LOOP QUANTUM GRAVITY

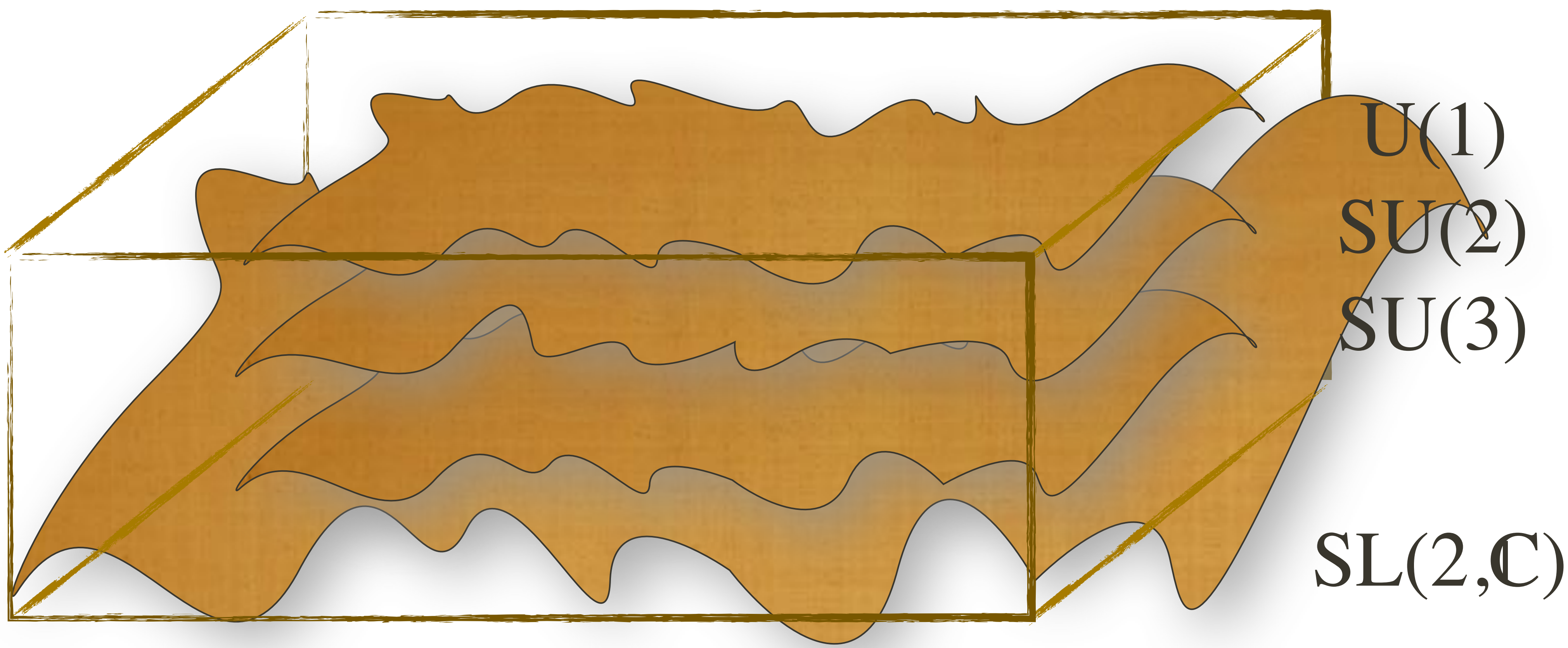
a general-covariant lattice gauge theory

Francesca Vidotto

UNIVERSITY OF THE BASQUE COUNTRY

Bad Honnef - August 2nd, 2018

- GENERAL RELATIVITY: background independence!



- FIELDS ⟷ GAUGE SYMMETRIES
    - GRAVITY AS AN INTERACTING GAUGE FIELD

IN THIS TALK

GAUGES IN GENERAL RELATIVITY $\longrightarrow \begin{cases} \text{- diffeos} \\ \text{- Lorentz} \end{cases} \longrightarrow \begin{cases} \text{- graph/lattice} \\ \text{- group variables} \end{cases} \longrightarrow$ LOOP QUANTUM GRAVITY

GAUGE VARIABLES are the handle for the possible interactions of a system

GAUGE

INVARIANT OR NOT?

Rovelli, Why gauge? 2013

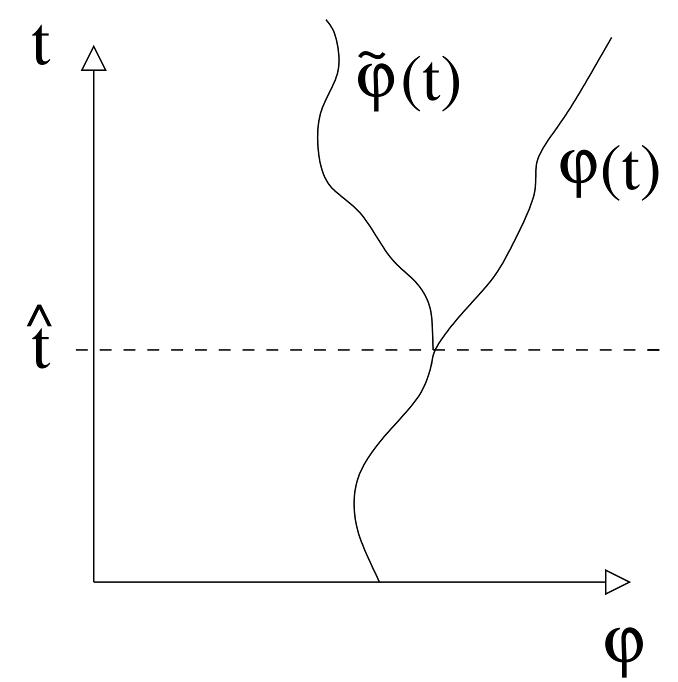
- $$S[e,\omega,A,\psi,\varphi] \;=\; S_{RG}[e,\omega] + S_{YM}[e,A] + S_f[e,\omega,A,\psi] + S_{sc}[e,A,\psi,\varphi]$$
  - Local Yang-Mills gauge transformations
  - Local Lorentz transformations
  - Diffeomorphism gauge transformations

- DIRAC

  A system is gauge invariant
  <u>if evolution is under-determined</u>.

- DETERMINISM

  Classical physics is deterministic
  → consider only gauge invariant quantities
  as "PHYSICAL"



