Space, Time, Matter in Quantum Gravity

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Bernhard Riemann 1854:

Die Frage über die Gültigkeit der Voraussetzungen der Geometrie im Unendlichkleinen hängt zusammen mit der Frage nach dem innern Grunde der Massverhältnisse des Raumes. . . .

Es muss also entweder das dem Raume zu Grunde liegende Wirkliche eine discrete Mannigfaltigkeit bilden, oder der Grund der Massverhältnisse ausserhalb, in darauf wirkenden bindenden Kräften, gesucht werden.

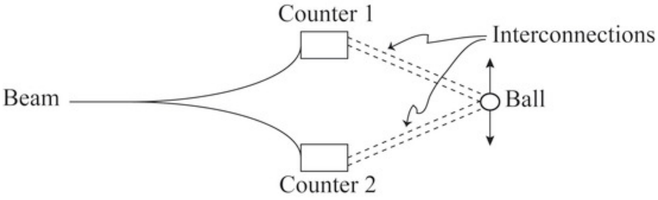
The question of the validity of the postulates of geometry in the indefinitely small is involved in the question concerning the ultimate basis of relations of size in space.. . .

Either then the actual things forming the groundwork of a space must constitute a discrete manifold, or else the basis of metric relations must be sought for outside that actuality, in colligating forces that operate upon it.

(Transl. by Henry S. White.)

Richard Feynman 1957:

. . . if you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment. . . . It may turn out, since we've never done an experiment at this level, that it's not possible . . . that there is something the matter with our quantum mechanics when we have too much *action* in the system, or too much mass—or something. But that is the only way I can see which would keep you from the necessity of quantizing the gravitational field. It's a way that I don't want to propose. . . .



Matvei Bronstein (1936):

The elimination of the logical inconsistencies connected with this [his thought experiments] requires a radical reconstruction of the theory, and in particular, the rejection of a Riemannian geometry dealing, as we see here, with values unobservable in principle, and perhaps also the rejection of our ordinary concepts of space and time, modifying them by some much deeper and nonevident concepts. *Wer's nicht glaubt, bezahlt einen Taler*.

Planck units

$$
\begin{aligned}
l_{\mathrm{P}} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-35}\ \mathrm{m} \\
t_{\mathrm{P}} &= \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ \mathrm{s} \\
m_{\mathrm{P}} &= \frac{\hbar}{l_{\mathrm{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-8}\ \mathrm{kg} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2
\end{aligned}
$$

Max Planck (1899):

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum . . . in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, "What is the question?"
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals including spin foams, asymptotic safety, . . .)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, . . .)
- String theory
- Fundamental discrete approaches
  (quantum topology, causal sets, group field theory, . . .);
  have partially grown out of the other approaches

C. Kiefer, *Quantum Gravity* (Oxford 2012)

Background independence

Albert Einstein:
Es widerstrebt dem wissenschaftlichen Verstande, ein Ding zu setzen, das zwar wirkt, aber auf das nicht gewirkt werden kann.

(It is contrary to the scientific mode of understanding to postulate a thing that acts, but which cannot be acted upon.)

There are no absolute fields in general relativity
(no background structure).

General relativity in canonical form

Einstein's equations can be written as a dynamical system (for the three-metric h_{ab} and its canonical momentum π^{ab} on a spacelike hypersurface Σ) of evolution equations together with constraints:

$$\begin{aligned}
\mathcal{H}_\perp &= 2\kappa\, G_{ab\,cd}\pi^{ab}\pi^{cd} - (2\kappa)^{-1}\sqrt{h}({}^{(3)}R - 2\Lambda) + \sqrt{h}\rho \approx 0 \\
\mathcal{H}^a &= -2\nabla_b \pi^{ab} + \sqrt{h}j^a \approx 0\,,
\end{aligned}$$

with the DeWitt metric

$$G_{ab\,cd} = \frac{1}{2\sqrt{h}}(h_{ac}h_{bd} + h_{ad}h_{bc} - h_{ab}h_{cd})$$

and

$$\kappa = 8\pi G/c^4$$

$H \approx 0$ is called "Hamiltonian constraint", $\mathcal{H}^a \approx 0$ are called "momentum (diffeomorphism) constraints".

