12th exercise sheet on Relativity and Cosmology I
Winter term 2013/14

Deadline for delivery: Thursday, 23rd January 2014 during the exercise class.

Exercise 30 (10 credit points): Fierz–Pauli Lagrange density

Consider the following Lagrange density (Fierz and Pauli 1939):

\[ L_{FP} = \frac{1}{8\pi G} (\psi_{\mu\nu, \sigma} \psi_{\mu\nu, \sigma} - 2 \psi_{\mu\nu, \rho} \psi_{\mu, \rho, \nu} \psi_{\rho, \nu} + 2 \psi_{\nu, \rho, v} \psi_{\rho, \nu}^* \psi_{\sigma, \rho}^* - \psi_{\mu\mu} \psi_{\nu\nu} + 2 T_{\mu\nu}) \psi_{\mu\nu}. \]

30.1 Show that the Lagrangian equations of motion following from this are equivalent to the linearized Einstein equations.

30.2 Calculate the canonical energy–momentum tensor

\[ t_{\mu\nu} = \frac{\partial L_{FP}}{\partial \psi_{\alpha\beta, \nu}} \psi_{\alpha\beta, \mu} - \eta_{\mu\nu} L_{FP}. \]

Exercise 31 (10 credit points): Quadrupole formula

In the lecture, the following expression for the energy flux was given for propagation in x-direction:

\[ f_x = \frac{1}{4\pi G} \left[ \frac{1}{4} (\dot{\psi}_{22} - \dot{\psi}_{33})^2 + \dot{\psi}_{23}^2 \right]. \]

Repeat the steps that lead to the quadrupole formula and give the calculational details. Show, in particular, the following relations for the components \( n^i \) of a unit vector \( \mathbf{n} \):

\[ \frac{1}{4\pi} \int_{S^2} n^l n^m d\Omega = \frac{1}{3} \delta_{lm}, \]

\[ \frac{1}{4\pi} \int_{S^2} n^k n^l n^m n^r d\Omega = \frac{1}{15} (\delta_{kl} \delta_{mr} + \delta_{km} \delta_{lr} + \delta_{kr} \delta_{lm}). \]