Rovelli, Why gauge? 2013

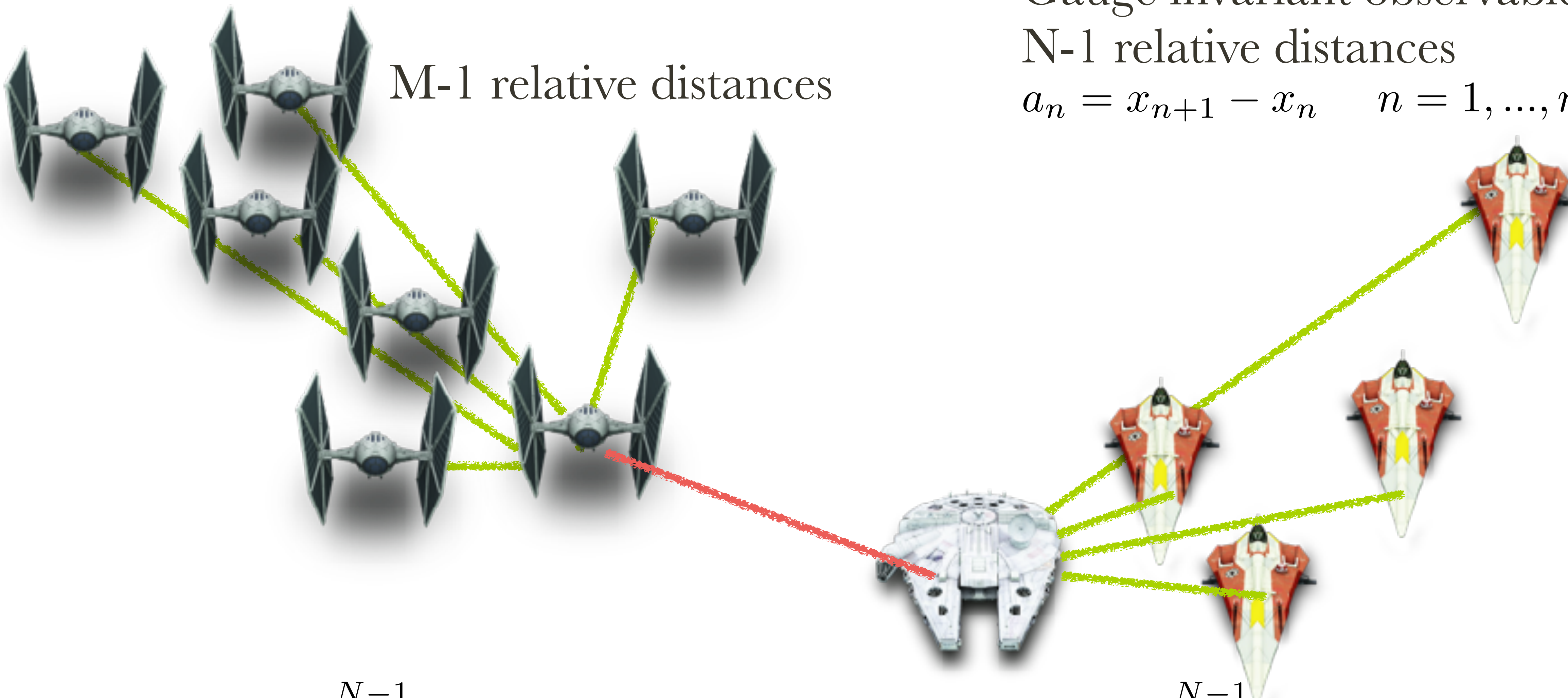
- REDUNDANCY: EFFECTIVENESS OF GAUGE UNEXPLAINED!
  - Arkani-Hamed, Cachazo, Kaplan ('10): gauge is just a complication!
  - AdS/CFT: physics coded on the *asymptotic* boundary
  - …
- GAUGE INVARIANCE: INTERPRETATION, NOT COUPLING
  - We interpret a physical system in terms of its gauge invariant objects
  - Necessity of coupling the system via non-gauge-invariant variables!
- COUPLING GAUGE SYSTEM
  - EM: $F = dA$ $\qquad L = \bar{\psi}(\gamma^\mu A_\mu)\psi$
  - Gravity: $g_{\mu\nu}$ $\qquad L = \sqrt{g} g_{\mu\nu} T^{\mu\nu}$

Gauge-invariant coupling from gauge-variant variables

ROVELLI'S SPACE FLEET

Rovelli, Why gauge? 2013

Gauge invariant observables:
N-1 relative distances
$a_n = x_{n+1} - x_n \qquad n = 1, ..., n-1$



$$L_1 = \frac{1}{2}\sum_{n=1}^{N-1}(\dot{y}_{n-1} - \dot{y}_n)^2 \qquad L_1 = \frac{1}{2}\sum_{n=1}^{N-1}(\dot{x}_{n-1} - \dot{x}_n)^2$$

$$L_{int} = \frac{1}{2}(\dot{y}_1 - \dot{x}_N)^2$$

EXTRA DOF: more gauge-inv observables than in the single systems

GAUGE AND RELATIONALITY

Rovelli, Why gauge? 2013

- RELATIONALITY
  The observables of a system are not its gauge-dependent quantities but rather the relation between them:

  Dirac observables are always relational

  In fact, we always deal with subsystems.

- HOLISM
  The observables of the coupled system are more than the sum of individual ones. Gauge-dependent variables contain information about how to couple systems.

- MEASUREMENT VS PREDICTIONS
  Gauge variables are what we measure but we cannot predict (**Partial Observables**). A couple of gauge variables is gauge invariant and it allows predictions.

- These insights applies directly to General relativity and Quantum Mechanics. To carry the space fleet analogy for Yang-Mills theory requires some care! (The, 2013)

Examples

- Time
- General Relativity
- Yang-mills Field

- BACKGROUND INDEPENDENCE
  Bodies are only localized with respect to one another.
  Bodies includes all dynamical objects, also the gravitational field.
  Spacetime is built up by contiguity relations:
  the fact of being "next to one another".

- GRAVITY AS A GAUGE THEORY
  The proper time over a world line is a partial observable.
  Only if I have a second clock I can make predictions.

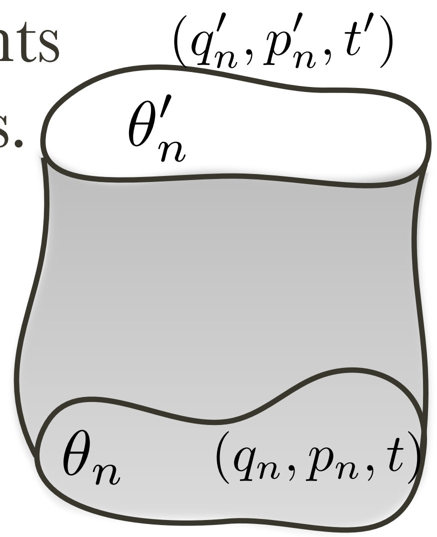
- COUPLING TO A MATERIAL REFERENCE SYSTEM
  The components of the gravitational field with respect to the directions defined by the matter system are gauge-invariant quantities of the coupled system; but they are gauge-dependent quantities of the gravitational field, measured with respect to a given external frame.

- VANISHING HAMILTONIAN
  Project on the physical space ignoring the partial observables, that are non-gauge-invariant.

Oeckl, 2003

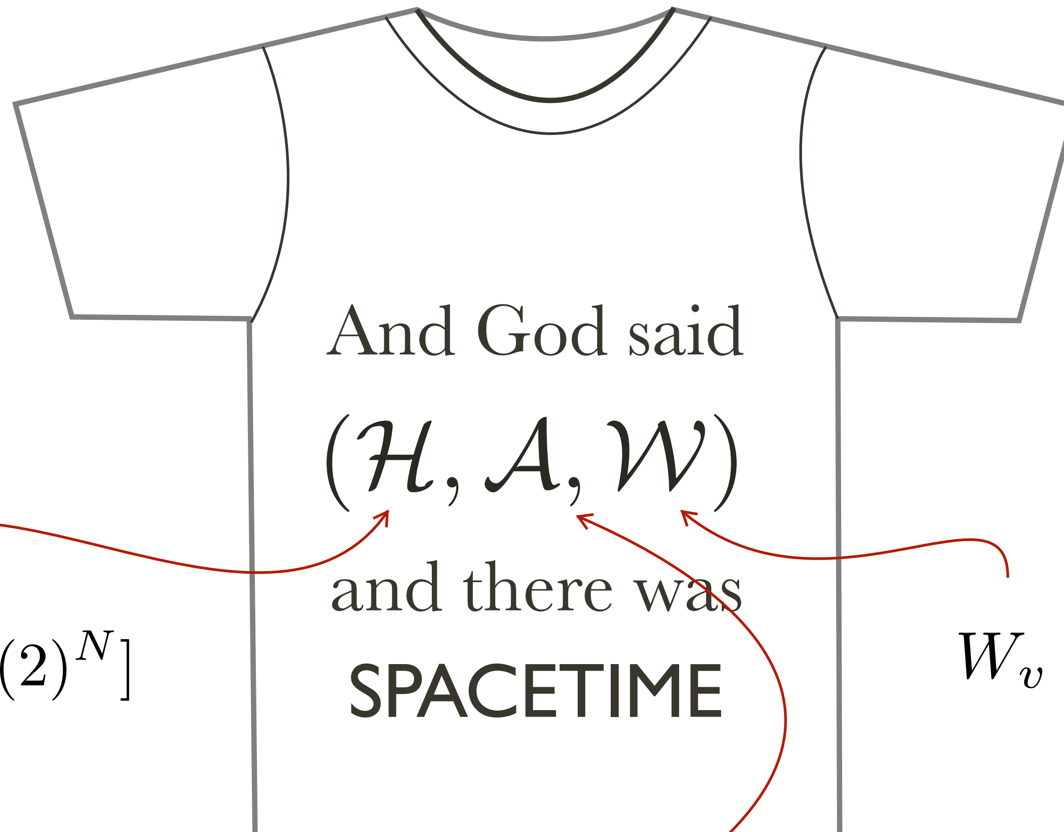
- Time is pure gauge, the Hamiltonian constraint determine time evolution
- Gauge-constraint for the internal gauge

- Democratisation of gauges: they all determine dynamical constraints among partial observables measured at the boundaries of a process.



