Quantum States and Phases in Dissipative Many-Body Systems with Cold Atoms

Cold Atoms
Engineering

Condensed Matter
Many-Body States

Quantum Optics
Dissipation/Driving

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Lecture Overview

Main theme:
Dissipation can be turned into a favorable, controllable tool in cold atom many-body systems.

Part I: Quantum State Engineering in Driven Dissipative Many-Body Systems

• Proof of principle: Driven Dissipative BEC
• Application I: Nonequilibrium phase transition from competing unitary and dissipative dynamics
• Application II: Cooling into antiferromagnetic and d-wave states of fermions


Part II: Dissipative Generation and Analysis of 3-Body Hardcore Models

• Mechanism
• Experimental prospects, ground state preparation
• Application I: phase diagram for attractive 3-hardcore bosons
• Application II: atomic color superfluid for 3-component fermions

Outline Part I:
Quantum State Engineering in Driven Dissipative Many-Body Systems

• Introduction: Open Systems in Quantum Optics

• Driven Dissipative BEC:
  - Mechanism for pure DBEC: Many-Body Quantum Optics
  - Physical Implementation of DBEC: Reservoir Engineering, Bogoliubov bath

• Application I: Competition of unitary vs. dissipative dynamics
  - first look: weak interactions
  - strong interactions: nonequilibrium phase transition

• Application II: Targeting pure fermion states
  - An excited many-body state: η-condensate
  - Antiferromagnetic and d-wave fermion states

References:
SD, A. Micheli, A. Kantian, B. Kraus, H.P. Büchler, P. Zoller, Nature Physics 4, 878 (2008);
Quantum State Engineering in Many-Body Systems

- **thermodynamic equilibrium**
  - standard scenario of condensed matter & cold atom physics

\[ H |E_g\rangle = E_g |E_g\rangle \quad \rho \sim e^{-H/k_B T} \overset{T \to 0}{\longrightarrow} |E_g\rangle \langle E_g| \]

Hamiltonian (many body)  cooling to ground state

- Hamiltonian Engineering:
  ✓ interesting ground states
  ✓ quantum phases

- **driven / dissipative dynamical equilibrium**
  - quantum optics

\[ \frac{d\rho}{dt} = -i [H, \rho] + \mathcal{L}\rho \]

competing dynamics

\[ \rho(t) \overset{t \to \infty}{\longrightarrow} \rho_{ss} \]

steady state

Liouvillian Engineering:
  ✓ many body pure states / driven quantum phases
  ✓ mixed states ~ “finite temperature”
  ✓ useful an interesting fermion states
Open Quantum Systems
Open Quantum Systems

\[ H = H_S + H_B + H_{\text{int}} \]

\[ H_B = \int d\omega \, \omega b_\omega^\dagger b_\omega \] continuum bath of harmonic oscillators

\[ H_{\text{int}} = i \int d\omega \kappa(\omega) \left[ b_\omega^\dagger J - b_\omega J^\dagger \right] \] linear bath operator coupling to the system

Three approximations:

1. Born approximation: \( \kappa(\omega) / \omega_0 \ll 1 \)
2. Markov approximation: \( \kappa(\omega) \approx \text{const.} \Rightarrow \kappa(t - t') \sim \delta(t - t') \)
3. Rotating wave approximation: \( \frac{\omega_0 - \nu}{\omega_0 + \nu} \ll 1 \)

\( \omega_0 - \nu = \Delta \) detuning

\[ H = (|e\rangle, |g\rangle) \left( \begin{array}{cc} \Delta & \Omega \\ \Omega & 0 \end{array} \right) \left( \begin{array}{c} \langle e| \\ \langle g| \end{array} \right) \]

\( \omega_0 \)

\( \kappa(\omega) \rightarrow \kappa_0 \)

\( \omega_0 - \vartheta \) reservoir bandwidth

\( \omega_0 + \vartheta \) system frequency
Open Quantum Systems

\[ \partial_t \rho_{\text{tot}} = -i[H_S + H_B + H_{\text{int}}, \rho_{\text{tot}}] \]

Eliminate bath degrees of freedom in second order time-dependent perturbation theory (Born approximation)

\[ \text{Tr}_{\text{bath}} \left\{ \begin{array}{c} \text{system} \\ \text{bath} \end{array} \right\} \]

effective system dynamics from Master Equation (zero temperature bath)

\[ \partial_t \rho = -i[H_S, \rho] + \kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger - \frac{1}{2} \{J_\alpha^\dagger J_\alpha, \rho\} \]

\[ \mathcal{L}[\rho] \] Liouvillian operator in Lindblad form

- Structure: second order perturbation theory
- mnemonic: norm conservation \( \partial_t \text{tr} \rho = 0 \)
- but: \( \partial_t \text{tr} \rho^2 \neq 0 \)

\( \Rightarrow \) Purity is not conserved
\( \Rightarrow \) go for \( \partial_t \text{tr} \rho^2 < 0 \)

pure state: \( \text{tr} \rho = \text{tr} \rho^2 = 1 \)

\[ \Rightarrow \text{tr} \rho^2 \quad \text{-- “purity”} \]
Open Quantum Systems

- Stochastic Interpretation: Quantum Jumps

\[ \partial_t \rho = -i[H, \rho] + \kappa \sum_{\alpha} J_{\alpha} \rho J_{\alpha}^\dagger - \frac{1}{2} \{ J_{\alpha}^\dagger J_{\alpha}, \rho \} \]

\[ = -i[H_{\text{eff}}, \rho]^* + \kappa \sum_{\alpha} J_{\alpha} \rho J_{\alpha}^\dagger \quad H_{\text{eff}} = H - i\kappa/2 \sum_{\alpha} J_{\alpha}^\dagger J_{\alpha} \]

\[ J_{\alpha} = |g\rangle \langle e| = \sigma^- \]

quantum jump operators

decay

damped Rabi oscillations

time evolution of upper state population of driven dissipative two-level system (single run)

