & \langle \mathbf{1}\rangle \end{bmatrix}$$

Properties of ME:

✓ trace conserving

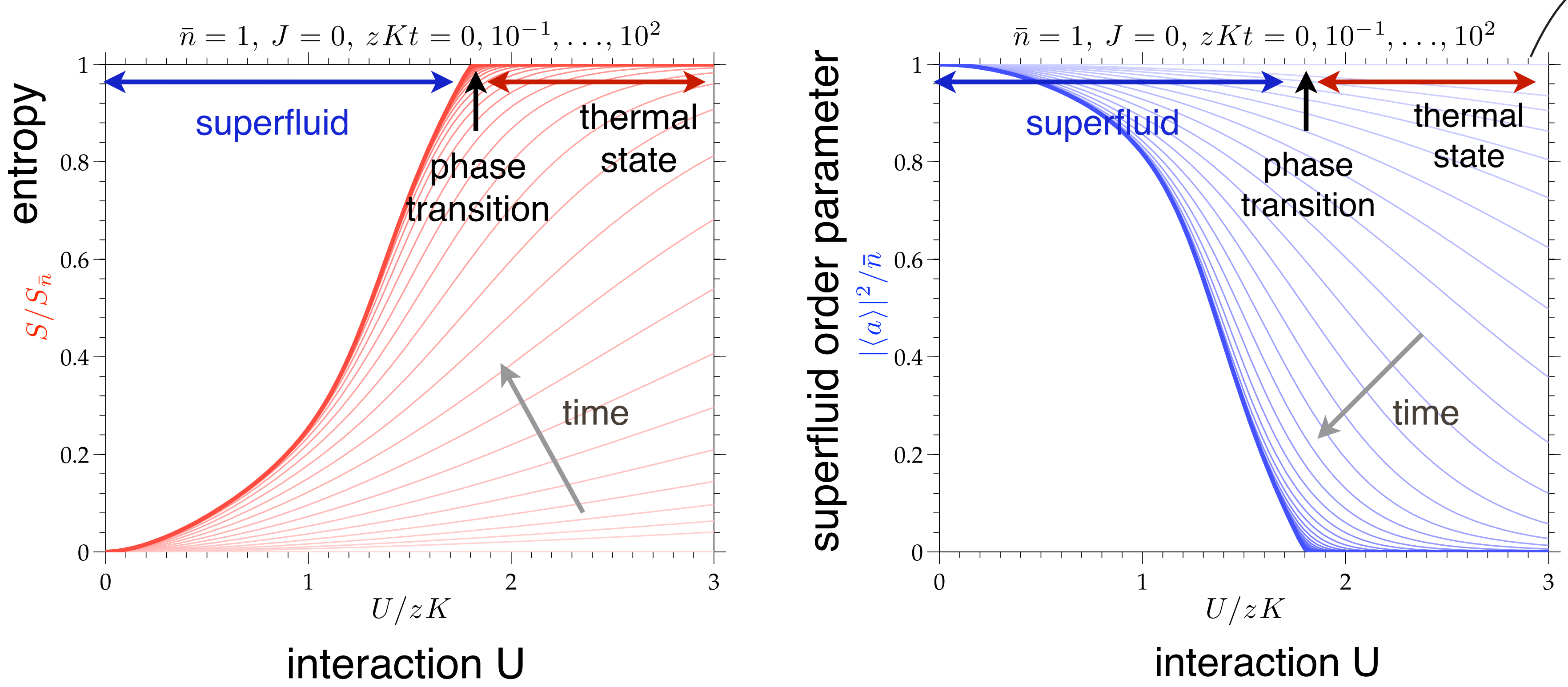
✓ mean particle number conserving

- Nonlinearity emerging in approximation to linear qm equation: similar GP equation

Driven Dissipative Phase Transition

$$\rho_{n,n} = \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}$$

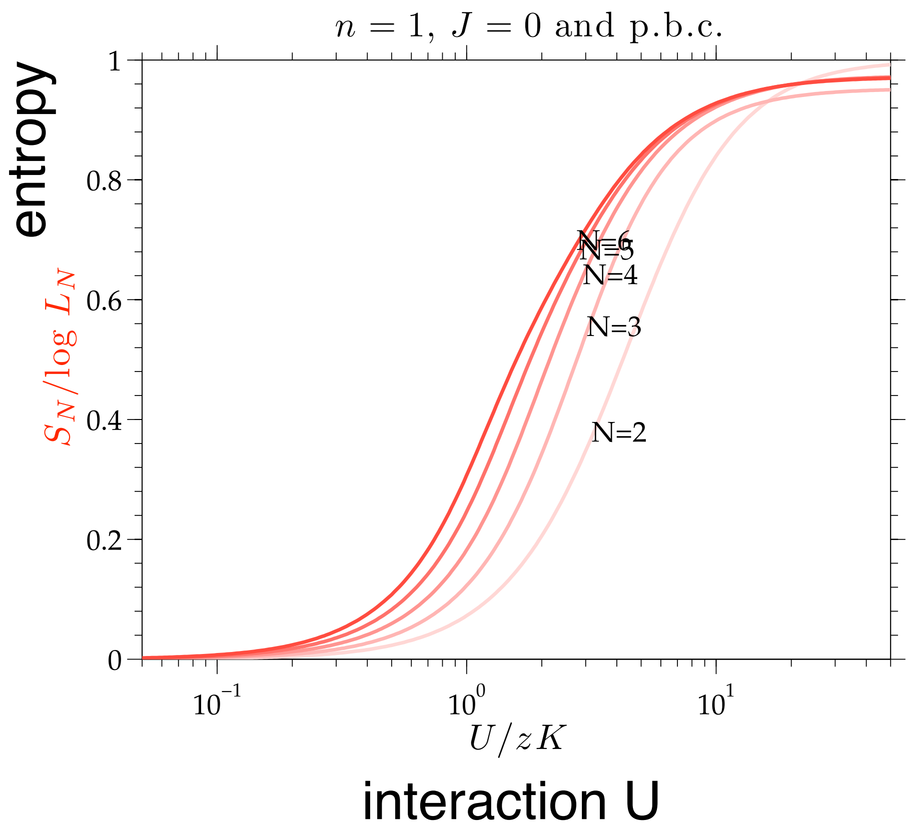
- Dynamic generation of the phase transition from initial coherent state

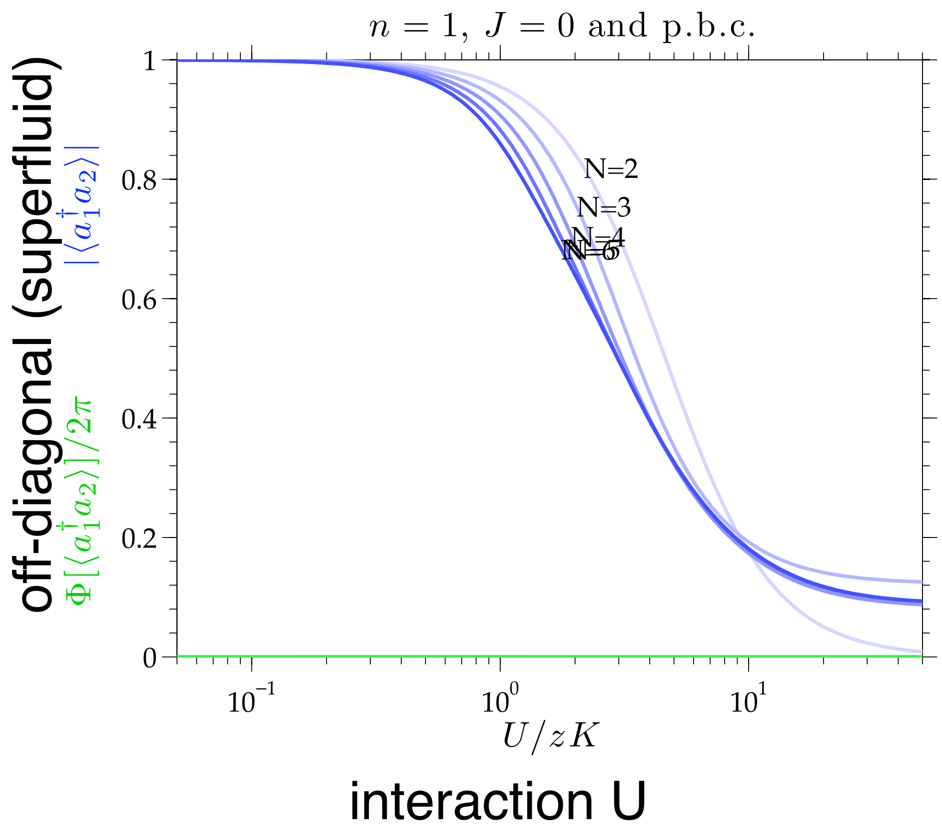


- $U \to 0$ pure coherent state solution
- Phase transition: Non-analyticity develops for $t \longrightarrow \infty$
- above critical point: thermal state: “fixed temperature” given by mean particle density N; no other scale appears
- No signatures of Mott physics due to strong mixing effect of U: unlike Bose-Hubbard case of two unitary tendencies at T=0:

A. Tomadin

Exact calculations for N=6 sites

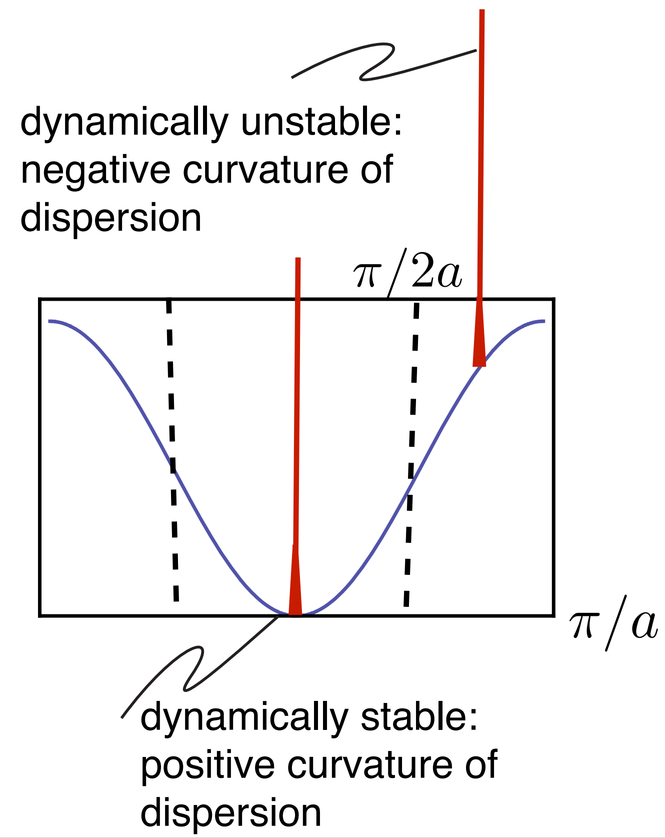


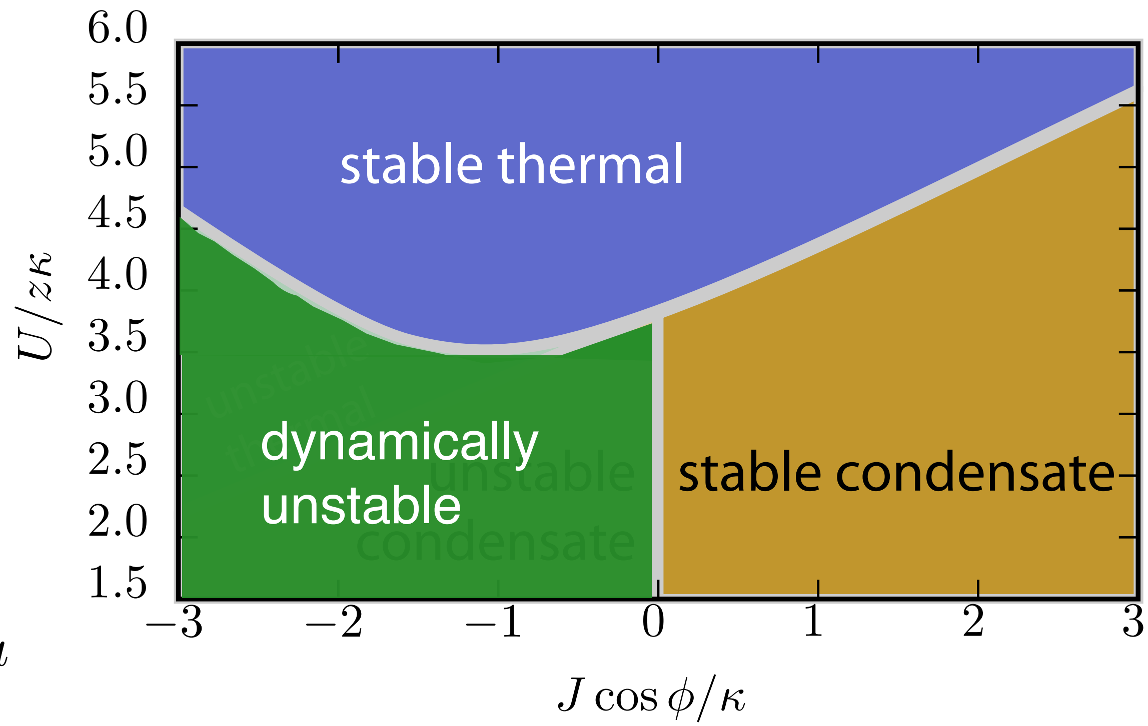


Nonequilibrium Phase Diagram

Classification?

- U/K transition:
  - interaction driven (like quantum PT)
  - terminates in thermal state (like classical finite temperature PT)
- Add negative J (via phase imprinting): further competition through dynamical instability
  - no stable equilibrium state (no dynamical fixed point)
  - dynamical limit cycle?





- Initialization: Coherent state, U=J=0
- follow time evolution of the system

Dissipative Driving of Fermions

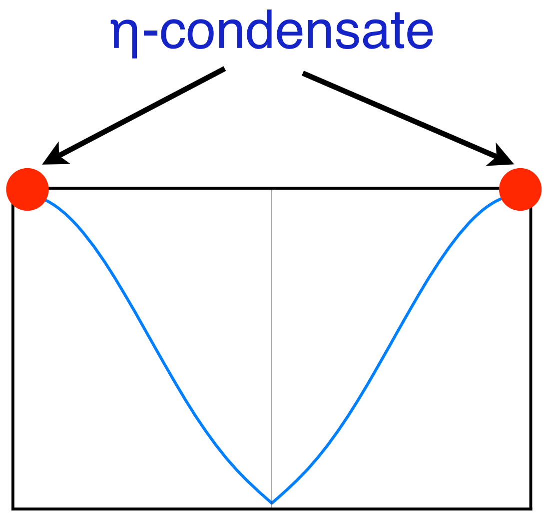
- Excited states: η Condensate
- Cooling into Antiferromagnetic and d-Wave States

Cooling to Excited States: η-Condensate

- η-state: exact excited (i.e. metastable) eigenstate of the two-species Fermi Hubbard Hamiltonian in d dimensions [Yang '89]

$$H = -J \sum_{\langle i,j \rangle, \sigma} f^\dagger_{i\sigma} f_{j\sigma} + U \sum_i f^\dagger_{i\uparrow} f^\dagger_{i\downarrow} f_{i\downarrow} f_{i\uparrow}$$

- local "doublon" $\eta^\dagger_i = f^\dagger_{i\uparrow} f^\dagger_{i\downarrow}$



- checkerboard superposition η-particle

$$\eta^\dagger = \frac{1}{M^{d/2}} \sum_i \phi_i \eta^\dagger_i \qquad \phi_i = \pm 1$$