- These constraints code the full content of a dynamical theory:
  - **Yang-Mills** constraint determines variable change *wrt a change of the* internal boundary frame.
  - **Diff** constraint determines variable change *wrt a change in the* location of the spatial boundary reference frame.
  - **Hamiltonian** constraint dertermines collective variable change *wrt a change in the* temporal location of the boundary *(time of measurement)*
- Indeterminacy $\longleftrightarrow$ arbitrariness of the frame choice.

- Dynamics is the study of relations between partial observables *that are gauge-dependent quantities of a system to which we can couple an apparatus.*



Hilbert Space:

$\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$

Transition Amplitude:

$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \; \psi_v)(\mathbb{1})$

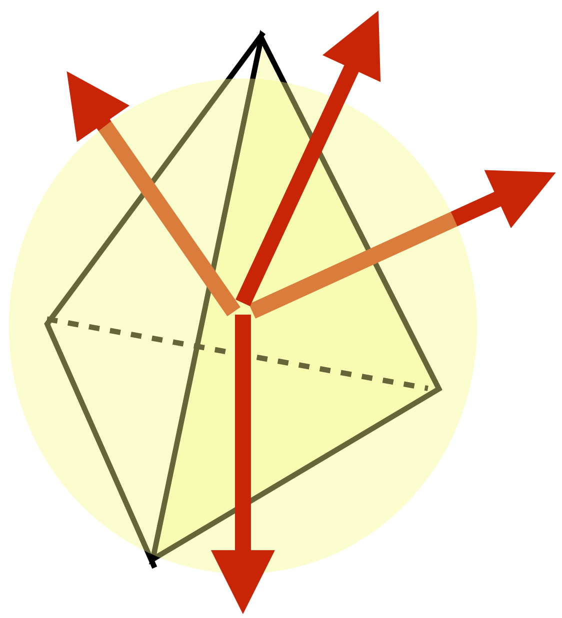
Operator Algebra:

$$[L_a^i, L_b^j] = i\delta_{ab}\ell^2\epsilon_k^{ij} L_a^k$$

$(\mathcal{H}, \mathcal{A}, \mathcal{W})$ defines a background independent quantum field theory

Loop Quantum Gravity

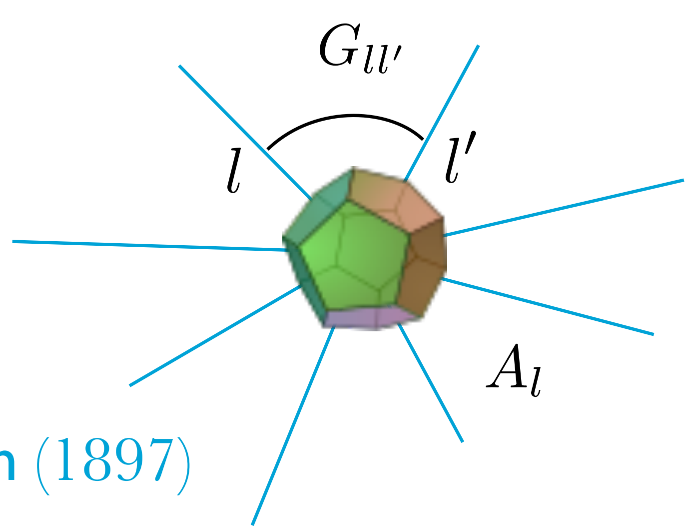
Kinematics



- $\vec{L}_l = \{L_l^i\}, i = 1,2,3$ SU(2) generators $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$

  gravitational field operator (tetrad)

- Gauge invariant operator $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ with $\sum_{l\in n} G_{ll'} = 0$



Penrose's **spin-geometry theorem** (1971), and **Minkowski theorem** (1897)

REPRESENTING GEOMETRIES

- Composite operators:
  - Area: $$A_\Sigma = \sum_{l\in\Sigma} \sqrt{L^i_l L^i_l}.$$
  - Volume: $$V_R = \sum_{n\in R} V_n, \qquad V_n^2 = \frac{2}{9}\,|\epsilon_{ijk} L^i_l L^j_{l'} L^k_{l''}|.$$
  - Angle: $$L^i_l L^i_{l'}$$

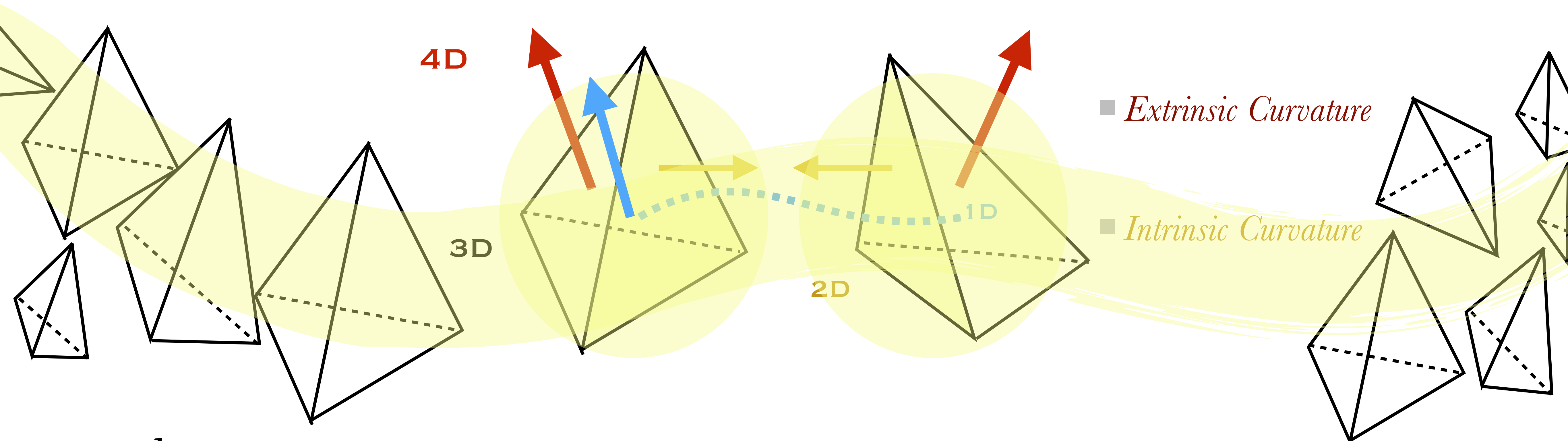
- Geometry is quantized:
  - DISCRETE: eigenvalues are discrete $\qquad \ell_P = \sqrt{\frac{\hbar G}{c^3}}$
  - FUZZY: the operators do not commute
  - quantum superposition *(coherent states)*

- Lorentz invariance is compatible with quantum discreteness!

(do not confuse with classical discreteness)

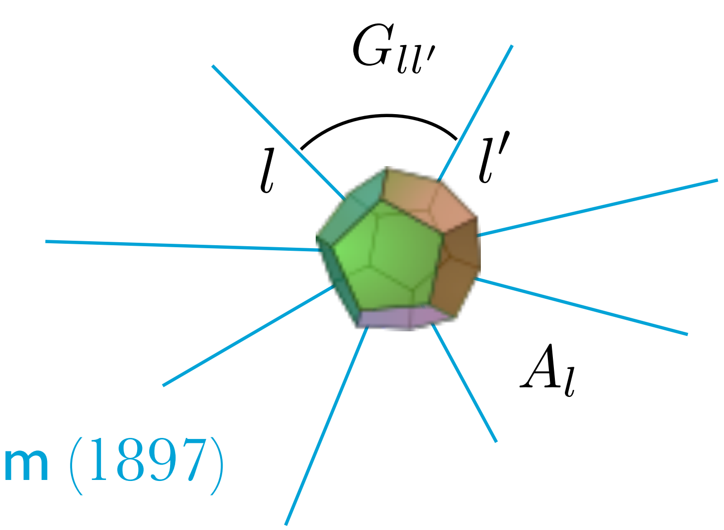
Same as the angular momentum: the discrete spectrum does not break the rotational symmetry.

