Gauging the spacetime metric,

looking back and forth a century later

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Hermann Weyl in 1918/19:

The general theory of relativity (GTR) should be built upon "conformal geometry" complemented by a

> *"transfer principle for the unit of length from a point P to an infinitesimally close one."* (Weyl 1918)

"Distant comparison" (Fernvergleich) is inadmissible in a pure "proximity geometry" (Nahegeometrie):

> . . . *"only segments at the same place can be measured against each other.* Gauging of segments is to be carried out at each single place of the world (Weltstelle), *this task* cannot be delegated to a central office of standards (zentrales Eichamt)." (Weyl 1919)

Scale invariance 50 and 100 years later:

Robert Dicke 1962

> *"It is evident that the particular values of the units of mass, length, and time employed are arbitrary and that the laws of physics must be invariant under a general coordinate dependent change of units."*

Gerard 't Hooft 2015:

> *"Small time and distance scales seem not to be related to large time and distance scales. (...) this is because we fail to understand the symmetry of scale transformations. (...) Since the world* appears *not to be scale invariant, this symmetry, if it exists, must be spontaneously broken."*

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I. Historical remarks

1. H. Weyl's purely infinitesimal geometry of 1918

Origin of ("local") gauge 1918: metric rescaling

- Riemannian geometry affected by a residuum of "distant geometry": direct comparison of lengths $l(p), l(q)$ at $p \neq q \in M$ possible.
- Geometry, based on field theory, demands a foundation on concepts which allow only "local" comparison (i.e., in inf. regions only).
- This is possible by a modest generalization of Riemannian geometry: conformal structure $\mathfrak{c} = [g]$ and a *prescription of how to compare length measurements* in infinitesimally close points p, p'.
- Mathematically realized by real valued differential form $\varphi = \varphi_\mu dx^\mu$ length (later scale) connection, depending on choice of $g \in \mathfrak{c}$, a *gauge* in the literal sense.
- Change of gauge $g \rightarrow \tilde{g} = \Omega^2 g$ accompanied by change of diff-form $\varphi \mapsto \widetilde{\varphi} = \varphi - d \log \Omega$: gauge transformation of scale connection.

Important mathematical features

- A Weylian metric is given by equivalence class of pairs $[g, \varphi]$ with equivalence by change of gauge as above.
- *Theorem (Weyl)*: $\exists$ unique compatible ("längentreu") affine connection, $\Gamma([g, \varphi])$.
- Curvature tensors $Riem, Ric = (R_{\mu\nu})$ are *scale invariant*, scalar curvature $R = R^{\nu}_{\nu}$ *scale covariant* of weight $w = -2$ (if g is considered of weight 2).
- In addition: curvature of scale connection, just $f = d\varphi$
  Warning: Weyl wrote $d\varphi$ *for our* φ, an "infinitesimal quantity" (Cartan notation for diff. forms was not yet in general use).
- Riemannian geometry is a special case, if
  (i) length(scale) connection is locally integrable, $f = d\varphi = 0$,
  (ii) first cohomology of M zero, i.e. φ globally integrable,
  (iii) and gauge chosen such that $\varphi = 0$, Riemann gauge.

Physical features (1918–1923)

- Weyl proposed a geometrically unified field theory of grav and em. Built upon it: a conjectural program for a field theorical matter theory (Mie–Hilbert–Weyl).
- Einstein's objection: In non-integrable Weyl geometry atomic frequencies should depend on the "history" of the atom.
- Weyl's counter: We first need a theory of measurement based on field concepts.
  Hypothesis: *Atomic frequencies* are not controlled by length transfer ("Übertragung") but by adaptation ("Einstellung") to the local field constellation.
  Ad hoc assumption: atomic frequencies adapt to the Weyl geometric scalar curvature R (scalar field of weight -2).
  $\rightarrow$ Measurement in scale gauge for which $R = const$ (Weyl gauge).