- N-η-condensate:

$$H(\eta^\dagger)^N |0\rangle = NU(\eta^\dagger)^N |0\rangle$$

exact eigenstate,
off-diagonal long range order

Cooling to Excited States: η-Condensate

- Small scale simulations (open BC) demonstrate η condensation for jumps

$$c_{ij}^{(1)} = (\eta_i^\dagger - \eta_j^\dagger)(\eta_i + \eta_j)$$

$$c_{ij}^{(2)} = n_{i\uparrow} f_{i\downarrow}^\dagger f_{j\downarrow} + n_{j\uparrow} f_{j\downarrow}^\dagger f_{i\downarrow}$$

- Interpretation: Quantum Jump picture

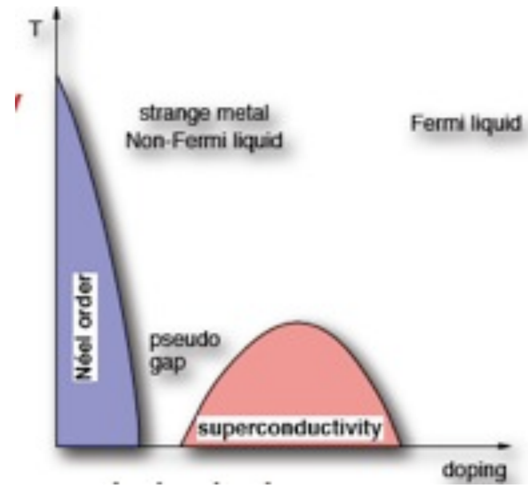
  - H generates spin-up and down configurations on each pair of sites (for any initial density matrix)
  - $c_{ij}^{(2)}$ associates into local doublons
  - $c_{ij}^{(1)}$ creates checkerboard superposition: η condensate

➡ May be conceptually interesting

➡ However, these jump operators are two-body: difficult to engineer

Motivation: Cooling Fermion Systems

Experimental phase diagram for cuprates



- High temperature superconductivity

- discovered in 1986 (Müller, Bednorz): cuprates show superconductivity at unconventionally high temperature
- riddle: attraction from repulsion
    - microscopically, strong Coulomb onsite repulsion
    - still, observe pairing of fermions with d-wave symmetry

- Minimal model: 2d Fermi-Hubbard model

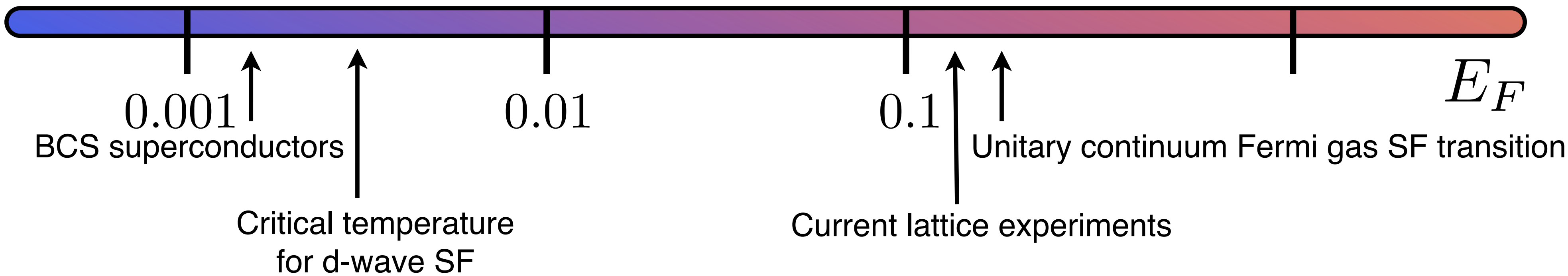
$$H_{\rm FH} = -J \sum_{\langle i,j \rangle, \sigma} c^\dagger_{i,\sigma} c_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} \qquad U \approx 10J$$

- realistic for cuprate high-temperature superconductors?
- hard to solve: strongly interacting fermion theory
    - no controlled analytical approach available
    - numerically (classical computer) intractable

➡ Quantum simulation of the Fermi-Hubbard model in optical lattices?

Quantum Simulation of Fermion Hubbard model

- Clean realization of fermion Hubbard model possible
  - Detection of Fermi surface in 40K (M. Köhl et al. PRL 94, 080403 (2005))
  - Fermionic Mott Insulators (R. Jördens et al. Nature 455, 204 (2008); U. Schneider et al., Science 322, 1520 (2008))
- Cooling problematic: small d-wave gap sets tough requirements



➡ Still need to be 10-100x cooler

- Existing proposal: Adiabatic quantum simulation (S. Trebst et al. PRL 96, 250402 (2006))
  - Start from a pure initial state of noninteracting model
  - Adiabatically transform to unknown ground state of interacting model
  - Concrete scheme: find path protected by large gaps:
    - prepare RVB ground state on isolated 2x2 plaquettes
    - couple these plaquettes to arrive at many-body ground state

Dissipative Quantum State Engineering Approach

- Roadmap:

(1) Precool the system (lowest Bloch band)

(2) Dissipatively prepare pure (zero entropy) state close to the expected ground state:
- energetically close
- symmetry-wise close
- spin-wise close

(3) Adapted adiabatic passage to the Hubbard ground state
- switch dissipation off
- switch Hamiltonian on

Precooling

Dissipative Cooling $H = 0, \quad \mathcal{L} \neq 0$

Adapted adiabatic passage

$H = H_{FH}, \quad \mathcal{L} = 0$

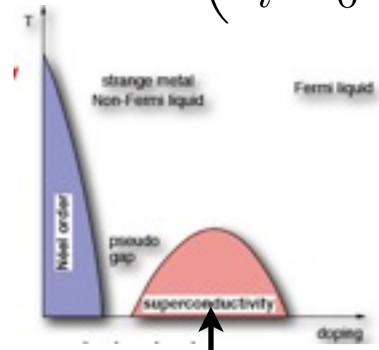
The State to Be Prepared

Pauli matrix $\sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$|\mathrm{D}\rangle = (D^\dagger)^N |\mathrm{vac}\rangle, \;\; D^\dagger = \sum_i (c^\dagger_{i+\mathbf{e}_x} - c^\dagger_{i+\mathbf{e}_y}) \sigma^{(y)} c^\dagger_i$$

mean field (product) state

$c_i = \begin{pmatrix} c_{i,\uparrow} \\ c_{i,\downarrow} \end{pmatrix}$ two-component spinor



d-wave SC

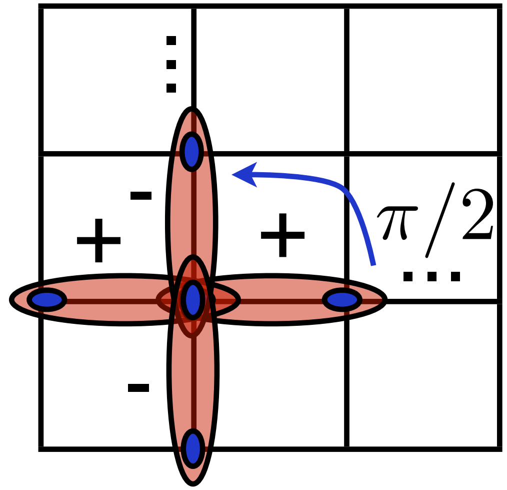
- What does the state have in common with the expected Hubbard ground state

  (1) Quantum numbers

  - pairing in the singlet channel
  - phase coherence: delocalization of singlet pairs
  - transformation under spatial rotations: “d-wave”

  ➡ The state shares the symmetries of Hubbard GS

  ➡ No phase transition will be crossed in preparation process



- in the talk, we mainly consider 1-dimensional analog for simplicity:

$$|\mathrm{D}_1\rangle = (D^\dagger)^N |\mathrm{vac}\rangle, \;\; D^\dagger = \sum_i c^\dagger_{i+1} \sigma^{(y)} c^\dagger_i$$

