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www.thp.uni-koeln.de/gravitation/courses/rci13.html

3rd exercise sheet on Relativity and Cosmology I

Winter term 2013/14

Deadline for delivery: Wednesday, 6th November 2013 at the end of the lecture.

Exercise 6 (5 credit points): *Inertial frames*

A rocket with a rest length L_0 moves with constant velocity radially away from Earth. From Earth a light pulse is emitted, which is then reflected by mirrors at the front as well as at the rear of the rocket. The first signal is received after the time t_A, the second after the time t_B.

Calculate the velocity at which the rocket moves in terms of L_0, t_A and t_B.

Determine at which distance from Earth the rocket is located when the first signal reaches Earth.

Exercise 7 (5 credit points): *Accelerated frame of reference*

Show that the equations

$$\begin{aligned}
t &= \frac{c}{g}\,\sinh\!\left(\frac{g\,t'}{c}\right) + \frac{x'}{c}\,\sinh\!\left(\frac{g\,t'}{c}\right)\,,\\
x &= \frac{c^2}{g}\,\left[\cosh\!\left(\frac{g\,t'}{c}\right) - 1\right] + x'\,\cosh\!\left(\frac{g\,t'}{c}\right)\,,\\
y &= y'\,,\\
z &= z'\,,
\end{aligned}$$

describe a transformation from an inertial frame to an accelerated frame of reference ($g = \text{const.}$). Calculate the components of the metric with respect to the frame (t', x', y', z').

Exercise 8 (10 credit points): *Rindler coordinates*

Consider the two-dimensional metric

$$\mathrm{d}s^2 = -\,v^2\,\mathrm{d}u^2 + \mathrm{d}v^2\,.$$

At which point in space do the components of the metric tensor exhibit a singularity?

Find a coordinate transformation which shows that this so-called Rindler space is only a part of the two-dimensional Minkowski space, which is usually represented by $\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \mathrm{d}x^2$.

Compare the Rindler coordinates with the coordinates from exercise **7**.

Give an illustrative interpretation of the Rindler coordinates (consider $u = \text{const.}$ and $v = \text{const.}$).

Determine the proper acceleration along the curve $v = \text{const.}$