Foundational and philosophical features (1921–1923)

(→ T. Ryckman's talk)

In 1921 Weyl started to doubt whether his UFT would be able to fulfill his hopes of 1918/19. He began to reconsider it from a philosophical/conceptual point of view. This resulted in his

- Mathematical analysis of the problem of space (PoS): characterization of very general features for the groups which can play the role of congruences and similarities in the infinitesimal, provided certain *apriori* principles hold ("analytic" ones or "synthetic" ones). Barcelona lectures (Feb 1922).
- In 1921 (app. I, 4th ed. *Raum-Zeit-Materie* and separate publication in *Gött Nachr.*) he argued that, assuming a Weyl geometric structure of spacetime, its metric is determined by the light rays and the inertial paths of test bodies. No rigid rods and stable clocks presupposed. — An intriguing structural argument with regard to the foundations of spacetime theory.

2. Revocation of scale gauge geometry by Weyl after 1927/29

Transfer of gauge idea from scale to phase in QM

- In 1925/26 Schrödinger, London et al. proposed to use gauge principle for wave function in QM, rather than for scale in gravity.
- Weyl 1927: approved it in 1st ed. *Gruppentheorie und QM*.
  Weyl 1929: Dirac spinor fields can be formulated in GTR (Einstein gravity), but with a point dependent $U(1)$ underdetermination of Dirac spinors.
  Natural introduction of electromagnetic field as connection with values in $Lie\,(U(1)) \cong \mathbb{R}$, parametrizing the phase of Dirac spinors.
- Weyl considered this as a strict and definitive alternative to his scale gauge geometry: *Gauge idea migrated from geometry to QM*.

($\rightarrow$ N. Straumann's talk)

Mathematical and physical automorphisms

In 1948/49 talk *Similarity and congruence* (similarly in Engl. ed. *Phil. of Math. and Nat. Science*) Weyl discussed automorphism groups, important for establishing objectivity of symbolic knowledge in mathematics and in physics. Clue to a *structural approach to knowledge*.

- "The physicist will question Nature to reveal him her true group of automorphisms."
  Physical automorphisms of classical physics induced from geometry: Galilei group with spatial reflections.
- Mathematical automorphisms largest group which respects the structure (normalizer of physical automorphisms).
  For classical geometry (physics): Similarities (extended Galilei group).
- Phys. automorphisms of relativistic physics (spacetime M): Diff(M) extended by point dependent operations in $G = SO(1,3) \times U(1)$ — gauge group $\mathfrak{G}(P)$ *ante letteram* of a principal bundle $\mathfrak{P}$ over M with group G.
- Math. automorphisms dito, but scale extended, $\widetilde{G} = G \times \mathbb{R}^+$.

Difficult distinction of phys. and math. automorphisms

- For classical mechanics the specification of phys. auto's was easy: material operations with rigid bodies (proper congruences).
  19th cent. physics (electromagnetism and crystallography) had to consider already structure of physical laws (space reflections).
- In relativistic physics *physicality* of automorphisms more indirect: central role of *conservation principles* (in particular charge),
- but in GTR *physicality of diffeos* already more *complicated*, although they generalize the translational symmetry.
  Equivalence principle invoked.

Weyl's revocation

- In 1940sff. Weyl considered scale no longer as part of physical automorphisms/symmetry (mathematical only):
- "*The atomic constants of charge and mass of the electron atomic constants and Planck's quantum of action* $\hbar$*, which enter the universal field laws of nature, fix an absolute standard of length, that through the wave lengths of spectral lines is made available for practical measurements.*" (1948/49 talk, similar in PMN)
- Taking up his language of 1919 the *laws of QM* and $(\hbar, e, m_e)$ have taken over the role of the *central office of standards*.
  A definitive good bye (for Weyl) to the idea of an atomic standard of length/time due to adaptation to local field constellations.

3. Retake of Weyl geometry in the 1970s

Foundations of spacetime: Ehlers/Pirani/Schild 1972 ff.

