Problem 1 (7 points): electrostatic multipole expansion
1. In the lecture, the multipole expansion was derived in spherical coordinates. Now carry out this derivation in Cartesian coordinates. To this end, expand the electrostatic potential
\[ \Phi(x) = k \int d^3x' \frac{\rho(x')}{|x-x'|} \]
into a Taylor series up to second order (corresponding to the quadrupole moment). Bring the expansion into the form
\[ \Phi(x) = k \left( \frac{Q}{r} + \frac{p \cdot x}{r^3} + \frac{1}{2} \frac{x^T Q x}{r^5} \right) \]
by defining the quadrupole moment \( Q \) in such a way that it is tracefree (\( Q_{ii} = 0 \)).

2. Consider an ellipsoid
\[ \left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^2 + \left( \frac{z}{c} \right)^2 \leq 1 \]
with homogeneously distributed total charge \( Q \) and half-axes \( a, b, \) and \( c \). Calculate the quadrupole moment and determine the potential in quadrupole approximation. (Hint: you may use exercise 5, problem 2.) In this approximation, consider the case \( \left( \frac{L}{L} \right) = 1 + \epsilon, |\epsilon| \ll 1 \) and discuss the result.

Problem 2 (8 points): magnetostatics
A homogeneously charged spherical shell of total charge \( Q \) is rotating at constant angular velocity \( \omega \) around the z-axis.

1. What is the current density \( j(x) \)?

2. Calculate the magnetic dipole moment.

3. Determine the vector potential \( A(x) \) and the magnetic induction \( B(x) \) everywhere.

(Hint: \( \int \xi \xi - \xi \xi - 1/2 d\xi = \frac{4}{3}\xi(\sqrt{\xi} + 1 - \sqrt{\xi} - 1) - \frac{2}{3}(\sqrt{\xi} + 1 + \sqrt{\xi} - 1) \).)