- "Without a deep revision of classical notions it seems hardly possible to extend the quantum theory of gravity also to [the short-distance] domain." Matvei Bronstein



- h_l "Holonomy of the Ashtekar-Barbero connection along the link"
- $\vec{L}_l = \{L_l^i\}, i = 1, 2, 3$ SU(2) generators $L^i\psi(h) \equiv \left.\dfrac{d}{dt}\psi(he^{t\tau_i})\right|_{t=0}$

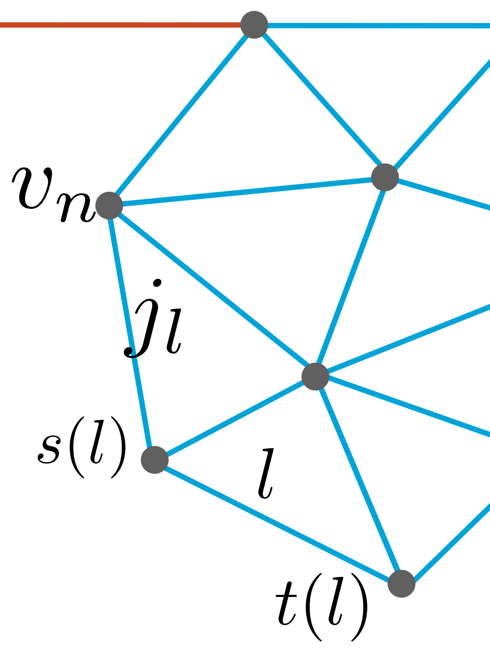
  gravitational field operator (tetrad)



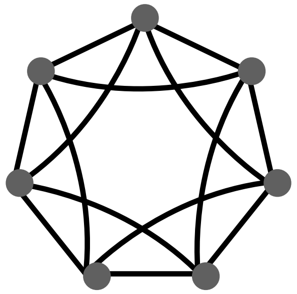
- Gauge invariant operator $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ with $\sum_{l \in n} G_{ll'} = 0$

Penrose's **spin-geometry theorem** (1971), and **Minkowski theorem** (1897)

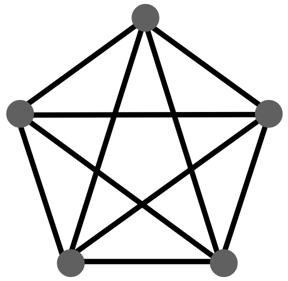
HILBERT SPACE



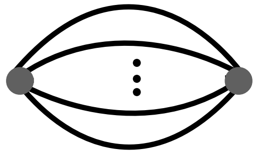
- Abstract graphs: $\Gamma = \{N, L\}$
- Group variables: $\begin{cases} h_l \in SU(2) \\ \vec{L}_l \in su(2) \end{cases}$
- Graph Hilbert space: $\mathcal{H}_\Gamma = L_2[SU(2)^L / SU(2)^N]$
- The space $\mathcal{H}_\Gamma$ admits a basis $|\Gamma, j_\ell, v_n\rangle$



- Restrict the states to a fixed graph with a finite number N of nodes. This defines an approximated kinematics of the universe, inhomogeneous but truncated at a finite number of cells.



- The graph captures the large scale d.o.f. obtained averaging the metric over the faces of a cellular decomposition formed by N cells.
- The full theory can be regarded as an expansion for growing N. For instance FRW cosmology corresponds to the lower order where there is only a regular cellular decomposition: the only d.o.f. is given by the volume.



- Different graphs can be useful to model different physical situations.

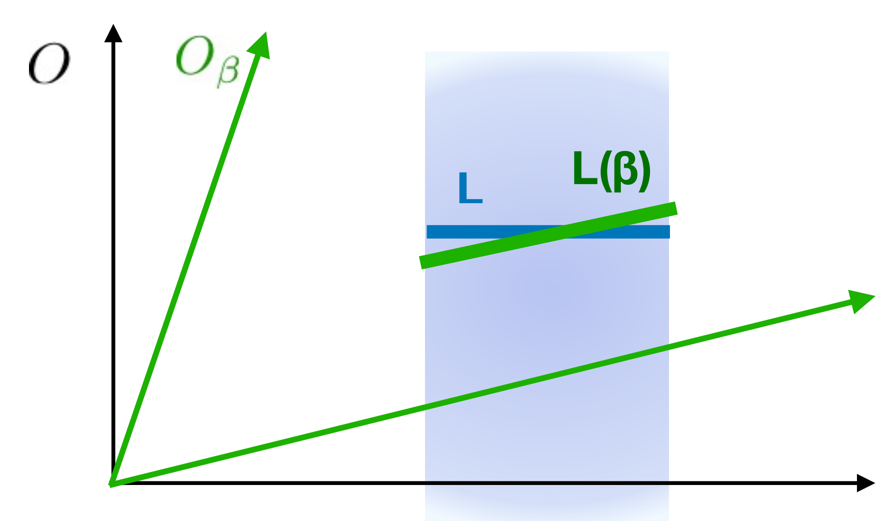
- Classical discreteness breaks Lorentz invariance. Quantum discreteness does not!
- Example: ROTATIONAL INVARIANCE
  - Classical: a vector with only discrete components breaks SO(3).
  - Quantum: quantum vector with discrete eigenvalues is compatible with a SO(3) invariant theory.

$$L_z|m\rangle = \hbar m|m\rangle$$

$$L_z(\theta)|m\rangle_\theta = R(\theta)L_z R(\theta)^{-1}|m\rangle_\theta = \hbar m|m\rangle_\theta$$

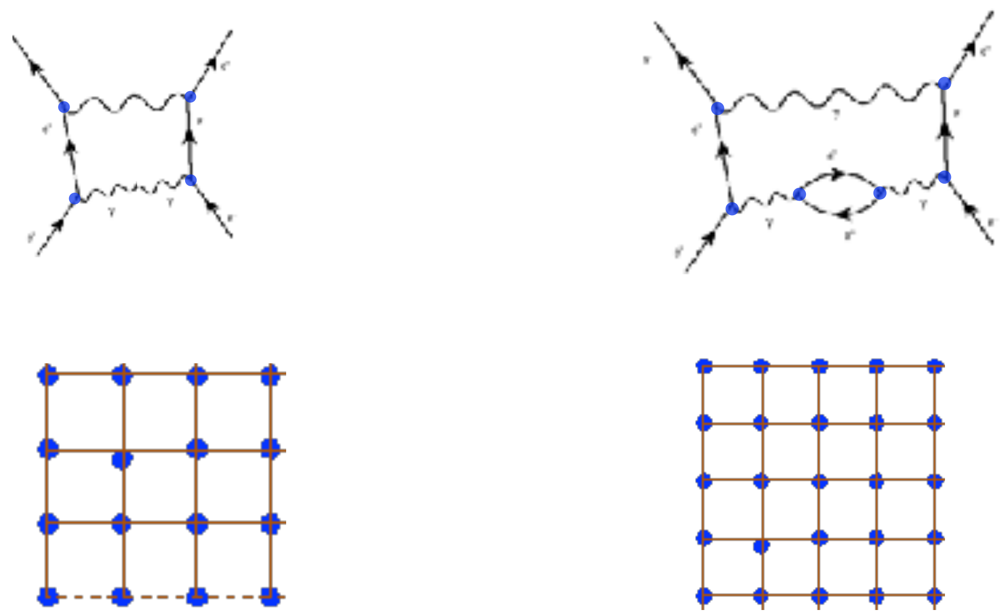
$$|m\rangle_\theta = R(\theta)|m\rangle = \sum_n R_{mn}(\theta)|n\rangle$$

- Geometry is a quantum geometry! A boost do not change the spectrum of geometrical quantities, only their probability to be measured.



CONVERGENCE BETWEEN QED & QCD PICTURES

- All physical QFT are constructed via a truncation of the d.o.f. (cfr: particles in QED, lattice in QCD)
- All physical calculation are performed within a truncation.
- The limit in which all d.o.f. is then recovered is pretty different in QED and QCD:



QUANTUM GRAVITY

Diff invariance !

[Rovelli, Ditt-invariance, 2011]

- The lattice is not on spacetime, it is spacetime!

Lattice = Feynaman diagram !!!