- Question: Do a causal (cone) structure and a projective path structure, satisfying a certain compatibility condition, determine a Weylian metric $[g, \varphi]$? – If so, it is unique (Weyl 1921).
- Existence discussed and (nearly) established.
  Assuming certain (plausible) principles serving as the basis of a "constructive axiomatics" EPS showed:
  $\exists$ a conformal structure $\mathfrak{c} = [g]$ and an affine connection ∇ which satisfy the following compatibility prop. (*EPS-compatibility*):
  geodesics of ∇, null at some point, remain on the null-cone of $\mathfrak{c}$.
- Question not posed by EPS (rather assumed as self-evident):
  EPS-compatibility equivalent to Weyl's compatibility condition ("Längentreue"), technically $\nabla g + 2\varphi \otimes g = 0$?
- Question posed, but not answered, by EPS:
  How can one come from the Weyl structure to a Riemannian metric?
  Understood as: Why is the Weylian metric integrable?
- During the following decades diverse follow up investigations in physics (at least 2 authors here present) and in phil. of spacetime.

Scalar field: Omote-Utiyama-Dirac (1971,1973) ff.

- Weyl geometric approaches to gravity re-opened independently: *M. Omote* (1971), *R. Utiyama* (1973), *A. Bregman* (1973) in Japan and *P.A.M Dirac* (1972/73) et al. in Europe.
- *Common idea*: Scale invariant modification of Hilbert term coupled to a scalar field ϕ (weight $w(\phi) = -1$) similar to JFBD (Jordan-Fierz-Brans-Dicke theory): $\mathfrak{L}_{HW} = -\epsilon_{sig} \frac{1}{2} \phi^2 R \sqrt{|g|}$, with R Weyl geom., $w(R) = -2$. ($\epsilon_{sig} = +1$ for *sign* $g = (+ - - -)$, $\epsilon_{sig} = -1$ else)
- Difference to JFBD (inf the following shorter BD): Kinetic term of scalar field with scale covariant derivative D, scale connection ("Weyl field") φ with kinetic term $-\frac{1}{4} f_{\mu\nu}\, f^{\mu\nu}$
- *Different physical contexts* and interpretations: Omote, Utiyama, Kugo, Bregman (Weyl-Cartan geometric) : *nuclear physics*, , ϕ "measuring field" φ massive "Weylon"; Dirac: *cosmology* (large number hypothesis !), *geophysics* (expanding earth !), ϕ^{-2} varying gravitational constant , φ electromagnetic potential (Proca field ?!).

Diverse follow up investigations until present

- 1970/80s direct successors of
  Utiyama: *Kugo, Hayashi*: Weylon φ mass close to Planck scale?
  Dirac: *Maeder, Canuto, N.Rosen/M. Israelit* (until 2010): attempts at cosmology, contributions of φ and/or ϕ to dark matter (Zwicky).
- First link to standard model SM (elementary particles):
  L. Smolin 1979, Hung Cheng 1988, W. Drechser/H Tann 1999ff: Scalar field ϕ as norm (expectation value) of Higgs field Φ?
- *Nishino/Rajpoot 2004ff.*: scale/conformal symmetry close to Planck scale E_P. Breaking of scale symmetry shortly below E_P, Weylon acquires mass; ϕ "Goldstone boson", different from Higgs ....
  Similar *Ohanian 2016*, but different symmetry breaking.
- *M. Novello 1992ff. and Brazilean school* until present: Palatini approach $\rightarrow$ integrable Weyl geometry (dynamically inert version). Study of non-scale invariant gravitational Lagrangians, diverse cosmological scenarios. ($\rightarrow$ C. Romero)

($\rightarrow$ ES, "Unexpected resurgence of Weyl geo. ..., arXiv:1703.03187)

New contexts (in comparison with 1920s)