The State to Be Prepared

Pauli matrix $\sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$|\mathrm{D}\rangle = (D^\dagger)^N |\mathrm{vac}\rangle, \;\; D^\dagger = \sum_i (c^\dagger_{i+\mathbf{e}_x} - c^\dagger_{i+\mathbf{e}_y}) \sigma^{(y)} c^\dagger_i$$

mean field (product) state

$c_i = \begin{pmatrix} c_{i,\uparrow} \\ c_{i,\downarrow} \end{pmatrix}$ two-component spinor

- What does the state have in common with the expected Hubbard ground state

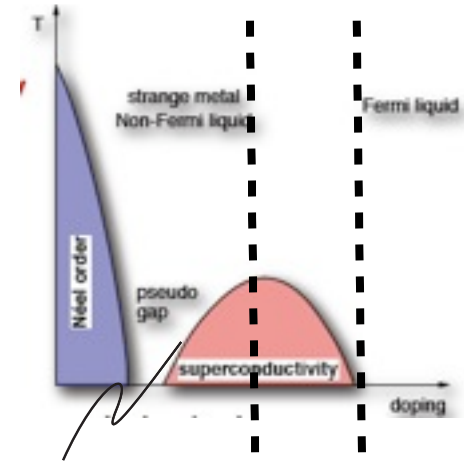
    (2) Energetically close? Not known, but:

    - off-site pairing $c^\dagger_{i+1}\sigma^{(y)}c^\dagger_i$ avoids excessive double occupancy

        cf onsite pairing: $c^\dagger_i \sigma^{(y)} c^\dagger_i$

    - the pairs are quasi-local, i.e. have a short coherence length in accord with observation in cuprates

doping not too close to AF



superfluidity decreases due to strong correlations

(A. Paramekanti, N. Trivedi, M. Randeria, PRB 70, 054504 (2004))

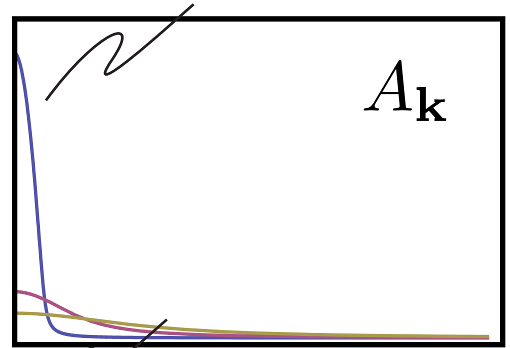
➡ State can be expected to be convenient starting point not too close to half filling

Relation to the BCS Wavefunction

- usually, fixed phase (coherent state) wave function

$$|\psi\rangle \propto \prod_{\mathbf{k}} (1 + A_{\mathbf{k}} c^\dagger_{-\mathbf{k},\uparrow} c^\dagger_{\mathbf{k},\downarrow})|\mathrm{vac}\rangle$$

$$= \exp\Big(\sum_{\mathbf{k}} A_{\mathbf{k}} c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow}\Big)|\mathrm{vac}\rangle = \sum_N \frac{1}{N!}\Big(\overbrace{\sum_{\mathbf{k}} A_{\mathbf{k}} c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow}}^{\text{Fixed particle number wavefunction}}\Big)^N |\mathrm{vac}\rangle$$



BCS amplitude

BCS gap

$$A_{\mathbf{k}} = \frac{v_{\mathbf{k}}}{u_{\mathbf{k}}} = \frac{\Delta_{\mathbf{k}}}{E_{\mathbf{k}} + \xi_{\mathbf{k}}}$$

$$E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$$

chemical potential

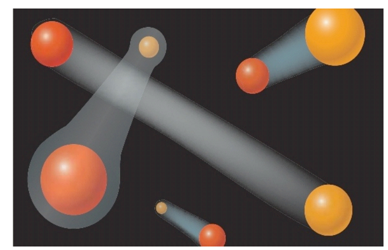
$$\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$$

dispersion

- distinct limits:

$\mu/\epsilon_F \to 1$
- localized in momentum space
- delocalized in position space

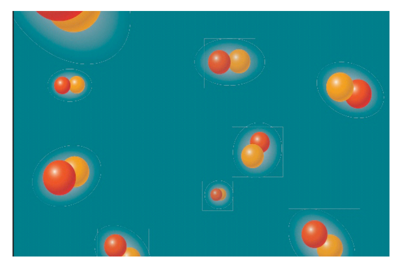
"BCS limit"



$\mu/\epsilon_F \to -\infty$
- delocalized in momentum space
- localized in position space

$A_{\mathbf{k}} \to \dfrac{\Delta_{\mathbf{k}}}{-\mu}$

"BEC / molecular limit"



- Relation to our state: $\sum_i c^\dagger_{i+1}\sigma^{(y)} c^\dagger_i = 2\sum_{\mathbf{k}} \cos\mathbf{k}\, c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{\mathbf{k},\downarrow}$ $\quad A_{\mathbf{k}} = 2\cos\mathbf{k}$

➡ State shares the symmetries, but can be energetically very different

Setting

- Goal: Construct jump operators with unique mean field dark states:

$$\mathcal{L}[\rho] = \sum_\ell j_\ell \rho j_\ell^\dagger - \tfrac{1}{2}\{j_\ell^\dagger j_\ell, \rho\}$$

$$j_\ell|\eta\rangle = 0 \;\forall\; \ell \qquad\qquad |\eta\rangle = \prod_a C_a^\dagger |\mathrm{vac}\rangle$$

dark state — mean field (product) state

solve: $\Rightarrow \boxed{[j_\ell, C_a^\dagger] = 0 \quad \forall \ell, a.}$ (sufficient for normal ordered jump operators)

- Requirements for implementation:

$$j_\ell = \sum_{\langle j|i\rangle \sigma,\sigma'} c_{j,\sigma'}^\dagger H_{\sigma,\sigma'} c_{i,\sigma}$$

- non-hermitian
- particle number conserving $[j_\ell, \sum_{i,\sigma} \hat{n}_{i,\sigma}] = 0 \,\forall\, \ell$
- quasi-local: j close central site i
- single-particle operation

this is what the eta operators suffered from!

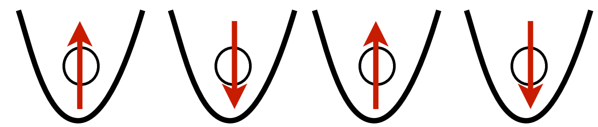
Antiferromagnetic Jump Operators

- Construct jump operators for antiferromagnetism as a preparation
- Antiferromagnetic “Neel state” is a product of AF “unit cell” operators

$$|\mathrm{AF}\pm\rangle = \prod_{i\in A}\hat S^{\pm}_{i-}|\mathrm{vac}\rangle = (-)^{M/2}\prod_{i\in B}\hat S^{\mp}_{i+}|\mathrm{vac}\rangle, \qquad C^\dagger_a = \hat S^{\pm}_{i\pm} = c^\dagger_{i\pm1}\sigma^{\pm}c^\dagger_i$$

doubly degenerate

bipartite lattice with sublattices A,B



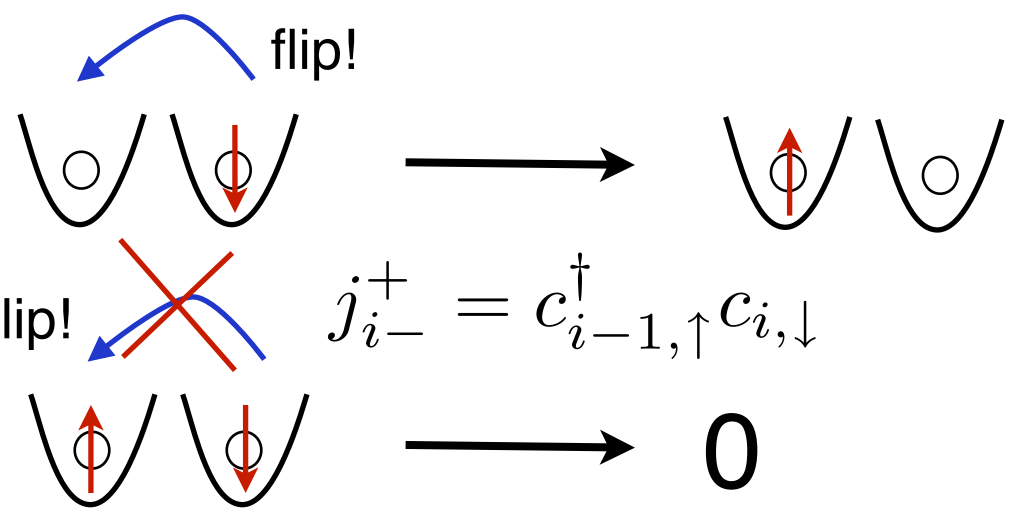
- Set of jump operators (one dimension):

$$\boxed{j_\ell = \{j^{\pm}_{i\pm},\ j^{z}_{i\pm}\}}$$

Pauli matrices

$$j^{\pm}_{i\pm} = c^\dagger_{i\pm1}\sigma^{\pm}c_i,\quad j^{z}_{i\pm} = c^\dagger_{i\pm1}(1\pm\sigma^z)c_i$$

