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www.thp.uni-koeln.de/gravitation/courses/rci1819.html

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Fourth exercise sheet on Relativity and Cosmology I

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Release: Mon, Nov. 12th **Submit**: Mon, Nov. 19th in lecture **Discuss**: Thu, Nov. 22nd

Exercise 11 (7 points): *Clocks*

Two atomic clocks are transported in two aeroplanes once around the Earth in either eastern or western direction. For simplicity, assume the aeroplanes fly directly above the equator, where the rotation speed of the Earth is about $v_{\mathrm{E}} \approx 1674\,\mathrm{km\,h^{-1}}$. Furthermore, use an average cruising speed of $v_{\mathrm{F}} \approx 800\,\mathrm{km\,h^{-1}}$ and a mean flying altitude of 10 km.
Calculate the respective time dilations the clocks exhibit compared to a clock which stayed on the ground, after the aeroplanes have landed. For this purpose, take into account the separate contributions due to the gravitational and the special relativistic velocity effect. Legitimate this separation.

Exercise 12 (6 points): *Motion in the gravitational field*

The equation of motion for a test particle in a gravitational field is given by

$$\ddot{x}^\mu + \Gamma^\mu{}_{\nu\kappa}\,\dot{x}^\nu\dot{x}^\kappa = 0\,, \tag{1}$$

where $\dot{x}^\mu = \mathrm{d}x^\mu/\mathrm{d}\tau$, τ is the proper time and $\Gamma^\mu{}_{\nu\kappa} = \frac{1}{2}\,g^{\mu\sigma}(\partial_\kappa\,g_{\sigma\nu} + \partial_\nu\,g_{\sigma\kappa} - \partial_\sigma\,g_{\nu\kappa})$.

12.1 Review briefly the derivation of eq. (1) from the variational equation $\delta\int\mathrm{d}\tau = 0$ as presented in the lecture. Why can the derivation not be used for photons?

12.2 Derive eq. (1) from the alternative variational equation

$$\delta\int g_{\mu\nu}\,\dot{x}^\mu\dot{x}^\nu\,\mathrm{d}\lambda \equiv \delta\int\mathcal{K}\,\mathrm{d}\lambda = 0\,,$$

where λ is an affine parameter and $\dot{x}^\mu = \mathrm{d}x^\mu/\mathrm{d}\lambda$.

Show that this derivation also holds for photons and determine $\mathcal{K}$ for the solution of eq. (1).

Exercise 13 (7 points): *Rotating reference frame*

Consider a coordinate chart, in which observers at "rest" in this chart orbit around the z-axis of an inertial Cartesian coordinate chart, with constant angular speed ω in the Newtonian approximation.

13.1 Find the terms corresponding to the centrifugal and the Coriolis forces in the geodesic equation. Calculate the Christoffel symbols of the first kind.

13.2 (Bonus) In **13.1**, the inertial effects have been separated from the gravitational effects, the latter of which are not present here. Can we do this in general?