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www.thp.uni-koeln.de/gravitation/courses/rci1718.html

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4th exercise sheet on Relativity and Cosmology I

Winter term 2017/18

Deadline for delivery: Thursday, 16th November 2017 during the exercise class.

Exercise 9: *Energy-momentum tensor for electromagnetic field*

Recall from the lecture course that the energy-momentum tensor for electromagnetic field reads

$$T^{\mu\nu} := \frac{1}{4\pi}\left(F^{\mu\lambda}F^{\nu}{}_{\lambda} - \frac{1}{4}\eta^{\mu\nu}F^{\lambda\rho}F_{\lambda\rho}\right) ,$$

where $F_{\mu\nu}$ has been defined in exercise **6**.

1. Express T^{00}, T^{0i} and T^{ij} in terms of $\vec{E}$ and $\vec{B}$. What is the physical meaning of T^{00} and T^{0i}?
2. Interpret the four conservation equations for $T^{\mu\nu}$ as well as the components T^{ij}. Use the results in item 1.

Exercise 10: *Accelerated frame of reference*

1. Show that the equations

$$\begin{aligned} t &= \frac{c}{g}\sinh\left(\frac{g\,t'}{c}\right) + \frac{x'}{c}\sinh\left(\frac{g\,t'}{c}\right) , \\ x &= \frac{c^2}{g}\left[\cosh\left(\frac{g\,t'}{c}\right) - 1\right] + x'\cosh\left(\frac{g\,t'}{c}\right) , \\ y &= y' , \\ z &= z' , \end{aligned}$$

describe a transformation from an inertial frame to an accelerated frame of reference ($g = \text{const.}$).

2. Calculate the components of the metric with respect to the frame (t', x', y', z').

Exercise 11: *Rindler coordinates*

Consider the two-dimensional metric

$$\mathrm{d}s^2 = -\,v^2\,\mathrm{d}u^2 + \mathrm{d}v^2 .$$

1. At which point in space do the components of the metric tensor exhibit a singularity?
2. Find a coordinate transformation which shows that this so-called Rindler space is only a part of the two-dimensional Minkowski space, which is usually represented by $\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \mathrm{d}x^2$.
3. Compare the Rindler coordinates with the coordinates from exercise **10**.
4. Give an illustrative interpretation of the Rindler coordinates (consider $u = \text{const.}$ and $v = \text{const.}$).
5. Determine the proper acceleration along the curve $v = \text{const.}$