- Action of jump operators
  - $j^{\pm}_{i\pm}$: Pauli blocking
  - j^{z}_{i} : spin transport



$$j^{+}_{i-} = c^\dagger_{i-1,\uparrow}c_{i,\downarrow}$$

$$S^{+}_{i-} = c^\dagger_{i-1,\uparrow}c^\dagger_{i,\downarrow}$$

d-Wave Jump Operators

- Rewrite the d-wave state in terms of AF unit cell operators:

shift invariance

$$|\mathrm{D}\rangle = (D^\dagger)^N |\mathrm{vac}\rangle, \;\; D^\dagger = \sum_i c^\dagger_{i+1}\sigma^{(y)}c^\dagger_i = \sum_i \hat{J}^\pm_i \qquad \hat{J}^\pm_i = \hat{S}^\pm_{i+} + \hat{S}^\pm_{i-}$$

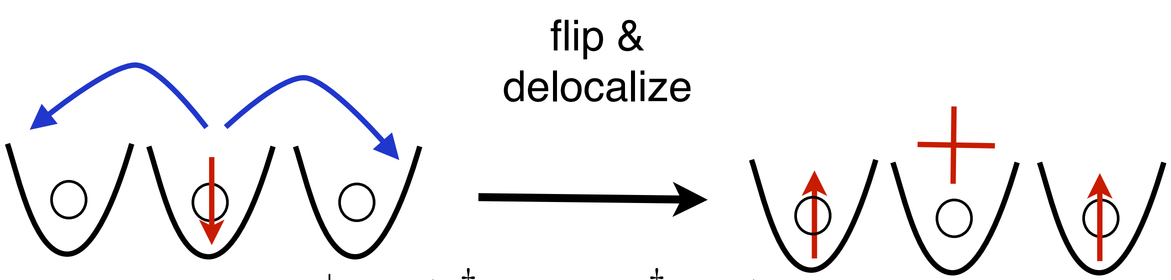
homogeneous product

but delocalized pairs

- Second equality: interpret the state as a symmetrically delocalized AF
- Set of jump operators:

$$\boxed{j_\ell = \{J^\pm_i, \;\; J^z_i\}} \qquad J^\pm_i \;\; = \;\; j^\pm_{i+} + j^\pm_{i-}, \qquad J^z_i = j^z_{i+} + j^z_{i-}$$

- Action of jump operators
  - $J^\pm_i$: Pauli blocking
  - J^z_i : spin transport
  - both: phase coherence via delocalization



$$J^+_i = (c^\dagger_{i-1,\uparrow} + c^\dagger_{i+1,\uparrow})c_{i,\downarrow}$$

➡ Combine fermionic Pauli blocking with delocalization as for bosons

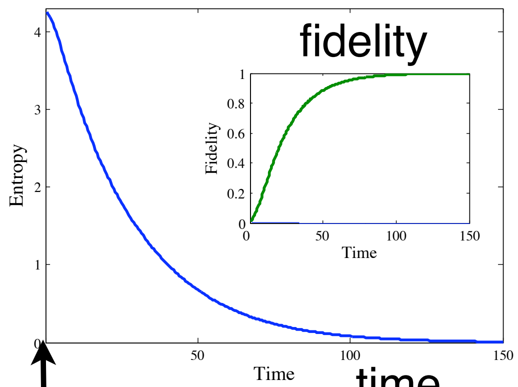
➡ Pauli blocking is the reason for single particle nature of operators

Uniqueness

- Recall: Unique dark state <-> state reached independent of initial condition
- Evidence for uniqueness from small scale numerical simulations

Antiferromagnetism

Entropy evolution

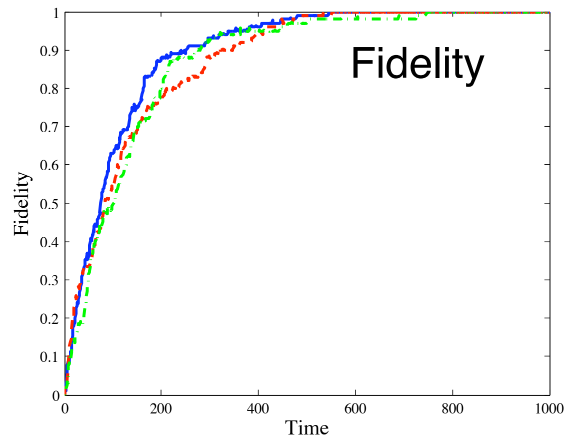


time

perfect mixture

6 sites,
6 particles

d-wave



time

3x3 sites, 4 particles, different random initial states

Entropy

$$S = \mathrm{tr}\rho(t)\log\rho(t)$$

Fidelity

$$\mathrm{tr}[\rho(t)|AF\pm\rangle\langle AF\pm|]$$

Uniqueness

- Understanding can be gained from symmetry considerations
  - Uniqueness of dark state equivalent to uniqueness of ground state (GS) of

$$H_\Delta = \sum_{i,\alpha=\pm,z} \Delta_\alpha J_i^{\alpha\,\dagger} J_i^\alpha \qquad \left[\mathcal{L}[\rho] = \sum_{\alpha,i} \kappa_\alpha J_i^\alpha \rho J_i^{\alpha\dagger} - \tfrac{1}{2}\{\underbrace{\kappa_\alpha J_i^{\alpha\dagger} J_i^\alpha}, \rho\} \right]$$

-> effective Hamiltonian

- H is semi-positive
- an exact GS is the above d-wave (E=0)
- unique iff no symmetry T such that

$$THT^{-1} = H, \quad T|\mathrm{D}\rangle \neq E|\mathrm{D}\rangle$$

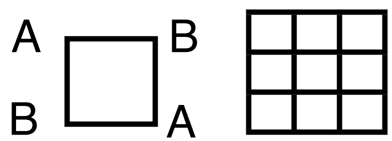
- Symmetries:
  - Translations
  - global phase rotations U(1)
  - global spin rotations SU(2) for $\Delta_z = \Delta_\pm/2$,

  d-wave is an eigenstate to these

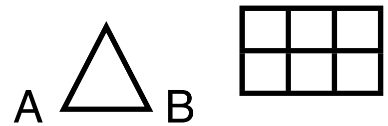
  - additional discrete symmetry on bipartite lattice for $\Delta_z = 0$ spoils uniqueness

$$T_d: \quad c_{i,\uparrow} \to -c_{i,\uparrow}; \quad c_{i,\downarrow} \to c_{i,\downarrow} \text{ for } i \in A,$$
$$c_{i,\uparrow} \to c_{i,\uparrow}; \quad c_{i,\downarrow} \to c_{i,\downarrow} \text{ for } i \in B$$

bipartite (periodic BC)



not bipartite (PBC)



SU(2) symmetry;
the jump operators
are SU(2) vectors

$$[S^\alpha, J_i^\beta] = \mathrm{i}\epsilon_{\alpha\beta\gamma} J_i^\gamma \quad \forall i$$