- Averaging over “quantum trajectories” generates all correlation functions

⇒ Engineer the jump operators \( J_{\alpha} \)

\[ [A, B]^* := AB - B^\dagger A^\dagger \]
Driven Dissipative BEC
Dark States in Quantum Optics

• Goal: pure BEC as steady state solution, independent of initial density matrix:

\[ \rho(t) \longrightarrow |BEC\rangle\langle BEC| \text{ for } t \rightarrow \infty \]

• Such situation is well-known quantum optics (three level system): optical pumping
  (Kastler, Aspect, Cohen-Tannoudji; Kasevich, Chu; ...)

\[ |g_+\rangle \quad \text{is a "dark state" decoupled from light} \]

\[ c_\alpha |g_+\rangle = 0 \]

\[ \Rightarrow \text{Dark state is Eigenstate of jump operators with zero Eigenvalue} \]

\[ \Rightarrow \text{Time evolution stops when system is in DS: pure steady state} \]
An Analogy

- **Λ-system: three electronic levels** (VSCPT by Aspect, Cohen-Tannoudji; Kasevich, Chu)

- **1 atom on 2 sites**

  \[(a_1^+ + a_2^+) |\text{vac}\rangle \quad (a_1^+ - a_2^+) |\text{vac}\rangle\]

  symmetric "in-phase"  
  anti-symmetric "out-of-phase"

  pumping into symmetric state

  ➡  "phase locking": like a BEC
Driven Dissipative lattice BEC

- Consider jump operator:

\[ c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j) \]

(1) BEC state is a dark state: \[ |BEC\rangle = \frac{1}{N!} \left( \sum_\ell a_\ell^\dagger \right)^N |\text{vac}\rangle \]

\[ c_{ij} |BEC\rangle = 0 \forall i \]

(2) BEC state is the only dark state:

- \((a_i^\dagger + a_j^\dagger)\) has no eigenvalues
- \((a_i - a_j)\) has unique zero eigenvalue

\[ (a_i - a_j) \forall i \rightarrow (1 - e^{i q e \lambda}) a_q \forall q \]
(3) **Uniqueness**: IBEC\(>\) is the only stationary state (sufficient condition)

If there exists a stationary state which is not a dark state, then there must exist a subspace of the full Hilbert space which is left invariant under the set \(\{c_\alpha\}\)

(4) **Compatibility** of unitary and dissipative dynamics

\[ |D\rangle \text{ be an eigenstate of } H, \quad H |D\rangle = E |D\rangle \]

\[ \rho(t) \xrightarrow{t \to \infty} |D\rangle \langle D| \]

- **Long range** order in many-body system from quasi-local dissipative operations
- **Uniqueness**: Final state independent of initial density matrix
- **Criteria** are general: jump operators for AKLT states (spin model), eta-states (fermions), d-wave states (fermions, next lecture)
Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission

\[ \omega \sim 2\pi \times 10^{14}\text{Hz} \]

- reservoir: vacuum modes of the radiation field (T=0)

- much lower energy scales...

Quantum optics ideas/techniques
Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission

- reservoir: vacuum modes of the radiation field (T=0)

- much lower energy scales...

Quantum optics ideas/techniques

(many body) cold atom systems

\[ \omega \sim 2\pi \times 10^{14} \text{Hz} \]
Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission
  - laser
  - atom
  - photon
  -光学光子
  - \( |g\rangle \)
  - \( |e\rangle \)
  - \( \Omega \)
  - \( \Gamma \)

- reservoir: vacuum modes of the radiation field (T=0)

- \( \omega \sim 2\pi \times 10^{14}\text{Hz} \)

- trapped atom in a BEC reservoir
  - BEC
  - “phonon”
  - laser assisted atom + BEC collision
  - \( \omega_{bd} \sim 2\pi \times k\text{Hz} \)

A. Griessner, A. Daley et al. PRL 2006; NJP 2007

(noninteracting atom)
Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission
- reservoir: vacuum modes of the radiation field (T=0)
  \( \omega \sim 2\pi \times 10^{14} \text{Hz} \)
- trapped atom in a BEC reservoir
- reservoir: Bogoliubov excitations of the BEC (at temperature T)
  \( \omega_{bd} \sim 2\pi \times k \text{Hz} \)

A. Griessner, A. Daley et al. PRL 2006; NJP 2007 (noninteracting atom)
Physical Realization

Schematic

Rabi frequency

\[ Ω b^\dagger (a_1 - a_2) + h.c. \]

antisymmetric

\[ c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j) \]

In practice

- level structure: optical superlattice
- coherent excitation: Raman laser

(1) Coherent excitation with opposite sign of Rabi frequency
Dissipative decay back: coupling of upper level to reservoir

\[ \kappa (a_1^\dagger + a_2^\dagger) b \sum_k (r_k + r_k^\dagger) \]

symmetric

\[ c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j) \]

- coupling to system: interspecies interaction
- short coherence length in bath provides quasi-local dissipative processes, but not mandatory for our setup to work

BEC = reservoir of Bogoliubov excitations

\[ T_{BEC} \ll \omega_{bd} \text{ effective zero temperature reservoir} \]
Physical Realization

(3) adiabatic elimination of auxiliary level, trace out the bath

Effective single band jump operators

\[ c_{12} = (a^\dagger_1 + a^\dagger_2)(a_1 - a_2) \]

Comments:

- Long range phase coherence from quasi-local dissipative operations
- Coherent drive: locks phases
- Dissipation: randomizes
- Conspiracy: purification
- The coherence of the driving laser is mapped on the matter system
- Setting is therefore robust

Many sites: Array of dissipative junctions
Competition of unitary vs. dissipative dynamics
Effects of finite interactions