- New interest in conformal transformations in high energy physics
  F. Bopp 1954, J. Wess 1959, H.-P. Dürr, W. Heisenberg et al. 1959 (Dirac field). H. *Kastrup 1962ff. ...*
  ($\rightarrow$ Kastrup, Ann.Phys. 2008).
- Localizing symmetries of SRT, not only of internal field symmetries, but also geometrical ones for generalized gravity,
  D. Sciama (1960)1962, T. Kibble 1961.
- Introduction of Cartan geometric ideas into gravity,
  F. Hehl 1966, A. Trautman 1973 and others.
  More general groups than Poincaré one ($\rightarrow$ F. Hehl's talk).
- Conformal trafos in BD gravity, ca. 1960ff.
- New riddles in cosmology, in particular expansion and initial singularity, approached by conformal or Weyl geometric methods.
  (*A. Maeder 1978f., C. Wetterich 1988, P. Mannheim 1990sff., Novello* et al. 1992ff. *Steinhart* et al. 2002)
  Dark matter (*Rosen/Israelit 1970sff., Cheng 1988, ...*)
- Important: *"Near" scaling invariance* of the SM, since 1970s.

Interlude

4. Terminology, concepts, notation

Weyl metric, Weyl structure, scale geometry

- Weylian manifold $(M, [g, \varphi])$ with unique compatible affine connection Γ, covariant derivative ∇, s.th. $\nabla g + 2\varphi \otimes g = 0$. Curvatures $Riem, Ric$ scale invariant, R scale covariant weight -2.
- Equivalent: Weyl structure $(M, \mathfrak{c}, \nabla)$, where ∇ compatible with $\mathfrak{c}$ in the sense: $\forall_{g \in \mathfrak{c}} \exists_{\varphi} \nabla g + 2\varphi \otimes g = 0$ (gauge trafos for φ-s follow).
- Scale bundle $M \times (\mathbb{R}^{+}, \cdot)$; scale group $\Omega \in \mathbb{R}^{+}$ operates on tangent, vector, tensor, spinor bundles over M with different weights $w = w(X)$, i.e. $X \mapsto \Omega^{w} X$.
  Then the scale covariant derivative is $DX = \nabla X + w\varphi \otimes X$.
- Compatibility of ∇ with g is $Dg = 0$, i.e., metricity in the Weyl geometric sense. It appears as "*semi-metricity*" only if judged from the (inadequately narrow) Riemannian perspective!

Geodesics, scalar field, scale gauges

- Geodesics, $\gamma(\tau), u = \dot{\gamma}$; *two variants*:
  scale invariant (Weyl), $\nabla_u \dot{u} = 0 \longleftrightarrow \ddot{\gamma}^\lambda + \Gamma^\lambda_{\mu\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu = 0$
  scale covariant, $w(u) = -1$, $D_u \dot{u} = 0$, then $g(u,u)$ scale invariant,
  (important for cosm. models/flow) $D_u \dot{u} = 0 \leftrightarrow \ddot{\gamma}^\lambda + \Gamma^\lambda_{\mu\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu - \varphi_\mu \dot{\gamma}^\mu \dot{\gamma}^\lambda = 0$.
- Weyl geometric gravity ("scalar tensor"): scalar field $\phi, w(\phi) = -1$,
  coupled to Hilbert term $(\pm\frac{1}{2}(\xi\phi)^2 R\sqrt{|g|}) +$ kinetic term $+V(\phi)$,
  ξ hierarchy factor between scale of ϕ and Planck scale.
- Scalar field gauge the one in which $\phi \doteq \phi_o$ (real) constant
  ("$\doteq$" equality in specified gauge).
  In particular Einstein gauge for $(\xi\phi_o)^2 = (8\pi G)^{-1} = E_P^2$.
- In integrable Weyl geometry (iWG) Riemann gauge,
  the one in which $\varphi \doteq 0$ (corresponds to "Jordan frame" in BD).
- Scale invariant observable quantities $\check{X}$ of fields X, $w = w(X)$,
  theoretically in any gauge by forming proportions: $\check{X} = \phi^w X$.
  Or in scalar field gauge; then $\check{X} \doteq X$ (up to constant factor).