➡ Avoid symmetries

➡ All three operators needed for uniqueness

Comments on the effective Hamiltonian

- Amusing parallel: Above Hamiltonian is a parent Hamiltonian for the d-wave state

$$H_\Delta = \sum_{i,\alpha} \Delta_\alpha J_i^{\alpha\,\dagger} J_i^\alpha = \sum_i h_i$$

  - H is semi-positive
  - an exact unique GS is the above d-wave state(E=0)
  - GS is GS for each h_i separately: projectors on GS

➡ completely analogous to e.g. AKLT model

➡ there, ground state is valence bond solid with exponentially decaying correlations

➡ different: state has long range order due to strong delocalization

➡ study excitations

  - mean field decoupling

$$\Delta_+ \sum_i J_i^{+\,\dagger} J_i^+ = \Delta_+ \sum_i c^\dagger_{i,\downarrow}(c_{i+1,\uparrow} + c_{i-1,\uparrow})(c^\dagger_{i+1,\uparrow} + c^\dagger_{i-1,\uparrow})c_{i,\downarrow} = \Delta_+ \sum_i c^\dagger_{i,\downarrow}(c^\dagger_{i+1,\uparrow} + c^\dagger_{i-1,\uparrow})c_{i,\downarrow}(c_{i+1,\uparrow} + c_{i-1,}$$

$$\approx \sum_{\mathbf{q}} \underbrace{\Delta^+ \cos\mathbf{q}}_{\text{single fermion gap}}\; c_{\mathbf{q},\downarrow} c_{-\mathbf{q},\uparrow} + h.c.$$

order parameter-like structure:
macroscopically populated
-> replace by c-number mean field
(-> loose particle number cons.)

  - "diagonal" contributions $\sim c^\dagger_{\mathbf{q}} c_{\mathbf{q}}$ from normal ordering J_i^z

➡ single fermion excitations are gapped: important for adabatic passage

Arbitrary phase coherent pairing states

- Any pairing product state can be characterized by 3 quantum numbers

$$O^{\dagger\,N}_{k,n,\mu}|\mathrm{vac}\rangle, \quad O^\dagger_{k,n,\mu} = \sum_i \exp ikx_i \; c^\dagger_{i+n}\sigma^\mu c^\dagger_i \qquad \sigma^\mu = (\mathbf{1}, \sigma^\alpha)$$

pairing momentum

pairing distance

- Examples:

$k=0,\; n=0,\; \mu=2$ s-wave BCS

$k=\pi,\; n=0,\; \mu=2$ eta-state

$k=0,\; n=1,\; \mu=2$ d-wave like state

- Jump operators constructed for all k, mu, and n >0 (displayed just for completeness...)

$$\begin{aligned}
\mu &= 0 : (c^\dagger_{i+n} - e^{ik} c^\dagger_{i-n})(\mathbf{1} \pm \sigma^z) c^\dagger_i, \; (c^\dagger_{i+n} - e^{ik} c^\dagger_{i-n})\sigma^y c^\dagger_i \\
\mu &= 1 : (c^\dagger_{i+n} - e^{ik} c^\dagger_{i-n})\sigma^\pm c^\dagger_i, \; (c^\dagger_{i+n} + e^{ik} c^\dagger_{i-n})\mathbf{1} c^\dagger_i \\
\mu &= 2 : (c^\dagger_{i+n} + e^{ik} c^\dagger_{i-n})\sigma^\pm c^\dagger_i, \; (c^\dagger_{i+n} + e^{ik} c^\dagger_{i-n})\sigma^z c^\dagger_i \\
\mu &= 3 : (c^\dagger_{i+n} - e^{ik} c^\dagger_{i-n})(\mathbf{1} \pm \sigma^z) c^\dagger_i, \; (c^\dagger_{i+n} - e^{ik} c^\dagger_{i-n})\sigma^x c^\dagger_i
\end{aligned}$$

➡ arbitrary n > 0 pairing states can be targeted

➡ d-wave not distinguished, but off-site pairing special

➡ symmetries of the state inherited by the parent Hamiltonian

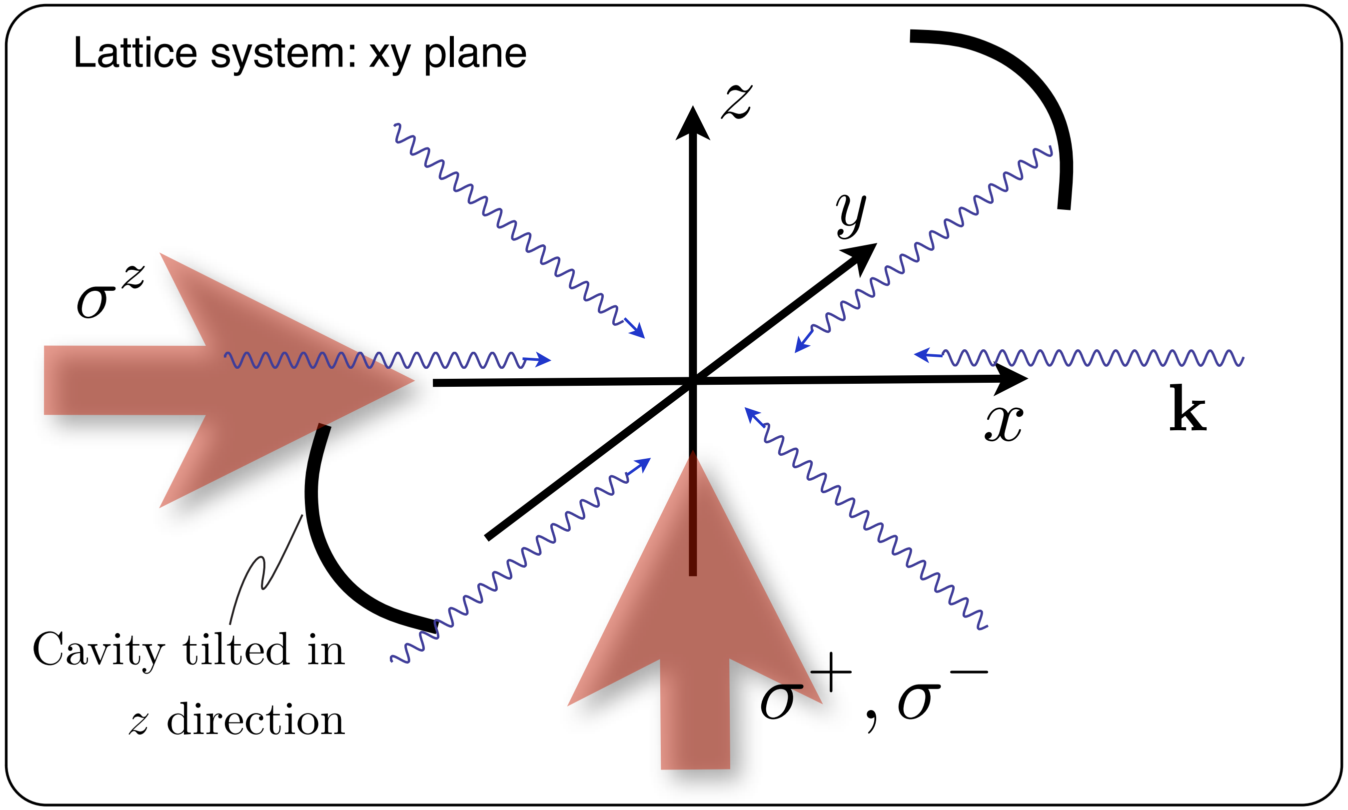
Implementation of d-wave jump operators

- Decisive property: single-particle nature of the jump operators
- Implement Fourier transformed operators:

$$\mathcal{L}[\rho] = \sum_{\alpha,i} J_i^\alpha \rho J_i^{\alpha\dagger} - \tfrac{1}{2}\{J_i^{\alpha\dagger} J_i^\alpha, \rho\} = \sum_{\alpha,\mathbf{k}} J_\mathbf{k}^\alpha \rho J_\mathbf{k}^{\alpha\dagger} - \tfrac{1}{2}\{J_\mathbf{k}^{\alpha\dagger} J_\mathbf{k}^\alpha, \rho\}$$

$$J_\mathbf{k}^\pm = \sum_\mathbf{q} \cos \mathbf{q}\, a_\mathbf{q}^\dagger \sigma^\pm a_{\mathbf{q}-\mathbf{k}}, \qquad J_\mathbf{k}^z = \sum_\mathbf{q} \cos \mathbf{q}\, a_\mathbf{q}^\dagger \sigma^z a_{\mathbf{q}-\mathbf{k}}$$