- dissipative dynamics favors pure BEC state
- interacting Hamiltonian dynamics not compatible

\[ H = -J \sum_{<i,j>} a_i^\dagger a_j + U \sum_i a_i^\dagger a_i^2 \]

\[ \frac{d\rho}{dt} = -i [H, \rho] + \mathcal{L}\rho \]

treating interactions in
- weak coupling
  - 3D: true (depleted) condensate, fixed phase: Bogoliubov theory
  - 1,2D: phase fluctuations destroy long range order: Luttinger theory
- Strong coupling, 3D
  - mixed state Gutzwiller Ansatz
Weak Coupling: Linearized jump operators

- momentum space jump operators are nonlocal nonlinear objects

\[ c_{q,\lambda} = \frac{1}{M^{d/2}} \sum_{k} (1 + e^{i k e_{\lambda}})(1 - e^{-i(k+q)e_{\lambda}}) a_{k}^{\dagger} a_{k+q} \]

- In a linearized theory the reduce to (any dimension)

\[ c_{q,\lambda} = f_{q,\lambda} a_{q} \quad f_{q,\lambda} = 2\sqrt{n}(1 - e^{-i q e_{\lambda}}) \]

- Interpretation:
  - bosonic mode operators: depopulation of momentum q in favor of condensate
  - zero mode explicit: \( f_{q=0,\lambda} = 0 \)
  - lead to momentum dependent decay rate

\[ \kappa_{q} = \sum_{\lambda} \kappa |f_{q,\lambda}|^2 \sim q^2 \]
Many-Body Master Equation

- Interpretation: How close are we to the GS of the Hamiltonian?
  - Diagonalize $H$
  - consider equation for single mode

Bogoliubov / hydrodynamic excitation

$$\partial_t \rho = -\frac{i}{2} [d^\dagger d, \rho]$$

$$+ 2\kappa (u^2 \rho d^\dagger + v^2 d^\dagger \rho d - uv(d^\dagger \rho d^\dagger + d \rho d))$$

“cooling” “heating” squeezing

$v_q^2, u_q^2 = v_q^2 + 1$ generalized Bogoliubov coefficients

$N, \quad N + 1$ cf. thermal reservoir

$\Rightarrow$ Intrinsic heating/cooling, though reservoir is at $T = 0$
Characterization of Steady State: Density Operator

- linearized ME exactly solvable: **Gaussian density operator** for each mode expressible as

\[
\rho_k = \exp \left( -\beta_k b_k^\dagger b_k \right)
\]

with squeezed operators \( b \) (Bogoliubov transformation)

➡ **mixed state** with

\[
\text{coth}^2 \left( \frac{\beta_k}{2} \right) = \frac{\kappa_k^2 + (\varepsilon_k + Un)^2}{\kappa_k^2 + E_k^2}
\]

- at low momenta, resemblance to **thermal state**:

\[
\beta_k \approx \frac{E_k}{T_{\text{eff}}}, \quad T_{\text{eff}} = \frac{Un}{2}
\]

▷ role of **temperature** played by interaction
Correlations in various dimension: 3D

- Steady state: condensate depletion:

\[ n_D = n - n_0 = \frac{1}{2} \int \frac{dq}{v_0} \frac{(Un)^2}{\kappa_q^2 + E_q^2} \]

- small depletion justifies Bogoliubov theory
- squeezing and mixing effects tied to interaction strength (unlike th. equilibrium)

- Approach to the steady state:

\[ n_{0,eq} - n_0(t) \sim \sqrt{\frac{Un}{8J}} \frac{1}{2\kappa n} t^{-1} \]

- power-law: Many-body effect due to mode continuum
- sensitive probe to interactions: cf. for noninteracting system

\[ n_{0,eq} - n_0(t) \sim t^{-3/2} \]

- universal at late times
Correlations in various dimension: 1/2D

• Steady State: quasi-condensates in low “temperature” phase

\[ \langle a_x^\dagger a_0 \rangle \sim \langle \exp i(\phi_x - \phi_0) \rangle \sim \begin{cases} e^{-\frac{T_{\text{eff}}}{8J_n}x}, & d = 1 \\ (x/x_0)^{-\frac{T_{\text{eff}}}{4T_{\text{KT}}}}, & d = 2 \end{cases} \]

\[ T_{\text{KT}} = \pi J_n \gg T_{\text{eff}} \quad T_{\text{eff}} = \frac{Un}{2} \quad x_0 = 2\kappa_n(T_{\text{eff}}J)^{-1/2} \]

Kosterlitz-Thouless temperature of 2D quasi-condensate

Dissipative coupling: only sets cutoff scale

• steady state well understood as thermal Luttinger liquid
• similar results for temporal correlations (from ME via quantum regression theorem)
• weak effect of dissipation on phase fluctuations:

\[ E_q \sim |q|, \kappa_q \sim q^2 \]
2D: Real Time Evolution

- Buildup of spatial correlations from disordered state

\[ \Psi_t(x,0) \sim \begin{cases} 
    e^{-|x|/\xi} & t = 0 \\
    \left(\frac{x}{x_0}\right)^{-\frac{1}{4}} \frac{T_{\text{eff}}}{4TK} e^{-\frac{x^2}{4\xi^2 \pi \kappa n t}} & t \to \infty 
\end{cases} \]

broadening of Gaussian governed by time-dependent length scale

\[ x_t = 2(\pi \xi^2 \kappa n t)^{1/4} \]
Strong Coupling: Nonequilibrium Phase Transition

• Analogy to Mott insulator / Superfluid quantum phase transition:
  - enhancement of superfluidity: Hopping $J$ driven dissipation $\kappa$
  - suppression of superfluidity: interaction $U$

  ➡ Expect phase transition as function of $J/U$, $\kappa/U$

• Differences:
  - Competition of two unitary evolutions vs. competition of unitary and dissipative evolution