Lattice site = small region of space = excitations of the gravitational field = quanta of space = quanta of the field

$$
\begin{tikzcd}
\begin{array}{c}\text{Exact quantum gravity}\\ \text{transition amplitudes}\\ W(h_l)\end{array} \arrow[r, "\hbar\to 0"] & \begin{array}{c}\text{General relativity}\\ \text{Hamilton function}\\ S(q)\end{array} \\
\begin{array}{c}\text{LQG}\\ \text{transition amplitudes}\\ W_{\mathcal{C}}(h_l)\end{array} \arrow[u, "\mathcal{C}\to\infty"] \arrow[r, "\hbar\to 0"] & \begin{array}{c}\text{Regge}\\ \text{Hamilton function}\\ S_\Delta(l_{i_b})\end{array} \arrow[u, "\Delta\to\infty"]
\end{tikzcd}
$$

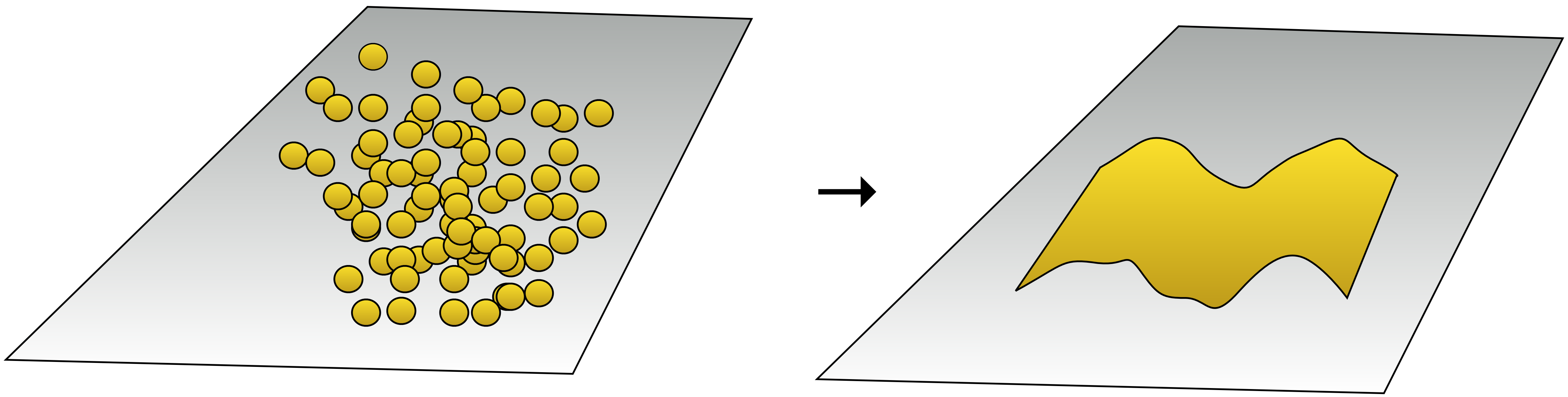
Continuous limit

Classical limit

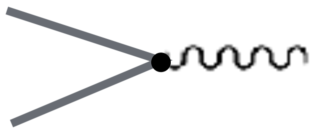
- No critical point
- No infinite renormalization
- Physical scale: Planck length

- Viability of the expansion: first radiative corrections are logarithmic (Riello'12)
- Regime of validity of the expansion: $L_{Planck} \ll L \ll \sqrt{1/R}$

LIMIT $\hbar \longrightarrow 0$



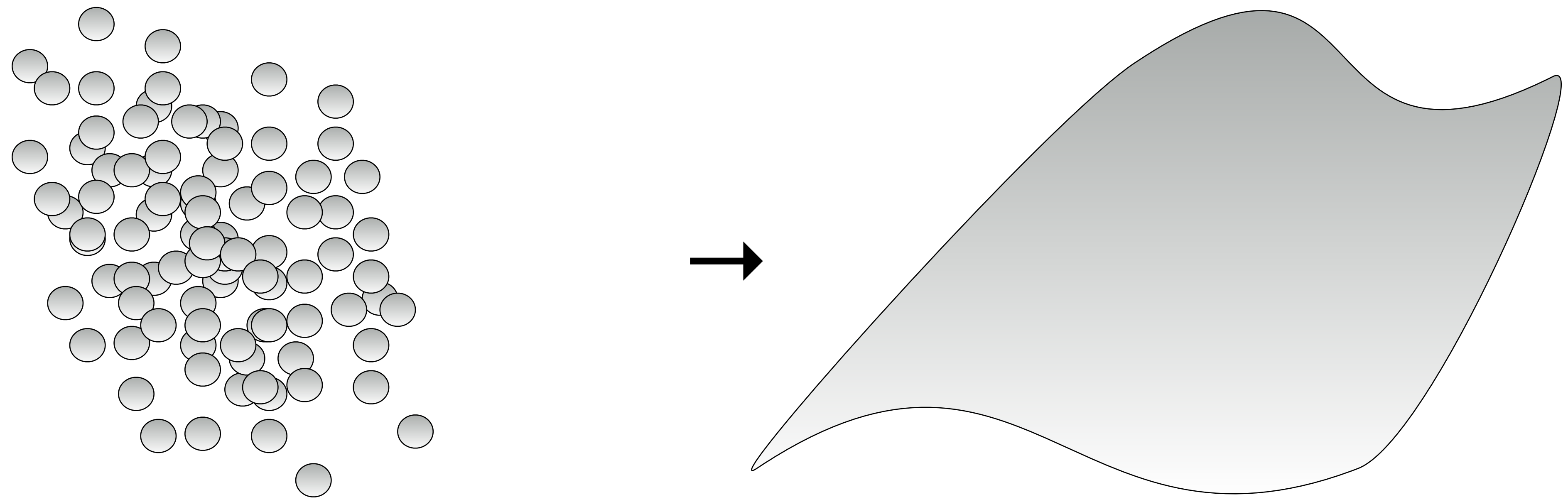
- DISCRETE
- FUZZY
- PROBABILISTIC



- NO DISCRETENESS
- NO FUZZYNESS

$$A_\mu(x) = \int d^3x\, (a(x)e^{+ikx} a^\dagger(x)e^{-ikx})$$

- A CLASSICAL FIELD $A_\mu(x)$

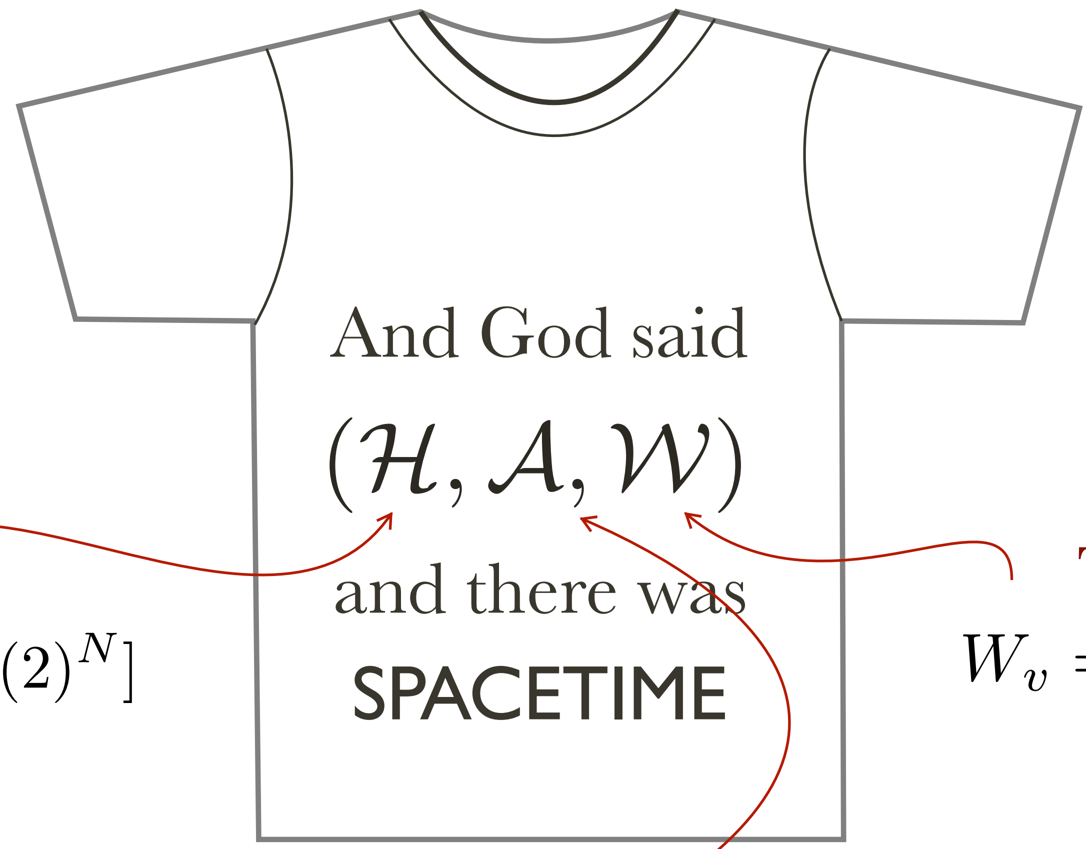


- DISCRETE $\ell_{Pl}^2 = \hbar G$
- FUZZY
- PROBABILISTIC

- NO DISCRETENESS $\ell_{Pl} \to 0$
- NO FUZZYNESS
- A CLASSICAL FIELD $E^i_a(x) \to g_{\mu\nu}(x)$