Constraints and evolution

I

Constraints are preserved in time $\Longleftrightarrow$ energy–momentum tensor of matter has vanishing covariant divergence

compare with electrodynamics: Gauss constraint preserved in time $\Longleftrightarrow$ charge conservation

II

Einstein's equations are the unique propagation law consistent with the constraints

compare with electrodynamics: Maxwell's equations are the unique propagation law consistent with the Gauss constraint

Problem of time I

Restriction to *compact* three-spaces Σ:

- The total Hamiltonian is a combination of pure constraints; all of the evolution will be generated by constraints;
- no external time parameter exists
- all physical time parameters are to be constructed from within our system, that is, as functional of the canonical variables; a priori there is no preferred choice of such an intrinsic time parameter

The absence of an extrinsic time and the non-preference of an intrinsic one is known as the problem of time in (classical) canonical gravity. Still, spacetime exists.

Structure of configuration space

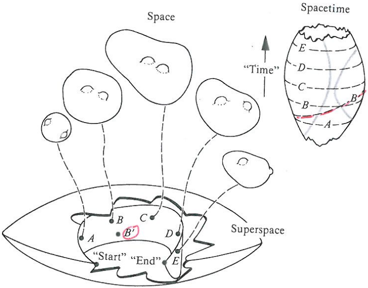
Superspace (Wheeler 1968):

$$\mathcal{S}(\Sigma) := \mathrm{Riem}\ \Sigma/\mathrm{Diff}\ \Sigma.$$

By going to superspace, the momentum constraints are automatically fulfilled. Whereas $\mathrm{Riem}\ \Sigma$ has a simple topological structure, the topological structure of $\mathcal{S}(\Sigma)$ is very complicated because it inherits (through $\mathrm{Diff}\ \Sigma$) some of the topological information contained in Σ.

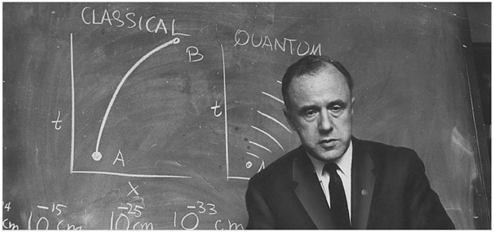
Important: DeWitt metric and its projection on superspace

Superspace



From: Misner, Thorne, Wheeler, Gravitation

Quantum geometrodynamics



(a) John Archibald Wheeler

(b) Bryce DeWitt

Application of Schrödinger's procedure to general relativity leads to

$$\hat{\mathcal{H}}_\perp \Psi \equiv \left(-16\pi G \hbar^2 G_{abcd} \frac{\delta^2}{\delta h_{ab} \delta h_{cd}} - (16\pi G)^{-1} \sqrt{h} \left({}^{(3)}R - 2\Lambda \right) + \sqrt{h} \hat{\rho} \right) \Psi = 0$$

Wheeler–DeWitt equation

$$\hat{\mathcal{H}}^a \Psi \equiv -2 \nabla_b \frac{\hbar}{\mathrm{i}} \frac{\delta \Psi}{\delta h_{ab}} + \sqrt{h} \hat{j}^a \Psi = 0$$

quantum diffeomorphism (momentum) constraint

Problem of time II

- Spacetime has disappeared, only space remains;
- Wheeler–DeWitt equation has the structure of a wave equation any may therefore allow the introduction of an ‘intrinsic time’;
- Hilbert-space structure in quantum mechanics is connected with the probability interpretation, in particular with probability conservation *in time* t; what happens with this structure in a timeless situation?
- What is an observable in quantum gravity?

Diffeomorphism constraints

Under

$$x^a \mapsto \bar{x}^a = x^a + \delta N^a(\mathbf{x}),$$

the three-metric transforms as

$$h_{ab}(\mathbf{x}) \mapsto \bar{h}_{ab}(\mathbf{x}) = h_{ab}(\mathbf{x}) - D_a \delta N_b(\mathbf{x}) - D_b \delta N_a(\mathbf{x}).$$

The wave functional then transforms according to

$$\Psi[h_{ab}] \mapsto \Psi[h_{ab}] - 2 \int \mathrm{d}^3 x \, \frac{\delta \Psi}{\delta h_{ab}(\mathbf{x})} D_a \delta N_b(\mathbf{x}).$$

Assuming the invariance of the wave functional under this transformation, one is led to

$$D_a \frac{\delta \Psi}{\delta h_{ab}} = 0.$$

Under large diffeomorphisms, the wave functional can acquire a phase.