  ✓ phase transition (temperature $T$)
  ✓ quantum phase transition ($g$)
Reminder: Mott Insulator-Superfluid Phase Transition

\[ H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j - \mu \sum_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i(\hat{n}_i - 1) \]

- Hopping J favors \textit{delocalization} in real space:
- Condensate (local in momentum space!)
- Fixed condensate phase: Breaking of phase rotation symmetry
- Interaction U favors \textit{localization} in real space for integer particle numbers:
  - Mott state with quantized particle no.
  - no expectation value: phase symmetry intact (unbroken)

\[ \langle b_i \rangle \sim e^{i\varphi} \]

\[ \rightarrow \text{Competition gives rise to a quantum phase transition as a function of} \]

\[ U/J \]
Reminder: Gutzwiller Ansatz

- Interpolation scheme encompassing the full range \( J/U \).
  - Main ingredient: product wave function ansatz

\[
|\psi\rangle = \prod_i |\psi_i\rangle, \quad |\psi_i\rangle = \sum_n f_n^{(i)} |n_i\rangle, \quad i \langle \psi | \psi_i \rangle = 1 \forall i
\]

- Limiting cases (homogeneous, drop site index, amplitudes chosen real):
  - Mott state with particle number \( m \): \( f_n = \delta_{n,m} \)
  - Coherent state: \( f_n = \sqrt{N/n!} e^{-N/2} \)

- Validity: approximation neglects all spatial correlations
  - becomes exact in infinite dimensions
  - reasonable in \( d=2,3 \) (T=0)
Mixed State Gutzwiller Approach

- Product ansatz for the density operator (instead of wave function)
  \[ \rho(t) = \prod_i \rho_i(t), \quad \rho_i(t) = \sum_{nm} |n_i\rangle \langle m| \rho^{(i)}_{nm}(t) \]
  Interpretation:
  ✓ off-diagonal: SF
  ✓ diagonal: atom statistics

- Project on on-site density operator:
  \[ \rho_k = \text{Tr}_{\neq k} \rho \]
  ➞ **Nonlinear** Mean Field Master Equation for reduced density operator (drop index)
  \[ \dot{\rho} = -i \left[ -ZJ(\langle b \rangle b^\dagger + \langle b^\dagger \rangle b) + \frac{1}{2} U b^\dagger b^2, \rho \right] \]
  \[ + Z\kappa \sum_{r,r'} \Gamma^{r,r'} \left\{ 2B^r \rho B^\dagger_{r'} - B^\dagger_{r'} B^r \rho - \rho B^\dagger_{r'} B^r \right\} \]

  with correlation matrix
  \[ \Gamma^{r,r'} = \begin{bmatrix}
  \langle \hat{n}^2 \rangle & \langle \hat{n}^\dagger \hat{n} \rangle & -\langle \hat{n} \hat{n} \rangle & -\langle \hat{n} \rangle \\
  \langle \hat{n} \hat{b} \rangle & \langle \hat{n} \rangle & -\langle \hat{b}^2 \rangle & \langle \hat{b} \rangle \\
  -\langle \hat{n} \hat{b}^\dagger \rangle & -\langle \hat{b}^\dagger b^2 \rangle & \langle \hat{n} \rangle + 1 & \langle \hat{b}^\dagger \rangle \\
  -\langle \hat{n} \rangle & -\langle \hat{b}^\dagger \rangle & \langle \hat{b} \rangle & \langle \mathbf{1} \rangle 
  \end{bmatrix} \]

  Properties of ME:
  ✓ trace conserving
  ✓ mean particle number conserving

- Nonlinearity emerging in approximation to linear qm equation: similar GP equation
Driven Dissipative Phase Transition

- Dynamic generation of the phase transition from initial coherent state

\[ \rho_{n,n} = \frac{\bar{n}^n}{(\bar{n} + 1)^{n+1}} \]

\[ \bar{n} = 1, J = 0, zKt = 0, 10^{-1}, \ldots, 10^2 \]

- \( U \to 0 \) pure coherent state solution
- Phase transition: Non-analyticity develops for \( t \to \infty \)
- above critical point: thermal state: “fixed temperature” given by mean particle density \( N \); no other scale appears
- No signatures of Mott physics due to strong mixing effect of \( U \): unlike Bose-Hubbard case of two unitary tendencies at \( T=0 \):
Exact calculations for N=6 sites

\[ \frac{S_N}{\log L_N} \]

\[ \frac{1}{2\pi} \left| \left\langle a_1^\dagger a_2 \right\rangle \right| \]

interaction U

\[ n = 1, J = 0 \text{ and p.b.c.} \]

\[ U/zK \]

\( n = 1, J = 0 \text{ and p.b.c.} \)

\( U/zK \)

A. Tomadin

Thursday, October 22, 2009
Nonequilibrium Phase Diagram

- **U/K transition:**
  - *interaction driven* (like quantum PT)
  - terminates in *thermal state* (like classical finite temperature PT)
(Nonequilibrium Phase Diagram)

- **Add negative J** (via phase imprinting): further competition through dynamical instability
  - no stable equilibrium state (no dynamical fixed point)
  - dynamical limit cycle?