Loop Quantum Gravity

Dynamics



Hilbert Space:

$$\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$$

Transition Amplitude:

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma\ \psi_v)(\mathbb{1})$$

Operator Algebra:

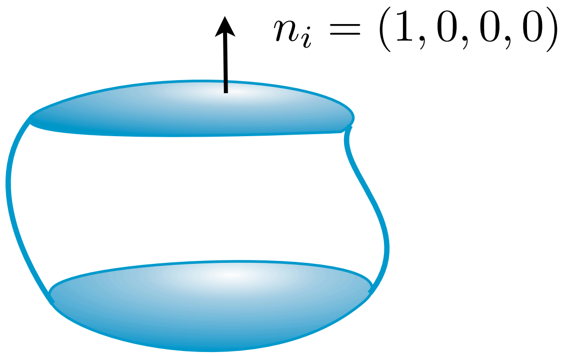
$$[L_a^i, L_b^j] = i\delta_{ab}\ell^2\epsilon^{ij}_k L_a^k$$

$(\mathcal{H}, \mathcal{A}, \mathcal{W})$ defines a background independent quantum field theory

- Variables $e = e_a dx^a \in \mathbb{R}^{(1,3)}$ and $\omega = \omega_a dx^a \in sl(2,\mathbb{C})$

- Action $S[e,\omega] = \int B[e] \wedge F[\omega]$ where $B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$

- Boundary
  gauge s.t. tetrads are diagonal
  $B^{oi} = K^i = \frac{1}{\gamma}\, e^o \wedge e^i$ and $B^{ij} = L^i = e^o \wedge e^i$



- Simplicity constraint $\vec{K} = \gamma \vec{L}$

$SL(2,\mathbb{C}) \to SU(2)$

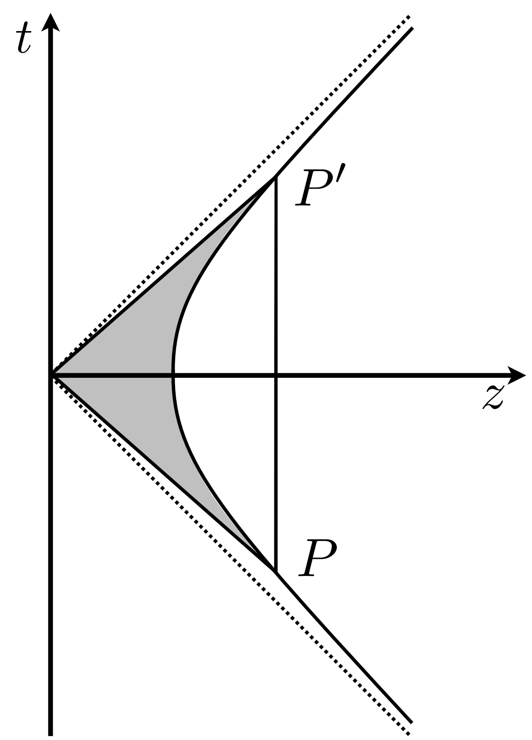
- **Lorentzian area** $A = \int_{\mathcal{R}} e^o \wedge e^i = \int_{\mathcal{R}} \gamma K^i = \int_{\mathcal{R}} L^i$

NO HIDDEN DOF

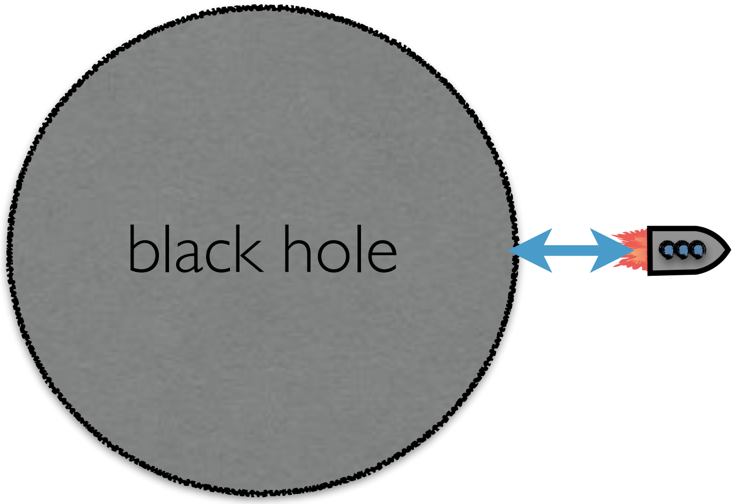
Spinfoam dynamics = constrained BF theory

$$B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$$

$$\vec{K} + \gamma \vec{L} = 0$$



- Constantly accelerated observer: $8\pi G = 1$
- K_z generators of boosts
- $E = aK_z$ generator of proper time evolution



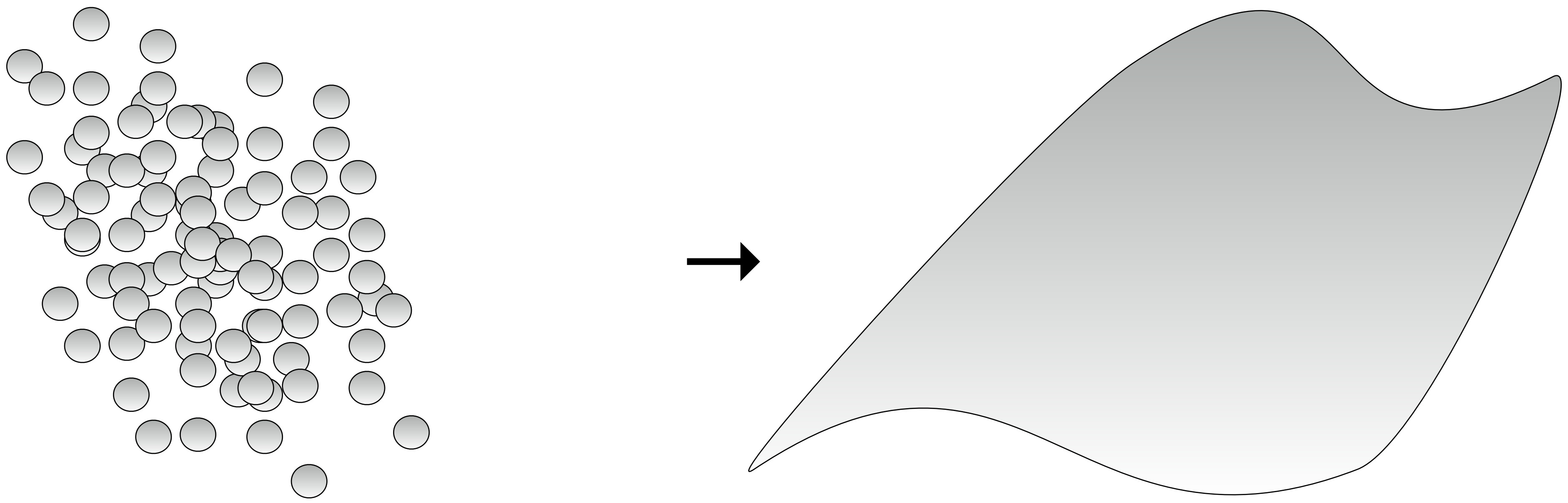
- **Frodden-Gosh-Perez '11:** $\frac{dE}{T} = \frac{a\,dA}{T} = dS$

- **Jacobson '95:** if $dE = a\,dA$ holds, then for any point and any observer the Einstein's equations follow.

