4th exercise sheet on Relativity and Cosmology I
Winter term 2017/18

Deadline for delivery: Thursday, 16th November 2017 during the exercise class.

Exercise 9: Energy-momentum tensor for electromagnetic field

Recall from the lecture course that the energy-momentum tensor for electromagnetic field reads

\[ T_{\mu \nu} := \frac{1}{4 \pi} \left( F_{\mu \lambda} F^{\nu \lambda} - \frac{1}{4} \eta_{\mu \nu} F_{\lambda \rho} F^{\lambda \rho} \right), \]

where \( F_{\mu \nu} \) has been defined in exercise 6.

1. Express \( T^{00}, T^{0i} \) and \( T^{ij} \) in terms of \( \vec{E} \) and \( \vec{B} \). What is the physical meaning of \( T^{00} \) and \( T^{0i} \)?
2. Interpret the four conservation equations for \( T^{\mu \nu} \) as well as the components \( T^{ij} \). Use the results in item 1.

Exercise 10: Accelerated frame of reference

1. Show that the equations

\[
\begin{align*}
  t &= \frac{c}{g} \sinh \left( \frac{g t'}{c} \right) + \frac{x'}{c} \sinh \left( \frac{g t'}{c} \right), \\
  x &= \frac{c^2}{g} \left[ \cosh \left( \frac{g t'}{c} \right) - 1 \right] + x' \cosh \left( \frac{g t'}{c} \right), \\
  y &= y', \\
  z &= z',
\end{align*}
\]

describe a transformation from an inertial frame to an accelerated frame of reference (\( g = \text{const.} \)).

2. Calculate the components of the metric with respect to the frame \((t', x', y', z')\).

Exercise 11: Rindler coordinates

Consider the two-dimensional metric

\[ ds^2 = -v^2 \, du^2 + dv^2. \]

1. At which point in space do the components of the metric tensor exhibit a singularity?
2. Find a coordinate transformation which shows that this so-called Rindler space is only a part of the two-dimensional Minkowski space, which is usually represented by \( ds^2 = -dt^2 + dx^2 \).
3. Compare the Rindler coordinates with the coordinates from exercise 10.
4. Give an illustrative interpretation of the Rindler coordinates (consider \( u = \text{const. and } v = \text{const.} \)).
5. Determine the proper acceleration along the curve \( v = \text{const.} \).