- **Classification?**

**Figure:**

- **Initialization:** Coherent state, U=J=0
- follow time evolution of the system

**Curvature of Dispersion:**

- **dynamically unstable:** negative curvature
- **dynamically stable:** positive curvature
Dissipative Driving of Fermions

- Excited states: $\eta$ Condensate
- Cooling into Antiferromagnetic and d-Wave States
Cooling to Excited States: $\eta$-Condensate

- $\eta$-state: exact excited (i.e. metastable) eigenstate of the two-species Fermi Hubbard Hamiltonian in $d$ dimensions [Yang ’89]

$$H = -J \sum_{\langle i, j \rangle, \sigma} f_{i\sigma}^\dagger f_{j\sigma} + U \sum_i f_{i\uparrow}^\dagger f_{i\downarrow}^\dagger f_{i\downarrow} f_{i\uparrow}$$

- local “doublon” $\eta_i^\dagger = f_{i\uparrow}^\dagger f_{i\downarrow}^\dagger$

- checkerboard superposition $\eta$-particle

$$\eta^\dagger = \frac{1}{M^{d/2}} \sum_i \phi_i \eta_i^\dagger \quad \phi_i = \pm 1$$

- $N$-$\eta$-condensate:

$$H(\eta^\dagger)^N |0\rangle = N U (\eta^\dagger)^N |0\rangle$$

exact eigenstate, off-diagonal long range order
Cooling to Excited States: $\eta$-Condensate

- Small scale simulations (open BC) demonstrate $\eta$ condensation for jumps

\[ c_{ij}^{(1)} = (\eta_i^\dagger - \eta_j^\dagger)(\eta_i + \eta_j) \]

\[ c_{ij}^{(2)} = n_{i\uparrow} f_{i\downarrow}^\dagger f_{j\downarrow} + n_{j\uparrow} f_{j\downarrow}^\dagger f_{i\downarrow} \]

- Interpretation: Quantum Jump picture

- $H$ generates spin-up and down configurations on each pair of sites (for any initial density matrix)

- $c_{ij}^{(2)}$ associates into local doublons

- $c_{ij}^{(1)}$ creates checkerboard superposition: $\eta$ condensate

- May be conceptually interesting

- However, these jump operators are two-body: difficult to engineer
Motivation: Cooling Fermion Systems

- High temperature superconductivity
  - discovered in 1986 (Müller, Bednorz): cuprates show superconductivity at unconventionally high temperature
  - riddle: attraction from repulsion
    - microscopically, strong Coulomb onsite repulsion
    - still, observe pairing of fermions with d-wave symmetry

- Minimal model: 2d Fermi-Hubbard model

\[ H_{FH} = -J \sum_{\langle i,j \rangle, \sigma} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} \]

- realistic for cuprate high-temperature superconductors?
- hard to solve: strongly interacting fermion theory
  - no controlled analytical approach available
  - numerically (classical computer) intractable

⇒ Quantum simulation of the Fermi-Hubbard model in optical lattices?
Quantum Simulation of Fermion Hubbard model

- Clean realization of fermion Hubbard model possible
  - Detection of Fermi surface in 40K (M. Köhl et al. PRL 94, 080403 (2005))
- Cooling problematic: small d-wave gap sets tough requirements
- Existing proposal: Adiabatic quantum simulation (S. Trebst et al. PRL 96, 250402 (2006))
  - Start from a pure initial state of noninteracting model
  - Adiabatically transform to unknown ground state of interacting model
  - Concrete scheme: find path protected by large gaps:
    - prepare RVB ground state on isolated 2x2 plaquettes
    - couple these plaquettes to arrive at many-body ground state

\[ E_F \]

Current lattice experiments

Still need to be 10-100x cooler
Dissipative Quantum State Engineering Approach

- Roadmap:

1. Precool the system (lowest Bloch band)
2. Dissipatively prepare pure (zero entropy) state close to the expected ground state:
   - energetically close
   - symmetry-wise close
   - spin-wise close
3. Adapted adiabatic passage to the Hubbard ground state
   - switch dissipation off
   - switch Hamiltonian on

\[ H = 0, \quad \mathcal{L} \neq 0 \]

\[ H = H_{FH}, \quad \mathcal{L} = 0 \]
The State to Be Prepared

\[ |D\rangle = (D^\dagger)^N |\text{vac}\rangle, \quad D^\dagger = \sum_i (c^\dagger_{i+e_x} - c^\dagger_{i+e_y}) \sigma^y c^\dagger_i \]

mean field (product) state

- What does the state have in common with the expected Hubbard ground state

(1) Quantum numbers

- pairing in the singlet channel
- phase coherence: delocalization of singlet pairs
- transformation under spatial rotations: “d-wave”

→ The state shares the symmetries of Hubbard GS
→ No phase transition will be crossed in preparation process

- in the talk, we mainly consider 1-dimensional analog for simplicity:

\[ |D_1\rangle = (D^\dagger)^N |\text{vac}\rangle, \quad D^\dagger = \sum_i c^\dagger_{i+1} \sigma^y c^\dagger_i \]

Pauli matrix \( \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \)

d-wave SC
The State to Be Prepared

\[ |D\rangle = (D^\dagger)^N |\text{vac}\rangle, \quad D^\dagger = \sum_i \left( c_{i+e_x}^\dagger - c_{i+e_y}^\dagger \right) \sigma(y) c_i^\dagger \]

mean field (product) state

Pauli matrix \( \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \)

c_i = \begin{pmatrix} c_{i,\uparrow} \\ c_{i,\downarrow} \end{pmatrix} two-component spinor

- What does the state have in common with the expected Hubbard ground state

(2) Energetically close? Not known, but:

- off-site pairing \( c_{i+1}^\dagger \sigma(y) c_i^\dagger \) avoids excessive double occupancy
  
  cf onsite pairing: \( c_i^\dagger \sigma(y) c_i^\dagger \)