Weyl geometric gravity, in particular iWG

- Generalized Einstein equation, scale independent ($w(T) = -2$)

$$Ric - \frac{R}{2} g = (\xi\phi)^{-2} T + {\scriptstyle (\epsilon_{sig} V(\phi)\phi^{-2} g)} \qquad (1)$$

- In *low energy regions* reasonably $d\varphi = 0$, "Weyl field" effectively trivial (mass of "Weylon" close to Planck scale): iWG.
- Even then, scalar field induces 1 additional degree of freedom, scale covariantly encoded by the pair (ϕ, φ).
- If $\phi \doteq e^{\omega}$ in Riemann gauge, $\varphi \doteq -d\omega$ in Einstein gauge. Geodesics in Einstein gauge influenced by potential ω. Dynamical consequences for inertial paths.
- "Energy conservation" for X free field $D_\nu T(X)^{\mu\nu} = 0$.
  In iWG and Riemann gauge ("Jordan frame"), $\nabla_\nu T(X)^{\mu\nu} \doteq 0$.
- Important for geodesic principle in Weyl geometric gravity ($\rightarrow$ D. Lehmkuhl's talk).

II. Why/in which respect still of interest today?

5. Standard model and gravity (effective, classical)

Two scalar fields, Φ (resp. H) and ϕ ...

- SM Lagrangian can be lifted from SRT to Weyl geom. framework,
  Here length/time weights, rather than mass weights, $w(\Phi) = -1$ ....
  Higgs field Φ, values in $SU(1)$ representation (bundle), isospin 1, etc., Φ^* adjoint.
  "Real norm" (vacuum expectation value) $H = \sqrt{<\Phi^*\Phi>}$.
  Conf. invariance of classical (low energy, effective mean) SM Lagrangian
  broken by Higgs mass term $-\frac{m_H^2}{2}H^2$ only.
- Coupling of Φ to ϕ makes mass term scale invariant: $-\frac{(\eta\phi)^2}{2}H^2$.
  η *ew-scale* hierarchy factor. – Higgs mass due to coupling of Φ with gravitational scalar field?
- With ∇_ν (respectively D_ν) and volume form $\sqrt{|g|}dx^{(4)}$, Lagrangian becomes scale invariant, of the form

$$\mathfrak{L} = \mathfrak{L}_{grav} + \mathfrak{L}_\phi + \mathfrak{L}_{SM} \tag{2}$$

(Smolin 1979, Cheng 1988, Drechsler 1999, Nishino/Rajpoot 2004ff., Cesare/Moffat/Sakellariadou 2018; similar although not Weyl geometric: Englert et a. 1975, Wetterich 1988 ....)
Hierarchy factors ξ, η added here (estimate $\rightarrow$ below).

...with a common biquadratic potential

- Potentials $V(H) = \frac{\lambda_{ew}^2}{4} H^4 - \frac{(\eta\phi)^2}{2} H^2$ and $V(\phi) = \beta\phi^4$ combined in a common biquadratic potential

$$V(\phi, H) = \left(\frac{\lambda_{ew}}{2} H^2 - \frac{(\eta\phi)^2}{2\lambda_{ew}} \right)^2 + \frac{\lambda_\phi}{4} \phi^4 \tag{3}$$

(→ Shaposhnikov/Zenhäusern 2009 for SR)