- Basic physical ingredients:
  - Dissipation: Emission in cavity
  - Use Earth Alkaline atoms in state dependent superlattice
- Engineering requirements:
  - Spin imprinting: Light Polarization
  - Momentum transfer: Laser angle (incoherent beams)
  - cos q dependence: Quantum Interference



Implementation of d-wave jump operators

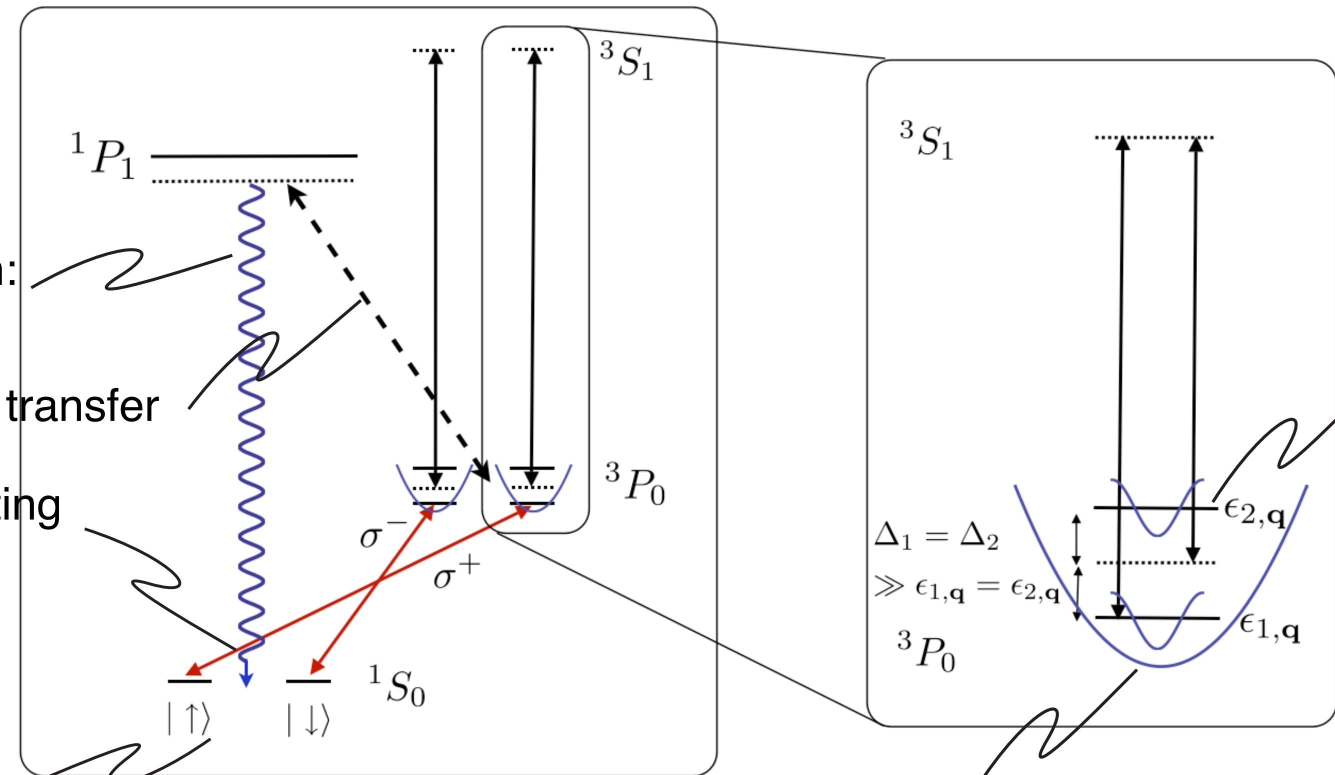
- Level scheme: Earth Alkaline atoms

spont. emission: cavity mode

momentum transfer

spin imprinting

physical spin



atom confinement via optical lattice

Bloch bands

Quantum Interference:

$$\frac{1}{\Delta + \epsilon_{\mathbf{q}}} + \frac{1}{-\Delta + \epsilon_{\mathbf{q}}} = \frac{2\epsilon_{\mathbf{q}}}{-\Delta^2 + \epsilon_{\mathbf{q}}^2} \approx -2\frac{\epsilon_{\mathbf{q}}}{\Delta^2}$$

cos q: onsite processes interfere destructively

$$J_{\mathbf{k}}^{\pm} = \sum_{\mathbf{q}} \cos \mathbf{q} \, a_{\mathbf{q}}^{\dagger} \sigma^{\pm} a_{\mathbf{q}-\mathbf{k}}$$

Adapted Adiabatic Passage

$$H \overset{?}{=} 0, \quad \mathcal{L} \neq 0$$

Adapted adiabatic passage $\downarrow$

$$H = H_{FH}, \quad \mathcal{L} = 0$$

- Assume we have prepared zero entropy d-wave
- Want to connect to Hubbard ground state
- Adiabatic passage (purely Hamiltonian dynamics):

$$H = \lambda(t) H_\Delta + (1-\lambda(t)) H_{\mathrm{FH}},$$
$$\lambda(t_{\mathrm{in}}) = 1, \ \lambda(t_{\mathrm{fin}}) = 0$$

ramping slowly: remain in ground state

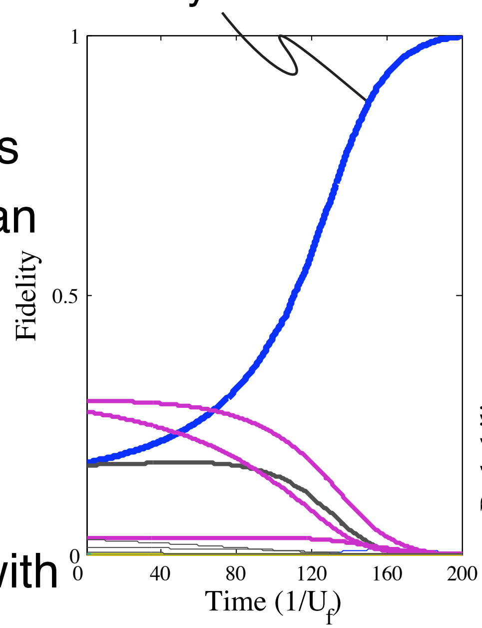
➡ Adapted adiabatic passage: Two ingredients

- gap protection from auxiliary Hamiltonian
  - parent Hamiltonian has d-wave eigenstate and is gapped: add detuning to the effective Hamiltonian

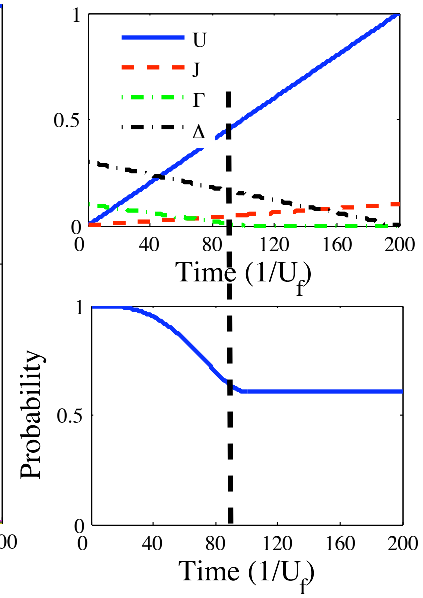
$$\gamma \to \gamma + \mathrm{i}\Delta$$