- the pairs are quasi-local, i.e. have a short coherence length in accord with observation in cuprates

\[ \text{superfluidity decreases due to strong correlations} \]
(A. Paramekanti, N. Trivedi, M. Randeria, PRB 70, 054504 (2004))

\[ \uparrow \text{ State can be expected to be convenient starting point not too close to half filling} \]

\[ \text{doping not too close to AF} \]

\[ \text{superfluidity decreases} \]

\[ \text{due to strong correlations} \]
Relation to the BCS Wavefunction

- usually, fixed phase (coherent state) wave function

\[ |\psi\rangle \propto \prod_k (1 + A_k c_{-k,\uparrow}^\dagger c_{k,\downarrow}^\dagger) |\text{vac}\rangle \]

\[ = \exp \left( \sum_k A_k c_{k,\uparrow}^\dagger c_{k,\downarrow}^\dagger \right) |\text{vac}\rangle = \sum N \frac{1}{N!} \left( \sum_k A_k c_{k,\uparrow}^\dagger c_{k,\downarrow}^\dagger \right)^N |\text{vac}\rangle \]

BCS amplitude

\[ A_k = \frac{u_k}{w_k} = \frac{\Delta_k}{E_k + \xi_k} \]

BCS gap

\[ E_k = \sqrt{\xi_k^2 + \Delta_k^2} \]

chemical potential

\[ \xi_k = \epsilon_k - \mu \]

dispersion

- distinct limits:

\[ \frac{\mu}{\epsilon_F} \to 1 \]

- localized in momentum space

- delocalized in position space

\[ \frac{\mu}{\epsilon_F} \to -\infty \]

- delocalized in momentum space

- localized in position space

- Relation to our state:

\[ \sum_i c_{i+1}^\dagger \sigma^{(\downarrow)} c_i^\dagger = 2 \sum_k \cos k c_{k,\uparrow}^\dagger c_{k,\downarrow}^\dagger \]

\[ A_k = 2 \cos k \]

- State shares the symmetries, but can be energetically very different
Setting

- **Goal:** Construct jump operators with unique mean field dark states:

\[
\mathcal{L}[\rho] = \sum_\ell j_\ell \rho j_\ell^\dagger - \frac{1}{2} \{j_\ell^\dagger j_\ell, \rho\}
\]

\[
j_\ell |\eta\rangle = 0 \quad \forall \ \ell
\]

Dark state

\[
|\eta\rangle = \prod_\alpha C_\alpha^\dagger |\text{vac}\rangle
\]

Mean field (product) state

\[
\Rightarrow [j_\ell, C_\alpha^\dagger] = 0 \quad \forall \ell, \alpha.
\]

(sufficient for normal ordered jump operators)

- **Requirements for implementation:**

\[
j_\ell = \sum_{\langle j|i\rangle_{\sigma,\sigma'}} c_{j,\sigma'}^\dagger H_{\sigma,\sigma'} c_{i,\sigma}
\]

- non-hermitian
- particle number conserving
- quasi-local: \( j \) close central site \( i \)

**single-particle operation**

\[
[ j_\ell, \sum_{i,\sigma} \hat{n}_{i,\sigma} ] = 0 \quad \forall \ell
\]

this is what the \( \eta \) operators suffered from!
Antiferromagnetic Jump Operators

- Construct jump operators for antiferromagnetism as a preparation

- Antiferromagnetic “Neel state” is a product of AF “unit cell” operators

\[ |\text{AF}^{\pm}\rangle = \prod_{i \in A} \hat{S}_{i-}^{\pm} |\text{vac}\rangle = (-)^{M/2} \prod_{i \in B} \hat{S}_{i+}^{\pm} |\text{vac}\rangle, \]

\[ C_{\alpha}^{\dagger} = \hat{S}_{i \pm}^{\pm} = c_{i \pm 1}^{\dagger} \sigma^{\pm} c_{i}^{\dagger} \]

- Set of jump operators (one dimension):

\[ \mathbf{j}_{\ell} = \{ \mathbf{j}_{i \pm}^{\pm}, \mathbf{j}_{i \pm}^{z} \} \]

- Action of jump operators
  - \( \mathbf{j}_{i \pm}^{\pm} \): Pauli blocking
  - \( \mathbf{j}_{i}^{z} \): spin transport

\[ S_{i -}^{+} = c_{i - 1}^{\dagger} \uparrow c_{i}^{\dagger}, \mathbf{j}_{i -}^{+} = c_{i - 1}^{\dagger} \uparrow c_{i}, \downarrow \]

\[ \downarrow \]

Thursday, October 22, 2009
d-Wave Jump Operators

• Rewrite the d-wave state in terms of AF unit cell operators:

\[ |D\rangle = (D^\dagger)^N |\text{vac}\rangle, \quad D^\dagger = \sum_i c_{i+1}^\dagger \sigma(y) c_i^\dagger = \sum_i \hat{J}_i^\pm \quad \hat{J}_i^\pm = \hat{S}_{i+}^\pm + \hat{S}_{i-}^\pm \]

  - Shift invariance
  - But delocalized pairs

• Second equality: interpret the state as a symmetrically delocalized AF

• Set of jump operators:

\[ \hat{J}_i = \{J_i^\pm, J_i^z\} \]

\[ J_i^\pm = \hat{J}_{i+}^\pm + \hat{J}_{i-}^\pm, \quad J_i^z = \hat{J}_{i+}^z + \hat{J}_{i-}^z \]

• Action of jump operators
  - \( J_i^\pm \): Pauli blocking
  - \( J_i^z \): spin transport
  - Both: phase coherence via delocalization

  ➡ Combine fermionic Pauli blocking with delocalization as for bosons
  ➡ Pauli blocking is the reason for single particle nature of operators
Uniqueness

- Recall: Unique dark state <-> state reached independent of initial condition
- Evidence for uniqueness from small scale numerical simulations

\[ S = \text{tr} \rho(t) \log \rho(t) \]

\[ \text{tr}[\rho(t)|AF \pm \rangle \langle AF \pm |] \]

Antiferromagnetism

\[ \begin{array}{c}
\text{Entropy evolution} \\
\text{perfect mixture}
\end{array} \]

d-wave

\[ \begin{array}{c}
\text{Fidelity} \\
3x3 \text{ sites, 4 particles, different random initial states}
\end{array} \]

