Problem 1 (3 points): circuit
Consider the following circuit with inductance $L$, resistance $R$, and capacitance $C$, powered by an AC voltage $U = U_0 e^{j\omega t}$.

Calculate the current $I(t)$ as well as the voltage drop across the capacitor and the inductor (neglect the ...). Determine the impedance $|Z|$ as well as the phase factor $\phi$ of the complex resistance $Z := U/I = |Z| e^{j\phi}$. Use the following data: $C = 7 \mu F$, $L = 2 mH$, $R = 2 R$, $U_0 = 220 V$ and $\omega = 7 MHz$.

Problem 2 (6 points): radiation field
1. In the lecture, the retarded potentials were derived in the radiation field dipole approximation. Calculate the corresponding fields $B(x,t)$ and $E(x,t)$ up to terms $O(1/r^2)$ (cf. the results of the lecture).

2. Calculate the energy density $\epsilon$ of the electromagnetic field, and show that $\epsilon$ integrated over the surface of a spherical shell is constant. What is the physical meaning of this? (3 points)

Problem 3 (11 points): antenna in dipole approximation
An AC current

\[ j(x, t) = j_0 \cos(\pi z/d) \cos(\omega t) \delta(x) \delta(y) \delta(z/2 - |z|e), \]

is flowing in a thin, linear antenna on the z-axis between $-d/2$ and $d/2$ ($\omega = \pi c/d$).

1. Determine the radiation field of the antenna without the dipole approximation. As usual in the radiation field, approximate $|x - x'| \approx r - (x \cdot x')/r$, and calculate the function $\mathbf{p}(t) = \int d^2 r' e^{-i(r' - r)/r} \mathbf{e} \cdot \mathbf{r}^2 \mathbf{E}(x,t)$ (analogous to $\mathbf{p}(t)$ in the dipole approximation). Using this, determine the vector potential $\mathbf{A}(x,t)$ and the fields $\mathbf{B}(x,t)$ and $\mathbf{E}(x,t)$. Calculate the Poynting vector $\mathbf{S}(x,t)$ as well as the average power emitted per solid angle $d\Omega$, given by $dP/d\Omega = r^2 n \cdot \mathbf{S}(x,t)$.

2. Calculate the vector potential $\mathbf{A}(x,t)$, the fields $\mathbf{B}(x,t)$ and $\mathbf{E}(x,t)$, $\mathbf{S}(x,t)$ as well as $dP/d\Omega$ for the antenna in dipole approximation.

3. Make sketches of $dP/d\Omega$ from parts 1 and 2 and compare the two. (1 point)