- In pot. min. ($H = H_o$) squared bracket vanishes, $\lambda_{ew}^2 H_o^2 = (\eta\phi)^2$. Norm of Higgs field proportional to ϕ, in any gauge. In particular

$$\phi \doteq const = \phi_o \quad \leftrightarrow \quad H_o \doteq const = h_o = 2^{-\frac{1}{2}} v \quad \approx 174 \;\text{GeV} \tag{4}$$

$$\text{Einstein gauge} \quad = \quad \text{Higgs gauge} \qquad \text{where} \quad \Phi = \frac{1}{2} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$\text{for} \quad \xi\phi_o = E_p \qquad , \qquad \underset{(\lambda_{ew}^{-1}\eta)\,\phi_o \,=\, h_o \,=\, \frac{v}{\sqrt{2}}}{\eta\phi_o = m_H} \quad \text{eff. Higgs mass/energy} \approx 126 \; GeV \tag{5}$$

- In pot. min. $V(\phi, H)$ reduces to quartic monomial $V(\phi) = \frac{\lambda_\phi}{4}\phi^4$ and contributes essentially to "*cosmological constant*". Question: Can *pot. min.* be considered as approx. ground state ? (Hope so!)

Weyl's *natural gauge* revisited

- Electron mass in SM results from interaction with Higgs field, scale covariantly $m_e = \mu_e h$; in Einstein/Higgs gauge $m_e \doteq \mu_e v$. m_e scales with h, thus weight $w(m_e) = -1$, as it must be.
- If $\hbar, e$ are treated as universal constants, *Rydberg parameter* $R_{yd} = \frac{m_e}{2\hbar^2} e^4 = \frac{\mu_e e^4}{2\hbar^2} h$ scales with h, $w(R_{yd}) = -1$.
- Then atomic frequencies, e.g. Balmer series $E_n = -R_{yd} n^{-2}$, also scale with weight $w = -1$.
- New SI conventions define physical units based on measurement of atomic frequencies ($Co_{\dots}$) and universal constants $c, \hbar, e, \dots$. Good reasons for identifying Higgs gauge with SI-measured quantities. Gravitational scalar field ϕ indirectly a "measuring field".
- Weyl's idea, adaptation of atomic clocks to local field constellation, *vindicated in new context* (Higgs field in place of scalar curvature)! If so: Weyl's *"natural" gauge = Higgs gauge = Einstein gauge*.

6. Cosmology and astrophysics (galaxy rotation)

“Cosmological constant”

- The *em tensor of the scalar field* $T^{(\phi)}$, resp. its total right hand side term $\Theta^{(\phi)} = (\xi\phi)^{-2} T^{(\phi)}$, contains a metric proportional component $\Theta^{(\phi ii)} = \Lambda g$, where $w(\Lambda) = -4$.
- Λ steps into the place role of the “cosmological constant”.
As part of $\Theta^{(\phi)}$ it is no longer an “absolute” element of the theory and no longer a true constant (like in other scalar tensor theories of gravity).
- Main contribution to Λ from *quartic pot. term* in $\mathfrak{L}_\phi$. Call it Λ_4.
In Einstein/Higgs gauge in fact *constant*, $\Lambda_4 \doteq \xi^{-2} \frac{\lambda_\phi}{4} \phi_o^2$.
Order of magnitude $\Lambda_4 \sim H_o^2$ (here H_o present Hubble parameter) leads to hierarchy sequence $\quad H_o \xrightarrow{\cdot\xi} \phi_o \xrightarrow{\cdot\xi} E_{Pl}$.
- Then ϕ_o roughly geometric mean of $H_o\,[\hbar]$ and E_{Pl} $\quad(\phi_o \sim 10^{-4}\, eV)$, with $\xi \sim 10^{31}$ and ew hierarchy factor $\eta \sim 10^{15}$.

Consequences ?

- Consequences for cosmological dynamics/ models depends on kinetic term of ϕ ($\rightarrow$ other scalar tensor theories).
- *Energy of small coherent wave packets*, wave vector k (null, weight -1, encoding frequency/energy), as seen by *cosmological observers* in timelike geodesic flow X ($w(X) = -1$), is given by $g(k, X)$. This is the clue to cosmological redshift ($\rightarrow$ Weyl 1923).
- $g(k, X)$ is scale invariant. This allows a *scale invariant representation of cosmological redshift* and relativizes the role of "expansion" for the latter's explanation (in addition to C. Wetterich's assumption of changing emission frequencies in cosmological rest frames).
- More detailed investigations needed.