Entropy evolution

6 sites, 6 particles

Fidelity
Uniqueness

Understanding can be gained from symmetry considerations

- Uniqueness of dark state equivalent to uniqueness of ground state (GS) of

\[ H_{\Delta} = \sum_{i, \alpha = \pm, z} \Delta_{\alpha} J_i^\alpha \dagger J_i^\alpha \]

\[ [\mathcal{L}[\rho] = \sum_{\alpha, i} \kappa_{\alpha} J_i^\alpha \rho J_i^{\alpha \dagger} - \frac{1}{2} \{\kappa_{\alpha} J_i^\alpha, J_i^{\alpha \dagger}, \rho\} \]

\[ \Rightarrow \text{effective Hamiltonian} \]

- H is semi-positive
- an exact GS is the above d-wave (E=0)
- unique iff no symmetry T such that

\[ T H T^{-1} = H, \quad T|D\rangle \neq E|D\rangle \]

- Symmetries:
  - Translations
  - global phase rotations U(1)
  - global spin rotations SU(2) for \( \Delta_z = \Delta_{\pm}/2 \),
  - additional discrete symmetry on bipartite lattice for \( \Delta_z = 0 \) spoils uniqueness

\[ T_d : \quad c_{i, \uparrow} \rightarrow -c_{i, \uparrow}; \quad c_{i, \downarrow} \rightarrow c_{i, \downarrow} \quad \text{for } i \in A, \]

\[ c_{i, \uparrow} \rightarrow c_{i, \uparrow}; \quad c_{i, \downarrow} \rightarrow c_{i, \downarrow} \quad \text{for } i \in B \]

bipartite (periodic BC)  
not bipartite (PBC)

\[ [S^{\alpha}, J_i^{\beta}] = i\epsilon_{\alpha \beta \gamma} J_i^{\gamma} \quad \forall i \]

Avoid symmetries

All three operators needed for uniqueness
Comments on the effective Hamiltonian

• Amusing parallel: Above Hamiltonian is a parent Hamiltonian for the d-wave state

\[ H_\Delta = \sum_{i,\alpha} \Delta_\alpha J_i^\alpha \dagger J_i^\alpha = \sum_i h_i \]

• H is semi-positive
• an exact unique GS is the above d-wave state (E=0)
• GS is GS for each \( h_i \) separately: projectors on GS

⇒ completely analogous to e.g. AKLT model
⇒ there, ground state is valence bond solid with exponentially decaying correlations
⇒ different: state has long range order due to strong delocalization
⇒ study excitations

• mean field decoupling

\[ \Delta_+ \sum_i J_i^+ \dagger J_i^+ = \Delta_+ \sum_i c_{i,\downarrow} \dagger (c_{i+1,\uparrow} + c_{i-1,\uparrow})(c_{i+1,\uparrow} \dagger + c_{i-1,\uparrow} \dagger) c_{i,\downarrow} = \Delta_+ \sum_i c_{i,\downarrow} \dagger (c_{i+1,\uparrow} \dagger + c_{i-1,\uparrow} \dagger) c_{i,\downarrow} (c_{i+1,\uparrow} + c_{i-1,\uparrow}) \]

\[ \approx \sum_q \Delta^+ \cos q \: c_{q,\downarrow} c_{-q,\uparrow} + h.c. \]

⇒ single fermion gap

• “diagonal” contributions \( \sim c_{q}^\dagger c_{q} \) from normal ordering \( J_i^z \)

⇒ single fermion excitations are gapped: important for adabatic passage
Arbitrary phase coherent pairing states

- Any pairing product state can be characterized by 3 quantum numbers

\[ O_{k,n,\mu}^\dagger \mathcal{N}_{\text{vac}}, \quad O_{k,n,\mu}^\dagger = \sum_i \exp ikx_i \ c_{i+n}^\dagger \sigma^\mu c_i^\dagger \quad \sigma^\mu = (1, \sigma^\alpha) \]

- Examples:
  - \( k = 0, \ n = 0, \ \mu = 2 \) \( \rightarrow \) s-wave BCS
  - \( k = \pi, \ n = 0, \ \mu = 2 \) \( \rightarrow \) eta-state
  - \( k = 0, \ n = 1, \ \mu = 2 \) \( \rightarrow \) d-wave like state

- Jump operators constructed for all \( k, \mu, \) and \( n > 0 \) (displayed just for completeness...)

\[
\begin{align*}
\mu &= 0 : (c_{i+n}^\dagger - e^{ik} c_{i-n}^\dagger)(1 \pm \sigma^z)c_i^\dagger, \quad (c_{i+n}^\dagger - e^{ik} c_{i-n}^\dagger)\sigma^y c_i^\dagger \\
\mu &= 1 : (c_{i+n}^\dagger - e^{ik} c_{i-n}^\dagger)\sigma^\pm c_i^\dagger, \quad (c_{i+n}^\dagger + e^{ik} c_{i-n}^\dagger)1 c_i^\dagger \\
\mu &= 2 : (c_{i+n}^\dagger + e^{ik} c_{i-n}^\dagger)\sigma^\pm c_i^\dagger, \quad (c_{i+n}^\dagger + e^{ik} c_{i-n}^\dagger)\sigma^z c_i^\dagger \\
\mu &= 3 : (c_{i+n}^\dagger - e^{ik} c_{i-n}^\dagger)(1 \pm \sigma^z)c_i^\dagger, \quad (c_{i+n}^\dagger - e^{ik} c_{i-n}^\dagger)\sigma^x c_i^\dagger
\end{align*}
\]

- arbitrary \( n > 0 \) pairing states can be targeted
- d-wave not distinguished, but off-site pairing special
- symmetries of the state inherited by the parent Hamiltonian
Implementation of d-wave jump operators

