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Third exercise sheet on Relativity and Cosmology I

Winter term 2018/19

Release: Mon, Nov. 5th **Submit**: Mon, Nov. 12th in lecture **Discuss**: Thu, Nov. 15th

Exercise 8 (6 points): *Energy-momentum tensor for electromagnetic field*

The energy-momentum tensor for the electromagnetic field reads (in Gaussian units with $c = 1$)

$$T^{\mu\nu} := \frac{1}{4\pi}\left(F^{\mu\lambda}F^{\nu}{}_{\lambda} - \frac{1}{4}\eta^{\mu\nu}F^{\lambda\rho}F_{\lambda\rho}\right) .$$

8.1 Express T^{00}, T^{0i} and T^{ij} in terms of $\vec{E}$ and $\vec{B}$. What is the physical meaning of T^{00} and T^{0i}?

8.2 Interpret the four conservation equations for $T^{\mu\nu}$ as well as the components T^{ij}. Use the results in **8.1**.

Exercise 9 (6 points): *Kottler–Møller coordinates*

Let g be a constant acceleration, (t, x, y, z) Cartesian coordinates in 4-dimensional Minkowski space. Consider the transformation ($c = 1$)

$$\begin{aligned} t &= \left(\frac{1}{g} + x'\right)\sinh gt' , \\ x &= \left(\frac{1}{g} + x'\right)\cosh gt' - \frac{1}{g} , \\ y &= y' , \qquad z = z' . \end{aligned}$$

9.1 Show that the transformation gives accelerated frames of reference.

9.2 Calculate the components of the metric with respect to the coordinates (t', x', y', z').

Exercise 10 (8 points): *Rindler coordinates*

Let a be a constant acceleration. Consider the 2-dimensional metric ($c = 1$)

$$\mathrm{d}s^2 = -\left(ax'\right)^2 \mathrm{d}t'^2 + \mathrm{d}x'^2 .$$

10.1 At which point in space do the components of the metric tensor exhibit a singularity?

10.2 Find a coordinate transformation which shows that this so-called Rindler space is only a part of the 2-dimensional Minkowski space, which in Cartesian coordinates reads $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2$.

10.3 Compare the Rindler coordinates with the Kottler–Møller coordinates.

10.4 Give an illustrative interpretation of the Rindler coordinates (consider $t' = \text{const.}$ and $x' = \text{const.}$).

10.5 Determine the proper acceleration along the curve $x' = \text{const.}$