A scale covariant version of RAQUAL

- Consequences for galaxy rotation curves ? Try kinetic L_ϕ term similar to MOND (RAQUAL) with fractional power $|D_\nu \phi D^\nu \phi|^{\frac{3}{2}}$.
  (Similar, adapted, in TeVes, superfluid approach etc.)

- This term plus conformally coupled quadratic kinetic term implies (after subtracting the trace of Einst. equ.) the scalar field equation

$$_g\nabla_\nu (|\nabla \omega| \partial^\nu \omega) \doteq 4\pi G\, \tilde{a}_o \, (\rho_m - 3 p_m) \qquad \text{in Einstein gauge} \qquad (6)$$

  ($\Phi \doteq e^\omega$ in Riem. gauge, $_g\nabla$ Levi-Civita in Einstein gauge, $\tilde{a}_o$ from coupling const. fractional power term)
  Covariant version of Milgrom equation (non-linear Poisson equ. MOND).

- Problems:
  Lensing needs additional energy momentum (dark matter).
  Moreover *screening* problem, like in other scalar field approaches.

A dark matter term coupled to ϕ ?

- (i) Lensing problem can be solved by an *additional Lagrange term* $\mathfrak{L}_{dm}$, coupled to ϕ, with sufficiently high energy momentum but *without* influence on the Milgrom equation.
  (ii) The "conspiracy" between *ordinary matter and dark matter* in galaxy rotation curves may then result from coupling of $\mathfrak{L}_{dm}$ to ϕ.
- A scalar field X with $X \sim \phi$ (because of $V_X = (X^2 - const \cdot \phi^2)^2$) and an unconventional (2nd order !) *Lagrangian* of the form

$$\mathfrak{L}_{dm+} = \beta\, \phi^{-2} X^2 D_\nu D^\nu X^2 \sqrt{|g|} \tag{7}$$

  realizes (i) and (ii).
  Example: $X^2 = H^2 = < \Phi^* \Phi >$ would bring ϕ into the "Higgs portal", although in a rather peculiar way.
- But the dynamical equation of X would no longer admit the potential minimum as ground state solution! (Approximately ??)

7. Final remarks

Comparison: Weyl geometric gravity 2018 — 1918

- *Recent/present work* in Weyl geometric gravity often enhances the gravitational sector by a scalar field ϕ of weight $w = -1$.
- Weyl's idea of *length transfer no longer central*.
  But φ important for calculus of scale covariant derivates.
- In *low energy regimes* with effectively supressed scale curvature, a closed scale connection φ with $d\varphi = 0$ plays a role, seemingly a "trivial" Weyl vector field.
- But even then a modification of Einstein gravity with consequences for the intertial dynamics is possible, if the *scalar field is not constant in Riemann gauge*.
- *Non trivial 'Weyl vector field"* close to the Planck regime explored by some authors (Ohanian 2016, ...).

Scaling as mathematical and/or physical automorphism

- A scale covariant formulation of gravity and matter fields (in part. SM, astrophysics, cosmology) seems to extend, at first, only the mathematical automorphism group of fundamental physics.
- Why should we do so ? (cf. Kretschmann's argument!)
- It is conceptually interesting (*philosophy of physics*).
  It even may *pave the way* towards extending physical insight.
- But if *new physical insight results* from extending the mathematical automorphism group (e.g., in cosmology (redshift, expansion etc.) or in astrophysics (dark matter effects etc.)), scale extension could become part of the physical automorphisms . . .
  . . . if one inderstands physicality of automorphims in a broader sense than suggested by the exclusive criterion of new *conserved quantities*, often considered as part of the gauge theoretic paradigm.