Gerard 't Hooft

Centre for Extreme Matter and Emergent Phenomena,
Science Faculty, Utrecht University,
POBox 80.089, 3508 TB, Utrecht
The Netherlands

Past and Future of Gauge Theory

Bad Honnef
678. WE-Heraeus-Seminar on "Hundred Years of Gauge Theory"
Opening talk, July 30, 2018

The Early Days.

Gauge invariance, introduced by Hermann Weyl, as an attempt to unify Einstein's General Relativity with electromagnetism, by adding local scale transformations, affecting the *gauge* of a weighing scale:

$$\mathrm{d}x^\mu \to e^{\omega(x)}\mathrm{d}x^\mu \ , \qquad g_{\mu\nu} \to e^{-2\omega(x)} g_{\mu\nu}$$

This did not quite work. Covariant derivative:

$$\partial_\mu \ \to \ D_\mu = \partial_\mu - \omega_\mu$$

But you do get electromagnetism if charged fields ψ transform as

$$\psi \to e^{i\omega(x)}\psi \ .$$

Quantum Electrodynamics, QED.

Quantising the fields with gauge invariance, leads to the correct quantum theory for electrically charged particles.


Schwinger,


Feynman,

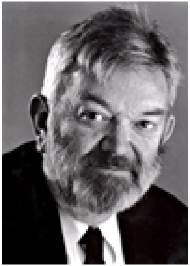

Tomonaga

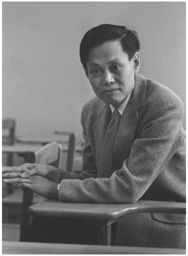
Infinities in the procedure could be made to cancel one an other, if done with sufficient care. *Renormalizable* theories contained *scalar* fields $\varphi(x)$, *fermionic* fields $\psi(x)$ and the *electromagnetic* fields $F_{\mu\nu}(x)$ and $A_\mu(x)$.

The weak interaction.

But the *weak interaction* also appeared to be mediated by a vector particle, just like the photon. There should be at least three types of weak photons, W^+, W^-, and Z, besides the photon, γ . In the 1960s, M. Veltman was convinced by the experimental evidence:

The weak interactions had to be some modification of a *Yang-Mills* theory.





C.N. Yang

R. Mills

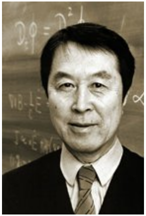
The Yang-Mills photons had to have *mass*, and the neutral component Z, if that existed at all, would couple differently to charged and neutral currents. Veltman attempted to formulate the renormalization procedure for the modified theory.

But his modified theory was not gauge-invariant.

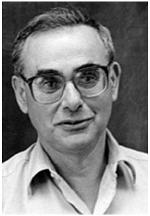
There appeared to be a problem. The *Nambu Goldstone theorem* states that

> *You cannot have spontaneous symmetry breaking without generating a massless particle.*

Indeed, *chiral isospin symmetry,* $SU(2)_{\mathrm{left}} \times SU(2)_{\mathrm{right}}$, comes with a "light" particle, the pion.



Y. Nambu



J. Goldstone

The pion mass is generated only because of an *explicit* breakdown of chiral isospin symmetry (the quark mass terms).

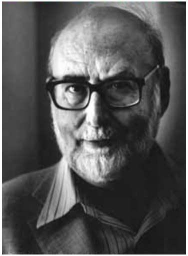
We learned that gauge invariance was not allowed to be messed with.

BEH

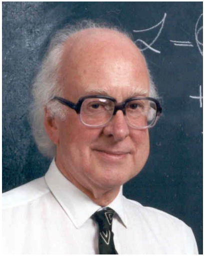
The *Brout-Englert-Higgs* mechanism (*not* a symmetry breaking !) must be invoked to represent the masses of the photons. Moreover, by allowing the Higgs field also to couple to the fermions, we could allow masses for charged leptons and neutral leptons to differ from one another.

So we solved not one, but two mass problems for the electro-weak theory.

R. Brout



F. Englert



P. Higgs

To be renormalizable, the *short distance structure* of the theory *must* be exactly that of a pure gauge theory. We need exact local gauge invariance.

The folklore of "the origin of mass"

If a mass term is gauge-invariant, we do not need the BEH mechanism to have such a mass term.

Therefore, the BEH mechanism is not "the origin of mass", but it is the origin of non-gauge-invariant mass !!

Today, it is easy to understand that a massive photon has 3 helicities, while a massless photon has only 2. One must understand this extra degree of freedom, also what it does at small distances.

In the old days this was hard, because Quantum Field Theory was not trusted as a valid approach.

In the language of quantum field theory, strictly speaking, the BEH mechanism should not be addressed as a "spontaneous symmetry breaking". The vacuum state is completely invariant under *local* gauge transformations.
But we pick a gauge by imposing a constraint such as $\partial_\mu A^\alpha_\mu = 0$, or $\phi_a = (0,\, 0,\, F + \phi_3')$, and describe the state for those 'field coordinates'.