- probabilistic ground state preparation
  - dissipative and Hubbard dynamics compete
  - focus on time before first jump: state prepared with probability

fidelity to Hubbard GS



ramp parameters



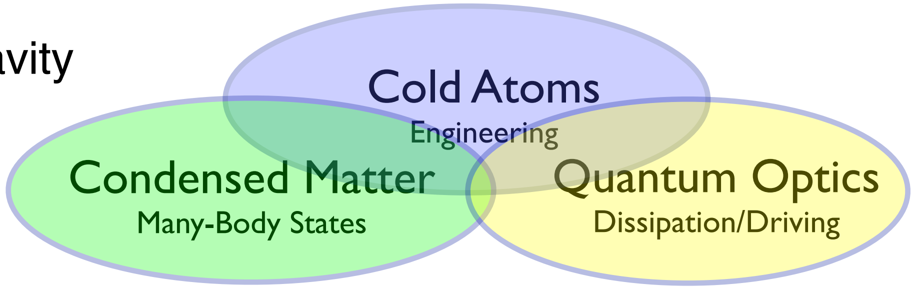
preparation probability

Summary Part I

By merging techniques from quantum optics and many-body systems:

Driven dissipation can be used as controllable tool in cold atom systems.

- Pure states with long range correlations from quasilocal dissipation
  - Many-body dark state, independent of initial density matrix
  - Laser coherence mapped on matter system
  - System steady state has zero entropy
- Nonequilibrium phase transition driven via competition of unitary and dissipative dynamics
  - driven by interactions (like quantum phase transition)
  - terminates into thermal state (like classical phase transition)
- Strong potential applications for fermionic quantum simulation
  - cool into zero entropy d-wave state as intial state for Fermi-Hubbard model
  - single particle operations due to Pauli blocking
  - realistic setting using earth alkaline atoms in a cavity



Optical Lattices

- AC-Stark shift
  - Consider an atom in its electronic ground state exposed to laser light at fixed position $\vec{x}$.
  - The light be far detuned from excited state resonances: ground state experiences a second-oder *AC-Stark shift*

$$\delta E_g = \alpha(\omega) I$$

with $\alpha(\omega)$ - dynamic polarizability of the atom for laser frequency ω, $I \propto \vec{E}^2$ - light intensity.

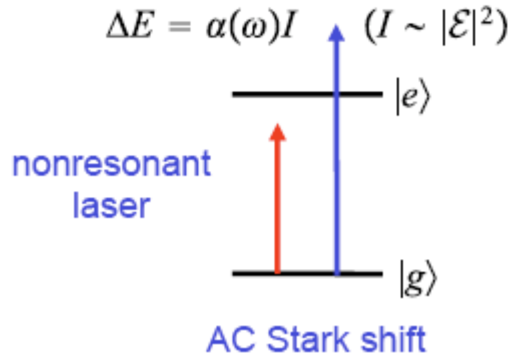
  - Example: two-level atom $\{|g\rangle, |e\rangle\}$.

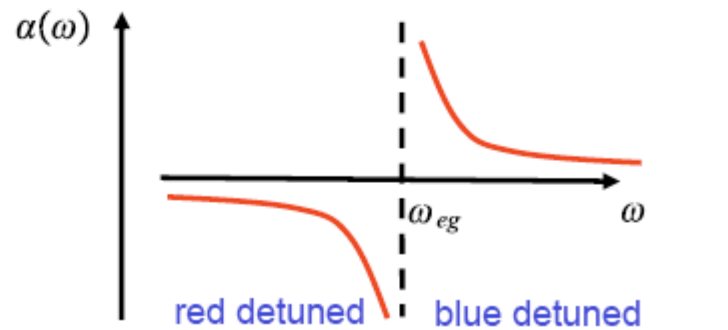
Rabi frequency

$$\delta E_g = \hbar \frac{\Omega^2}{4\Delta}$$

detuning from resonance $\Delta = \omega - \omega_{eg}$

$\Omega \ll \Delta$

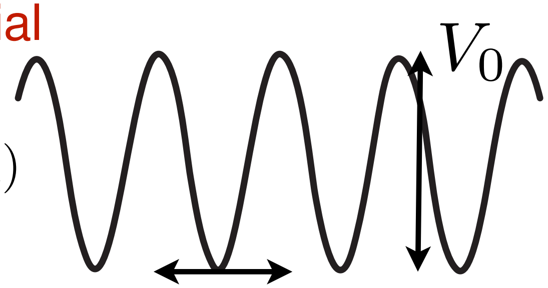




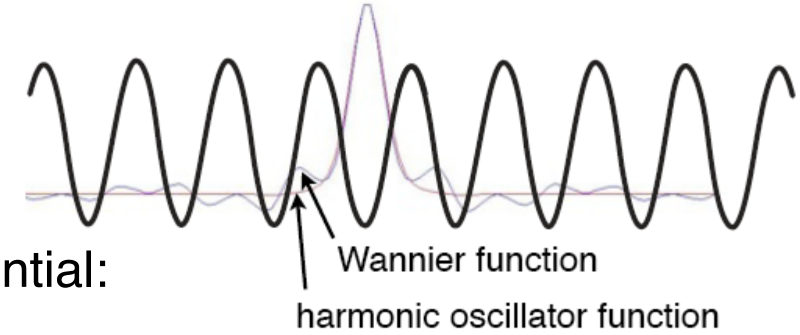
$\Delta < 0,\ \delta E_g < 0$ $\Delta > 0,\ \delta E_g > 0$

- For standing wave laser configuration $\vec{E}(\vec{x}, t) = \vec{e}\mathcal{E} \sin kx\, e^{-i\omega t} + \text{h.c.}$, AC-Stark shift is a function of position: It generates an optical potential

$$V_{\text{opt}}(\vec{x}) \equiv \delta E_g(\vec{x}) = \hbar \frac{\Omega^2(\vec{x})}{4\Delta} \equiv V_0 \sin^2 kx \qquad (k = 2\pi/\lambda)$$



Effective Lattice Hamiltonian



- Start from our model Hamiltonian, add optical potential:

$$H = \int_{\mathbf{x}} [a^\dagger_{\mathbf{x}}(-\frac{\triangle}{2m} - \mu + V(\mathbf{x}) + V_{\mathrm{opt}}(\mathbf{x})\ )a_{\mathbf{x}} + g\hat{n}^2_{\mathbf{x}}]$$

- Periodicity of the optical potential suggests expansion of field operators into localized lattice periodic Wannier functions (complete set of orthogonal functions)

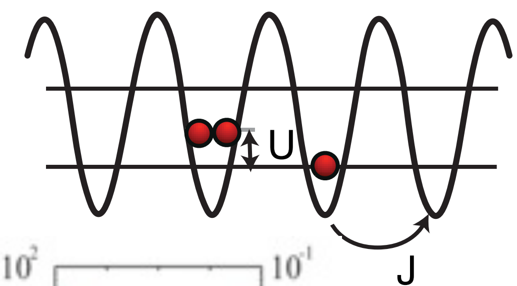
$$a_{\mathbf{x}} = \sum_{i,n} w_n(\mathbf{x} - \mathbf{x}_i) b_{i,n}$$

band index

minimum position

- For low enough energies (temperature), we can restrict to lowest band:

$$T, U, J \ll \sqrt{4V_0 E_R}, E_R = k^2/(2m) \rightarrow n = 0$$



- Then we obtain the single band Bose-Hubbard model

$$H = -J \sum_{\langle i,j \rangle} b^\dagger_i b_j - \mu \sum_i \hat{n}_i + \sum_i \epsilon_i \hat{n}_i + \tfrac{1}{2} U \sum_i \hat{n}_i(\hat{n}_i - 1)$$

$$\hat{n}_i = b^\dagger_i b_i$$

$$J = -\int dx w^*_0(x)(-\hbar^2/2m\triangle - V_{\mathrm{opt}}(x)) w_0(x - \lambda/2)$$

$$U = g \int dx |w_0(x)|^4$$