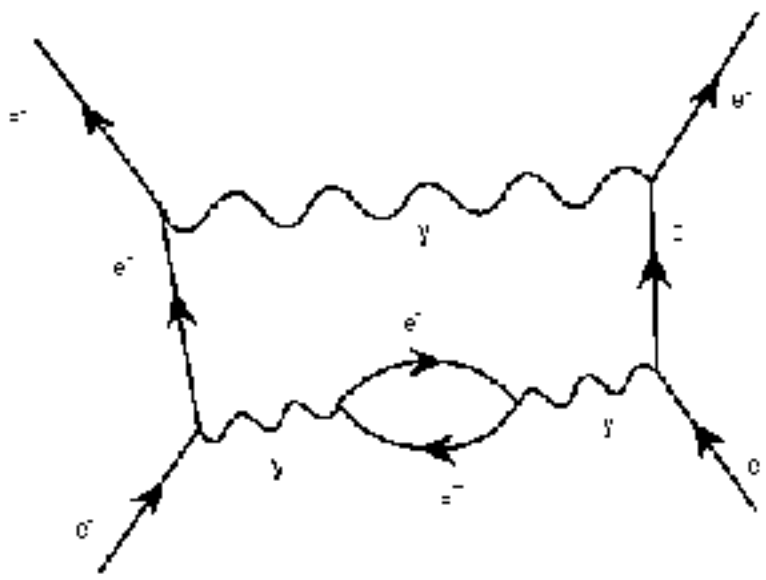
simplicity constraint + Lorentz invariance + general covariance = GR

[Baez-Bunn '15 , Chirco,-Haggard,-Riello,-Rovelli, '14]

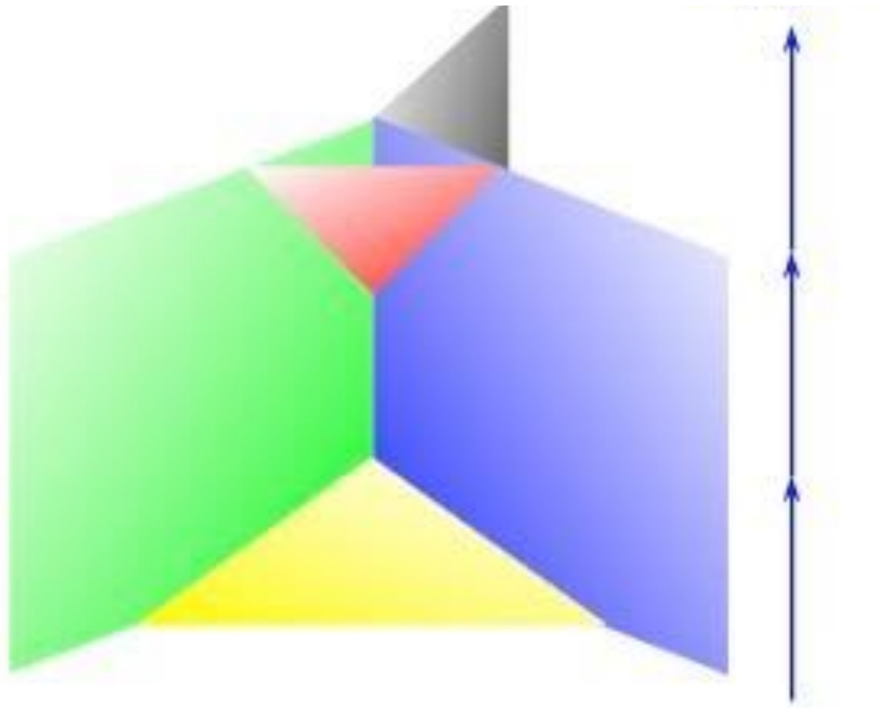
LIMIT $\hbar \longrightarrow 0$



- DISCRETE $\ell_{Pl}^2 = \hbar G$
- FUZZY
- PROBABILISTIC
- NO DISCRETENESS $\ell_{Pl} \to 0$
- NO FUZZYNESS
- A CLASSICAL FIELD $E_a^i(x) \to g_{\mu\nu}(x)$

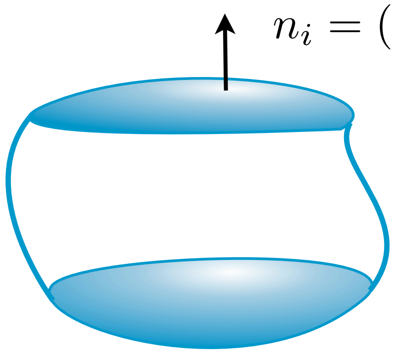


FEYNMAN GRAPH



SPINFOAM

- Variables $e = e_a dx^a \in \mathbb{R}^{(1,3)}$ and $\omega = \omega_a dx^a \in sl(2,\mathbb{C})$
- Action $S[e,\omega] = \int B[e] \wedge F[\omega]$ where $B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$
- Boundary
  gauge s.t. tetrads are diagonal $B^{oi} = K^i = \frac{1}{\gamma}\, e^o \wedge e^i$ and $B^{ij} = L^i = e^o \wedge e^i$
- Simplicity constraint $\boxed{\vec{K} = \gamma \vec{L}}$

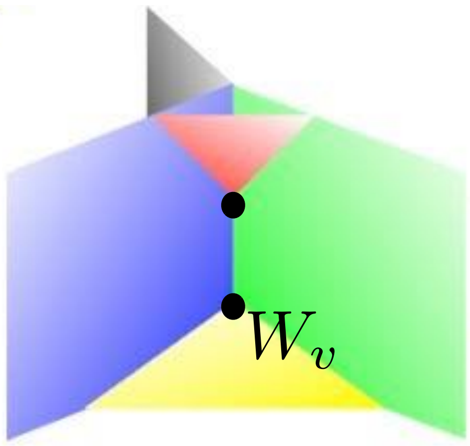


$SL(2,\mathbb{C}) \to SU(2)$

- **Lorentzian area** $A = \int_{\mathcal{R}} e^o \wedge e^i = \int_{\mathcal{R}} \gamma K^i = \int_{\mathcal{R}} L^i$

$$\mathcal{W}(\psi) = \sum_{\sigma} \prod_{f} d_{j_f} \prod_{v} W_v$$

σ "spinfoam": two-complex with faces f and edges e
colored with spins and intertwiners,
bounded by ψ



σ : spinfoam

$$d_j = 2j + 1$$

$$W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \ \psi_v)(\mathbb{1})$$

$$Y_\gamma : \mathcal{H}_j \longmapsto \mathcal{H}_j \subset \mathcal{H}_{(p=\gamma(j+1),\, k=j)}.$$

$SU(2)$ rep $\qquad$ $SL(2,C)$ rep

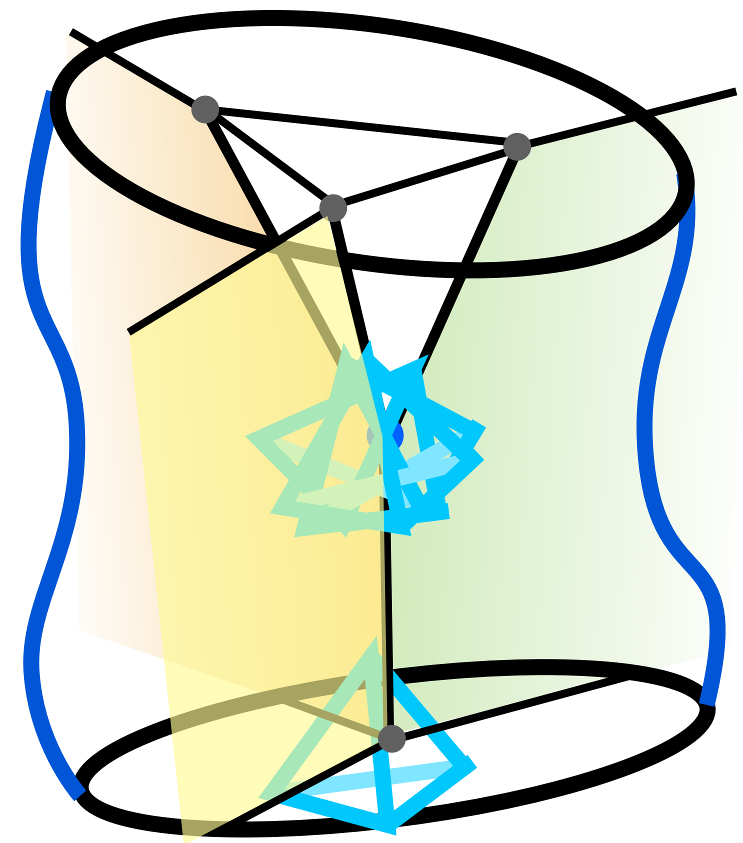
SPINFOAM AMPLITUDES

[Engle-Pereira-Livine-Rovelli, Freidel-Krasnov '08]

Probability amplitude $P(\psi) = |\langle W|\psi\rangle|^2$

Amplitude associated to a state ψ of a boundary of a 4d region

- Superposition principle
- Locality: vertex amplitude
- Lorentz covariance
- Unitary irreducible representations
- Simplicity constraint
- Classical limit: GR

