

4th exercise sheet on Relativity and Cosmology II

Summer term 2013

Deadline for delivery: Wednesday, 8th May 2013 at the end of the lecture.

Exercise 8 (12 credit points): *Reissner–Nordström solution*

The aim of this exercise is to calculate the gravitational field outside of a static, spherically symmetric charge distribution with mass M and charge Q .

For this purpose, use the ansatz

$$ds^2 = -e^{\nu(t,r)} dt^2 + e^{\lambda(t,r)} dr^2 + r^2 d\Omega^2 \quad (1)$$

for the metric and calculate the electro-magnetic field strength tensor $F^{\mu\nu}$, for example, by using the Maxwell equations in source-free regions

$$F^{\mu\nu}{}_{;\nu} = 0.$$

(Check: $F^{0r} = -F^{r0} = e^{-(\lambda+\nu)/2} Q/r^2$, otherwise $F^{\mu\nu} = 0$.)

Afterwards, compute the components of the energy-momentum tensor given by

$$T_{\mu\nu} = \frac{1}{4\pi} \left(F_{\mu\lambda} F_{\nu}{}^{\lambda} - \frac{1}{4} g_{\mu\nu} F_{\kappa\sigma} F^{\kappa\sigma} \right).$$

Now calculate the metric functions ν and λ by means of the Einstein equations $G^{\mu}{}_{\nu} = \kappa T^{\mu}{}_{\nu}$.

(Use the components of the Einstein tensor $G^{\mu}{}_{\nu}$ arising from the ansatz (1) given in the lecture course.)

Choose the integration constant appropriately such that you obtain the correct Newtonian limit and compare the resulting metric to the Schwarzschild solution.

Exercise 9 (8 credit points): *Classical tests of GR for the Reissner–Nordström solution*

Find a coordinate transformation $\bar{r}(r)$ to transform the Reissner–Nordström metric into its isotropic form

$$ds^2 = -A(\bar{r}) dt^2 + B(\bar{r}) (d\bar{r}^2 + \bar{r}^2 d\Omega^2)$$

and calculate the coefficients $\alpha_1, \alpha_2, \beta_1, \beta_2$ introduced in the lecture by means of $A(\bar{r})$ and $B(\bar{r})$.

Use this result to analyze how a charge Q of the Sun would influence the classical tests of General Relativity in the solar system.