A simple analogy is Gauss's law in QED (or its generalizations to the non-Abelian case). The quantized version of the constraint $\nabla \mathbf{E} \approx 0$ reads

$$\frac{\hbar}{\mathrm{i}} \nabla \frac{\delta \Psi[\mathbf{A}]}{\delta \mathbf{A}} = 0,$$

from which invariance of Ψ with respect to gauge transformations of the form $\mathbf{A} \rightarrow \mathbf{A} + \nabla \lambda$ follows.

Constraint algebra

$$
\begin{aligned}
\{\mathcal{H}_\perp(\mathbf{x}), \mathcal{H}_\perp(\mathbf{y})\} &= -\sigma\delta_{,a}(\mathbf{x},\mathbf{y})\left(h^{ab}(\mathbf{x})\mathcal{H}_b(\mathbf{x}) + h^{ab}(\mathbf{y})\mathcal{H}_b(\mathbf{y})\right) \\
\{\mathcal{H}_a(\mathbf{x}), \mathcal{H}_\perp(\mathbf{y})\} &= \mathcal{H}_\perp(\mathbf{x})\delta_{,a}(\mathbf{x},\mathbf{y}) \\
\{\mathcal{H}_a(\mathbf{x}), \mathcal{H}_b(\mathbf{y})\} &= \mathcal{H}_b(\mathbf{x})\delta_{,a}(\mathbf{x},\mathbf{y}) + \mathcal{H}_a(\mathbf{y})\delta_{,b}(\mathbf{x},\mathbf{y})
\end{aligned}
$$

Important: are there central terms (anomalies) $\propto \hbar^n$ in the quantum theory? If yes, not all of the quantum constraints would hold (compare string theory).

Dirac consistency

The problem has been investigated by J. Schwinger (Phys. Rev. 132 (1962) 1317). Schwinger uses a different notation, based on the dynamical co-ordinates $q^{rs} = K^2 e^{rs}$, with appropriate momenta conjugate to them, but his equations are equivalent to the ones given here.

Schwinger makes his attack more powerful by considering the quantum analogue, not of ϕ_L, but of $K^n\phi_L$, where n is some number at our disposal. Classically, $K^n\phi_L$ satisfies, as the conditions replacing Eqs (14) and (15)

$$[K^n\phi_L, \phi'_u] = (K^n\phi_L)_{,u}\,\delta(x-x') + (n+1)K^n\phi_L\,\delta_{,u}(x-x') \tag{18}$$

$$[K^n\phi_L, K'^n\phi'_L] = -K^n K'^n\{e^{rs}\phi_r + e'^{rs}\phi'_r\}\,\delta_{,s}(x-x') \tag{19}$$

Schwinger assumes for the quantum $K^n\phi_L$ the Hermitian expression

$$(K^n\phi_L)_Q = p^{rs}K^{n-1}(g_{rs}\,g_{ab} - g_{ra}\,g_{sb})p^{ab} + K^{n+1}R_3 \tag{20}$$

It satisfies Eq. (18) for any n, since this equation holds for any quantity that transforms suitably under transformations of the co-ordinates x^1, x^2, x^3 of the surface.

Schwinger proceeds to examine what value of n, if any, would make $(K^n\phi_L)_Q$ satisfy Eq. (19) with the quantum expressions (17) for the ϕ_u's on the right-hand side, with their coefficients all on the left. The calculation involves quantities of the form $\delta(x-x')\delta_{,r}(x-x')$. There does not exist any general method for handling such quadratic quantities in the δ-function, free from inconsistencies. However, by using simple and plausible but non-rigorous methods, Schwinger is able to show that the condition is satisfied with n = 3. Since the right-hand side of Eq. (19) must be Hermitian, it must then be equally possible to have all the coefficients of the ϕ_u's on the right.

The problem of the quantization of the gravitational field is thus left in a rather uncertain state. If one accepts Schwinger's plausible methods, the problem is solved. But one cannot be happy with such methods without having a reliable procedure for handling quadratic expressions in the δ-function.

The sort of inconsistency one must prevent is illustrated by the following calculation with $\delta(y)$ for one variable y. We have

$$y\,\delta(y) = 0 \tag{21}$$

Differentiating, we get

$$y\,\delta'(y) = -\delta(y) \tag{22}$$

Multiplying Eq. (21) by $\delta'(y)$, we get

$$y\,\delta(y)\,\delta'(y) = 0$$

and multiplying Eq. (22) by $\delta(y)$, we get

$$y\,\delta(y)\,\delta'(y) = -\{\delta(y)\}^2$$

end of paper!