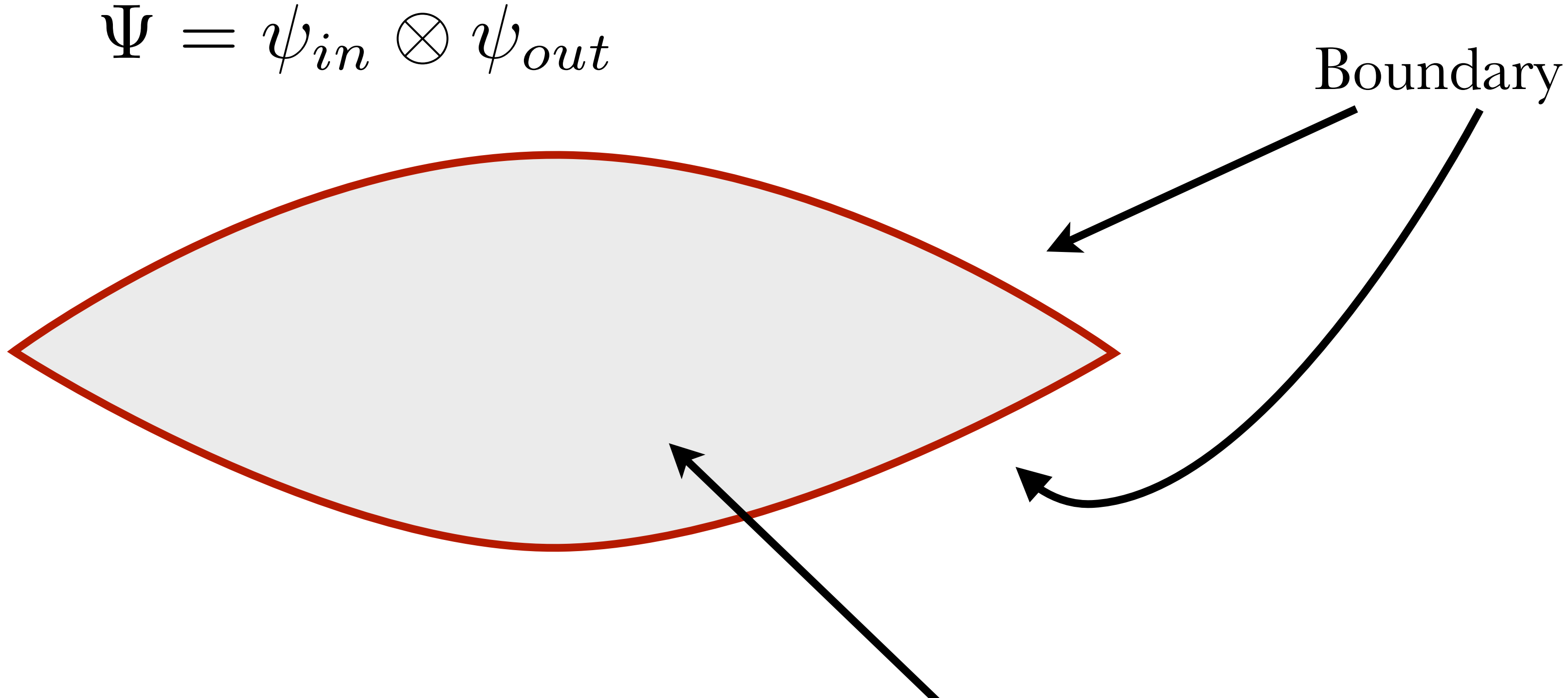


www.cpt.univ-mrs.fr/~rovelli/IntroductionLQG.pdf

QUANTUM MECHANICS		GENERAL RELATIVITY
Process	$\leftarrow$ Locality $\rightarrow$	Spacetime region
State		Boundary, space region

- Spacetime is a process, a state is what happens at its boundary.

Boundary state $\Psi = \psi_{in} \otimes \psi_{out}$



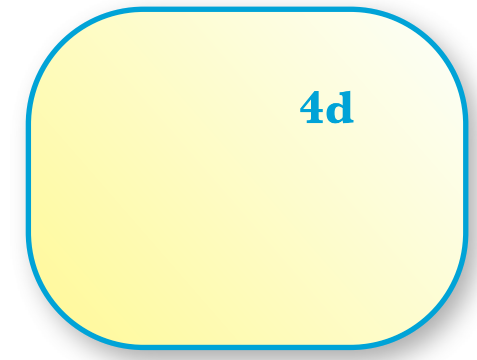
Amplitude of the process $A = W(\Psi)$

SPINFOAM AMPLITUDES

[Engle-Pereira-Livine-Rovelli, Freidel-Krasnov '08]

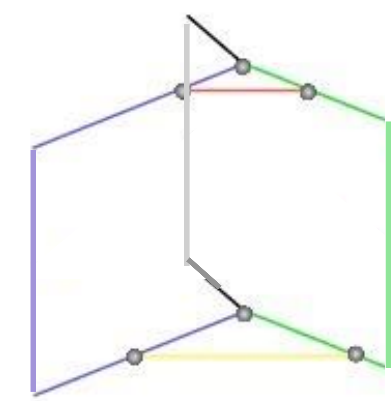
Probability amplitude $P(\psi) = |\langle W|\psi\rangle|^2$

Amplitude associated to a state ψ of a boundary of a 4d region

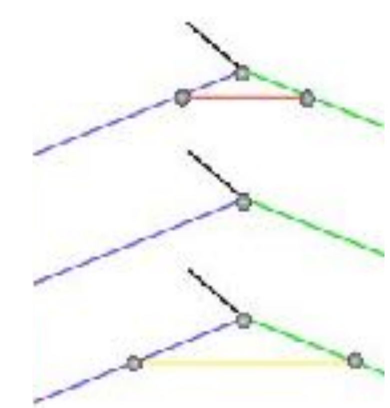


3d boundary

- Superposition principle $\quad \langle W|\psi\rangle = \sum_\sigma W(\sigma)$
- Locality: vertex amplitude $\quad W(\sigma) \sim \prod_v W_v.$
- Lorentz covariance $\quad W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma \ \psi_v)(\mathbb{1})$
- Unitary irreducible representations
- Simplicity constraint $\quad \vec{K} = \gamma \vec{L}$
- Classical limit: GR $\quad$ Barrett, Dowall, Fairbain, Gomes, Hellmann, Alesci...'09



boundary graph



a spin network history

- FROM QUANTUM TO CLASSICAL
  The *classical* limit is $\hbar \longrightarrow 0$, the limit for ∞ quanta is relevant for the *continuous* limit

  No thermodynamical limit is needed at that stage.

- EMERGENCE OF SPACETIME IS STANDARD CLASSICAL EMERGENCE
  just as the electromagnetic field emerges from photons

- SPACETIME IN THE QUANTUM REGIME IS MADE OF QUANTA
  - there is no classical spacetime in the quantum regime
  - same as in Q.E.D. where there are photons

- SPACETIME IN THE QUANTUM REGIME IS A QUANTUM PROCESS
  - states are defined by the continuity relations between quanta
  - a spinfoam is a quantum interaction, but also a spacetime region

CONCLUSIONS

- REALITY NOT MADE BY GAUGE-INVARIANT QUANTITIES ONLY

  gauge-variant quantities are not a mathematical redundancy: systems couple via gauge-dependent quantities
  gauge variables are components of relational observables which depend on more than a single component

- LOOP QUANTUM GRAVITY

GAUGES IN GENERAL RELATIVITY $\longrightarrow$ $\begin{cases} \text{- diffeos} \\ \text{- Lorentz} \end{cases}$ $\longrightarrow$ $\begin{cases} \text{- graph/lattice} \\ \text{- group variables} \end{cases}$ $\longrightarrow$ LOOP QUANTUM GRAVITY

  - GRAPHS are a natural diff invariant structures
  - no Lorentz invariance breaking

  lattice site = small region of space = excitations of the gravitational field = quanta of space = quanta of the field

  - KINEMATICS is based on group variables and gauge invariant states
  - discrete fuzzy geometry

  - LORENTZIAN DYNAMICS obtained quantizing the simplicity constrain
  - it maps the SU(2) boundary states into the Lorentzian bulk


CARLO ROVELLI AND FRANCESCA VIDOTTO
COVARIANT LOOP QUANTUM GRAVITY
AN ELEMENTARY INTRODUCTION TO QUANTUM GRAVITY AND SPINFOAM THEORY

Gauge is ubiquitous.
It is not unphysical redundancy of our mathematics. It reveals the relational structure of our world.

Carlo Rovelli