

9th exercise sheet on Relativity and Cosmology I

Winter term 2012/13

Deadline for delivery: Thursday, 13th December 2012 during the exercise class.

Exercise 23 (8 credit points): *Dust*

Derive the continuity equation and the Euler equation for dust given in the lecture within the framework of special relativity.

Furthermore, show that in an arbitrary reference frame it follows from the conservation of the energy-momentum tensor of dust that dust particles move on geodesics.

Exercise 24 (12 credit points): *Ideal fluid*

The energy-momentum tensor of an ideal fluid is given by

$$T_{\mu\nu} = \rho u_{\mu}u_{\nu} + P (u_{\mu}u_{\nu} + g_{\mu\nu}),$$

where u_{μ} is the four-velocity, ρ is the density and P is the pressure of the fluid.

Use the fact that the energy-momentum tensor of an ideal fluid is divergence-free to derive the continuity equation and the Euler equation.

Furthermore, write out the continuity equation for the metric

$$g_{\mu\nu} = \text{diag} \left[-1, a(t)^2, a(t)^2, a(t)^2 \right].$$