Paul Dirac (1968)

Loop quantum gravity

- Particular role of connections
- Disappearance of spacetime as in geometrodynamics
- Prediction of discreteness at the Planck scale (at the kinematical level)
- Potential problems with the semiclassical limit (recovery of spacetime and general relativity)

Example: quantization of a Friedmann universe

Closed Friedmann–Lemaître universe with scale factor a, containing a homogeneous massive scalar field ϕ (two-dimensional *minisuperspace*)

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2$$

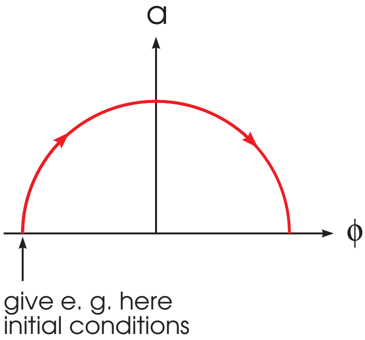
The Wheeler–DeWitt equation reads (with units $2G/3\pi = 1$)

$$\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial \phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2\right)\psi(a,\phi) = 0$$

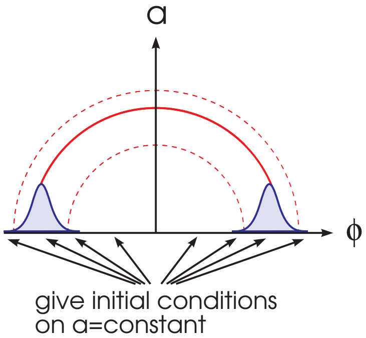
Factor ordering chosen in order to achieve covariance in minisuperspace

Determinism in classical and quantum theory

Classical theory



Quantum theory



Recollapsing part is deterministic successor of expanding part

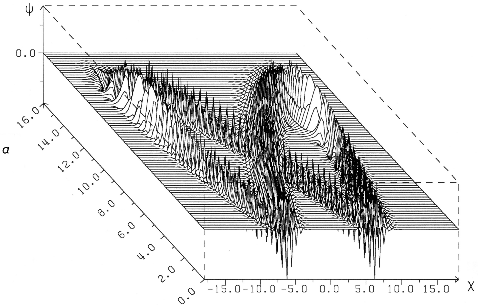
'Recollapsing' wave packet must be present 'initially'

No intrinsic difference between 'big bang' and 'big crunch'!

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



C.K. (1990)

Semiclassical approximation

From the timeless Wheeler–DeWitt equation, one can derive the limit of quantum field theory on a curved spacetime by using a Born–Oppenheimer type of approximation. In this way, an approximate semiclassical (WKB) time emerges.

In this limit, one has the usual Hilbert space structure and the associated probability interpretation.

Higher orders of this approximation allow the derivation of quantum-gravitational corrections terms, which for example give corrections to the CMB anisotropy spectrum.

Experimental tests of quantum gravity?

Example: Transition rate from the $3d$ level to the $1s$ level in the hydrogen atom due to the emission of a graviton:

$$\Gamma_{\rm g} = \frac{G m_{\rm e}^3 c \alpha^6}{360 \hbar^2} \approx 5.7 \times 10^{-40}\ {\rm s}^{-1}$$

This corresponds to a life-time of

$$\tau_{\rm g} \approx 5.6 \times 10^{31}\ {\rm years}\ .$$

Other possibility: Test of the superposition principle à la Feynman ('gravcat states')?

The CMB spectrum from the PLANCK mission

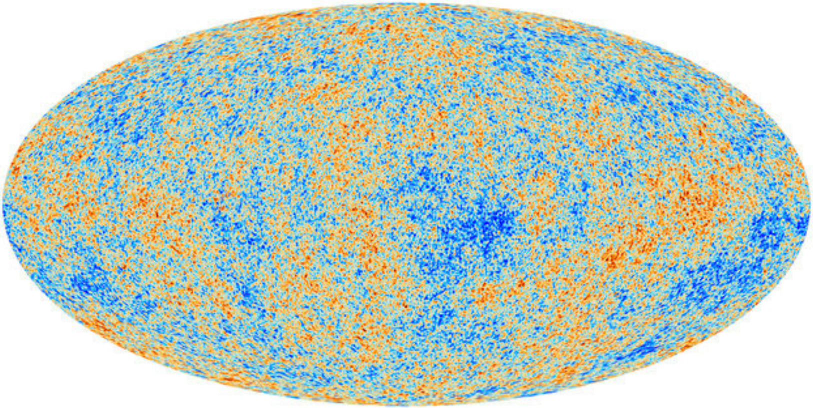


Figure: PLANCK mission

Quantum origin of perturbations

Power spectrum for the scalar modes (inflaton plus metric):