- Decisive property: **single-particle nature** of the jump operators
- Implement Fourier transformed operators:

\[
\mathcal{L}[\rho] = \sum_{\alpha, i} J_i^\alpha \rho J_i^{\alpha\dagger} - \frac{1}{2} \{ J_i^\alpha J_i^{\alpha\dagger}, \rho \} = \sum_{\alpha, k} J_k^\alpha \rho J_k^{\alpha\dagger} - \frac{1}{2} \{ J_k^{\alpha\dagger} J_k^\alpha, \rho \}
\]

\[
J_k^\pm = \sum_q \cos q a_q^\dagger \sigma^\pm a_{q-k}, \quad J_k^z = \sum_q \cos q a_q^\dagger \sigma^z a_{q-k}
\]

- Basic physical ingredients:
  - Dissipation: Emission in cavity
  - Use Earth Alkaline atoms in state dependent superlattice

- Engineering requirements:
  - Spin imprinting: Light Polarization
  - Momentum transfer: Laser angle (incoherent beams)
  - \( \cos q \) dependence: Quantum Interference
Implementation of d-wave jump operators

- Level scheme: Earth Alkaline atoms

\[ J_{\pm}^{k} = \sum_{q} \cos q a_{q}^{\dagger} \sigma_{q}^{\pm} a_{q-k} \]

spont. emission: cavity mode
momentum transfer
spin imprinting
physical spin

atom confinement via optical lattice

Bloch bands
Quantum Interference:
\[ \frac{1}{\Delta + \epsilon_{q}} + \frac{1}{-\Delta + \epsilon_{q}} \approx -2 \frac{\epsilon_{q}}{\Delta^{2}} \]
cos q: onsite processes interfere destructively
Adapted Adiabatic Passage

- Assume we have prepared zero entropy d-wave
- Want to connect to Hubbard ground state
- Adiabatic passage (purely Hamiltonian dynamics):
  \[ H = \lambda(t) H_\Delta + (1 - \lambda(t)) H_{FH}, \]
  \[ \lambda(t_{\text{in}}) = 1, \quad \lambda(t_{\text{fin}}) = 0 \]
  ramping slowly: remain in ground state

- Adapted adiabatic passage: Two ingredients
  - gap protection from auxiliary Hamiltonian
    - parent Hamiltonian has d-wave eigenstate and is gapped: add detuning to the effective Hamiltonian
    \[ \gamma \rightarrow \gamma + i\Delta \]
  - probabilistic ground state preparation
    - dissipative and Hubbard dynamics compete
    - focus on time before first jump: state prepared with probability

\[ H = 0, \quad \mathcal{L} \neq 0 \]
Adapted adiabatic passage

\[ H = H_{FH}, \quad \mathcal{L} = 0 \]
Summary Part I

By merging techniques from quantum optics and many-body systems:
Driven dissipation can be used as controllable tool in cold atom systems.

- **Pure states** with long range correlations from quasilocal dissipation
  - Many-body dark state, independent of initial density matrix
  - Laser coherence mapped on matter system
  - System steady state has zero entropy
- **Nonequilibrium phase transition** driven via competition of unitary and dissipative dynamics
  - driven by interactions (like quantum phase transition)
  - terminates into thermal state (like classical phase transition)
- **Strong potential applications for fermionic quantum simulation**
  - cool into zero entropy d-wave state as intial state for Fermi-Hubbard model
  - single particle operations due to Pauli blocking
  - realistic setting using earth alkaline atoms in a cavity


**Optical Lattices**

- **AC-Stark shift**
  - Consider an atom in its electronic ground state exposed to laser light at fixed position \( \vec{x} \).
  - The light be far detuned from excited state resonances: ground state experiences a second-order *AC-Stark shift*

\[
\delta E_g = \alpha(\omega) I
\]

with \( \alpha(\omega) \) - dynamic polarizability of the atom for laser frequency \( \omega \), \( I \propto \vec{E}^2 \) - light intensity.

- Example: two-level atom \( \{|g\rangle, |e\rangle\} \).

  - Rabi frequency
  
  \[
  \delta E_g = \hbar \frac{\Omega^2}{4\Delta}
  \]
  
  detuning from resonance \( \Delta = \omega - \omega_{eg} \)

  \( \Omega \ll \Delta \)

- For standing wave laser configuration \( \vec{E}(\vec{x}, t) = \vec{E}_0 e^{-i\omega t} + h.c. \), AC-Stark shift is a function of position: It generates an optical potential

\[
V_{opt}(\vec{x}) \equiv \delta E_g(\vec{x}) = \hbar \frac{\Omega^2(\vec{x})}{4\Delta} \equiv V_0 \sin^2 kx \quad (k = \frac{2\pi}{\lambda})
\]
Effective Lattice Hamiltonian

- Start from our model Hamiltonian, add optical potential:

\[ H = \int x \left[ a_x^\dagger \left( -\frac{\Delta}{2m} - \mu + V(x) + V_{opt}(x) \right) a_x + g \hat{n}_x^2 \right] \]

- Periodicity of the optical potential suggests expansion of field operators into localized lattice periodic Wannier functions (complete set of orthogonal functions)

\[ a_x = \sum_{i,n} w_n(x - x_i) b_{i,n} \]

- For low enough energies (temperature), we can restrict to lowest band:

\[ T, U, J \ll \sqrt{4V_0E_R}, E_R = k^2/(2m) \rightarrow n = 0 \]

- Then we obtain the single band Bose-Hubbard model

\[ H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j - \mu \sum_i \hat{n}_i + \sum_i \epsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i(\hat{n}_i - 1) \]

\[ J = -\int dx w_0^*(x) \left( -\frac{\hbar^2}{2m\Delta} - V_{opt}(x) \right) w_0(x - \lambda/2) \]

\[ U = g \int dx |w_0(x)|^4 \]