In contrast, when we have a *global* symmetry, the vacuum state can be *degenerate*, so we can choose this state to fluctuate, asymmetrically, around a vector $(0,\, 0,\, F)$.

Our original proofs of renormalizability were based on various techniques. One was the cutting rules for Feynman diagrams,

Unitarity: $S \cdot S^\dagger = \mathbb{I}$,

and

causality:

$[\phi(x_1),\ \phi(x_2)] = 0$ if $(x_1 - x_2)^2 > 0$, ($x_1 - x_2$ is space-like.)

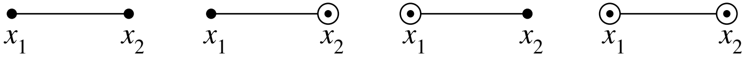
$$\text{Let } \Delta(k) = \frac{1}{2\pi i}\,\frac{1}{k^2 + m^2 - i\varepsilon} \quad , \quad \Delta^\pm(k) = \theta(\pm k_0)\,\delta(k^2 + m^2)\ .$$

$$\text{Let } \Delta(x) = (2\pi)^{-3}\int \mathrm{d}^4k\, e^{ik\cdot x}\Delta(k) \quad , \quad \Delta^\pm(x) = (2\pi)^{-3}\int \mathrm{d}^4k\, e^{ik\cdot x}\Delta^\pm(k)\ .$$

$$\text{Then} \qquad \Delta(x) = \theta(x_0)\Delta^+(x) + \theta(-x_0)\Delta^-(x)\ ;$$
$$\Delta^*(x) = \theta(x_0)\Delta^-(x) + \theta(-x_0)\Delta^+(x)\ .$$

In deriving this, use $$\theta(z) = \frac{1}{2\pi i}\int_{-\infty}^{\infty}\mathrm{d}\tau\,\frac{e^{i\tau z}}{\tau - i\varepsilon}\ .$$

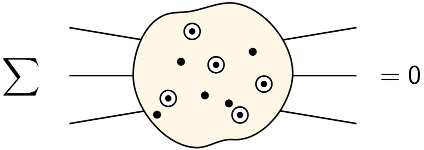
These identities can now be used to prove combinatorial relations between Feynman diagrams:



$\Delta(x_2 - x_1) \qquad -\Delta^{+}(x_2 - x_1) \qquad -\Delta^{-}(x_2 - x_1) \qquad \Delta^{*}(x_2 - x_1)$

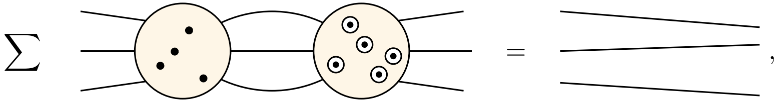
The sum of all four expressions always zero.

Take a Feynman diagram with given topology, then sum over dots. One finds:



(except if there are no vertices at all).

Pulling the dotted vertices apart:



which stands for:

$$S \cdot S^{\dagger} = \mathbb{I} \ .$$

From this, one derives that the Feynman rules for theories with only scalar particles add up to deliver a unitary scattering matrix. The importance of this is that, now, one can read off how infinite subtractions can be employed to make diagrams finite without violating unitarity and causality.

In gauge theories, we have to *fix the gauge*: $C(A, \varphi) = 0$, where φ stands for some scalar fields such as the Higgs, by adding the appropriate *Faddeev-Popov ghost* Lagrangian:

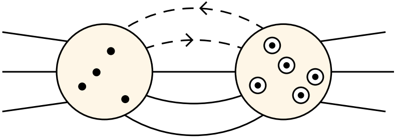
$$\Delta\mathcal{L} = -\tfrac{1}{2} C(A, \varphi)^2 + \overline{\eta}\, \frac{\partial C(A, \varphi)}{\partial \Lambda} \eta \ ,$$

and inspect unitarity for this.
Here, η and $\overline{\eta}$ are *anti-commuting, scalar* fields.

For the original proofs that these theories are renormalizable, we used the cutting rules to note that only physical particles survive in the intermediate lines connecting S to $S^\dagger$. By combining the Feynman rules, they were found to obey the non-Abelian generalisation if the Ward-Takahashi identities, a symmetry relating the diagrams.

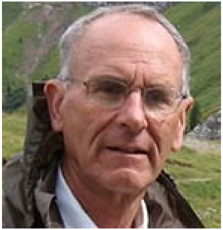
However, we did not recognise this as a symmetry between the *fields*.