$$\Delta_{\rm s}^2(k) = \frac{1}{8\pi^2}\,(t_{\rm P}H)^2\,\epsilon^{-1} \approx 2\times 10^{-9}$$

ϵ: slow-roll parameter

Tensor-to-scalar ratio: $r := \frac{\Delta_{\rm t}^2}{\Delta_{\rm s}^2} = 16\epsilon$

Knowing r, one knows the energy scale of inflation,

$$\mathcal{E}_{\rm inf} \approx 1.06\times 10^{16}\ {\rm GeV}\left(\frac{r}{0.01}\right)^{1/4}$$

Gravitons from the early Universe

Gravitons are created out of the vacuum during an inflationary phase of the early Universe ($\sim 10^{-34}$ s after the big bang); the quantized gravitational mode functions $h_{\mathbf{k}}$ in de Sitter space obey

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle = \frac{4}{k^3} \left(t_{\mathrm{P}} H\right)^2 \delta(\mathbf{k} + \mathbf{k}') \equiv P_{\mathrm{t}} \; \delta(\mathbf{k} + \mathbf{k}')$$

Power spectrum:

$$\Delta_{\mathrm{t}}^2(k) := \frac{k^3}{2\pi^2} P_{\mathrm{t}} = \frac{2}{\pi^2} \left(t_{\mathrm{P}} H\right)^2$$

(H is evaluated at Hubble-horizon exit, i.e. at $|k\eta| = 1$)

First observational test of quantum gravity

- Within the inflationary scenario, the observed CMB fluctuations can only be understood from quantized metric plus scalar field modes.
- This is an indirect test of linearized quantum gravity (formulated by Bronstein in 1936).
- The observation of primordial B-modes would be an indirect confirmation of the existence of gravitons.
- The difference in the duration of inflation between the 'cold spots' and the 'hot spots' in the CMB spectrum is only of the order of the Planck time.

Decoherence in quantum cosmology

In quantum cosmology, arbitrary superpositions of the gravitational field and matter states can occur. How can we understand the emergence of an (approximate) classical universe?

- ‘System’: global degrees of freedom (scale factor, inflaton field, ...)
- ‘Environment’: small density fluctuations, gravitational waves, ...

(Zeh 1986, C.K. 1987)

Example: scale factor a of a de Sitter universe ($a \propto \mathrm{e}^{H_{\mathrm{I}}t}$) (‘system’) experiences decoherence by gravitons (‘environment’) according to

$$\rho_0(a,a') \to \rho_0(a,a')\exp\left(-CH_{\mathrm{I}}^3 a(a-a')^2\right)\ ,\ C>0$$

The Universe assumes classical properties at the beginning of inflation

(Barvinsky, Kamenshchik, C.K. 1999)

Hermann Weyl (Raum-Zeit-Materie):

In dem Dunkel, welches das Problem der Materie annoch umhüllt, ist vielleicht die Quantentheorie das erste anbrechende Licht.

In the darkness, which still wraps up the problem of matter, perhaps quantum theory is the first dawning light.

The role of matter

- Recall attempts by Weyl and Einstein: resolve the duality between spacetime and matter
- Quantum general relativity: no unification (yet) with non-gravitational fields – can the origin of mass be understood in this framework?
- Quantization of gauge theories of gravity; role of fermions
- Conformal invariance at high energy (early Universe)? Origin of mass through conformal symmetry breaking?
- Alternative framework: string theory
- Problem of divergences

Conclusion

- The wave functional in quantum geometrodynamics depends on the three-dimensional metric (not the connection!) and effectively on the three-geometry (superspace); external time and therefore spacetime have disappeared.
- In string theory, (part of) space may have diappeared, too.
- Emergence of mass in quantum gravity is largely open