$\sum_{\text{ghosts}}$



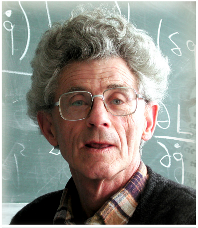
$= 0\;.$

However, it is a *super*-symmetry between fields:



Becchi

Rouet



Stora

Tyutin:

$$\mathcal{L} = \mathcal{L}^{\rm inv} + \lambda^a(x) C^a(A,x) + \overline{\eta}^a(x) \frac{\partial C^a(x)}{\partial \Lambda^b(x')} \eta^b(x') \; + f(\lambda^a) \; .$$

$$\begin{aligned} \delta A^a(x) &= \; \overline{\varepsilon} \frac{\partial A^a(x)}{\partial \Lambda^b(x')} \eta^b(x') \; ; \\ \delta \eta^a(x) &= \; \tfrac{1}{2} \overline{\varepsilon} \, f^{abc} \, \eta^b(x) \, \eta^c(x) \; ; \\ \delta \overline{\eta}^a(x) &= \; -\overline{\varepsilon} \, \lambda^a(x) \; ; \\ \delta \lambda^a(x) &= \; 0 \; , \end{aligned}$$

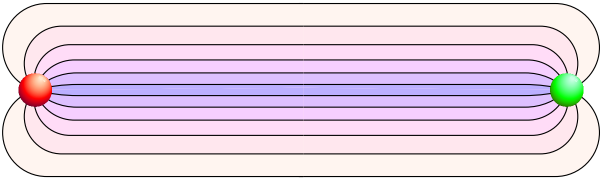
$\rightarrow \quad \delta S = 0 \; .$

skip ?

In the early days, it was not realised that, actually, gauge theories *without* a BEH type of spontaneous symmetry breaking, are a lot harder to understand than the BEH theories.

This is because these theories contain strongly interacting, massless particles: *gluons*

The prime example of such a theory is Quantum Chromo Dynamics.
In this theory, it is generally agreed that the magnetic dual of the BEH mechanism takes place. This causes quarks to be confined by the formation of electric vortex configurations.



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General conjecture for the vacuum structure of all local gauge theories: if the local gauge group is $SU(3)$, the vacuum is one of 3 possible phase configurations:
(1) The standard BEH mechanism with a spin ½ Higgs field: all gauge particles then are massive photons,
(2) The vacuum may be in the electric/magnetic dual of the BEH state, where we see massive glueball particles playing the role of gauge bosons, while all particles in non-trivial representations of the gauge group are confined, or
(3) An explicit or effective isospin 1 BEH mechanism: one massless $U(1)$ photon survives (this is an electric/magnetic self-dual mode)

When the gauge group is larger, various subgroups could condense in different ways, yielding a vacuum state that combines several of the above condensation modes.

With the Large Hadron Collider, we are now seeing a glimpse of the future of quantum field theory !

The mass of the Higgs particle – finally found – is very close to the value that *flattens off* the curves of the running coupling constants

as if we are approaching a domain with scale invariance,
more precisely: local conformal symmetry.

This may mean that we can approach *quantum gravity* more quickly than expected.
Gravity <u>is</u> a theory with local conformal invariance!

$$\mathcal{L}_{\rm EH} = \frac{\sqrt{-g}}{16\pi G}\, g^{\mu\nu} R_{\mu\nu} \ ;$$

$$\mathcal{L} = \mathcal{L}_{\rm EH} + \sqrt{-g}\Big(-\tfrac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \cdots\Big)$$

$g_{\mu\nu}$ is a dynamical field, therefore the *local conformal transformation*

$$\begin{aligned} g_{\mu\nu} &\rightarrow \omega^2(x)\, g_{\mu\nu}\ , \\ \sqrt{-g} &\rightarrow \omega^4(x)\sqrt{-g}\ , \\ \varphi(x) &\rightarrow \omega^{-1}(x)\,\varphi(x)\ , \\ A^a_\mu(x) &\rightarrow A^a_\mu(x)\ , \quad \text{etc.} \end{aligned}$$

is a genuine local gauge symmetry.

By turning fron the unitarity gauge, $\varphi_1(x) = 1$, to a renormalizable gauge, one can *almost* obtain a renormalizble theory of gravity !

How do we address the hierarchy problem?

When we transform to a distance scale 10^{-20} times the Standard Model scale, all fields presently contained in the SM, appear to be strictly massless. This means that the action S is invariant when we add or subtract *constants* to these fields:

$$\begin{aligned} \varphi_a(x) &\rightarrow \varphi_a(x) + C_a^s \ , \\ A_\mu^a(x) &\rightarrow A_\mu^a(x) + C_\mu^{v\,a}, \\ \psi_k(x) &\rightarrow \psi_k(x) + \eta_k \ . \end{aligned}$$

The constants C^s, C^v, $\cdots$, are all *generators of symmetries*. The anticommuting fields η_k generate *super symmetries.*

The commutators are higher order effects and cannot be derived today, so we know little about these symmetries.

Something to speculate about.