Exercise 17 (4 credit points): Accretion disks

Give an estimate for the characteristic energy that is emitted by an accretion disk with radius $R$ around a compact spherically symmetric object. For simplicity (even though this is not totally realistic), assume that the luminosity is that of a black body of radius $R$ and temperature $T$ and that it amounts to a given fraction $\varepsilon$ of the Eddington luminosity. (At the end, use $\varepsilon \approx 0.5$.)

Exercise 18 (4 credit points): Redshift in case of a gravitational collapse

Consider an observer on the surface of a collapsing spherical star who emits radial light signals in short proper time intervals $\Delta s$, i.e. with a constant frequency $\omega_s = 2\pi/\Delta s$. These signals are received by a stationary observer at large distance $r = r_R$, i.e. with a frequency $\omega_R = 2\pi/\Delta t_R$, where $\Delta t_R$ refers to the Schwarzschild time. Calculate the dependence of the frequency ratio $\omega_R/\omega_s$ on $t_R$. Indicate the time scale of the redshift in terms of seconds if you measure $M$ in solar masses.

**Hint:** Use Eddington–Finkelstein coordinates (as discussed in the lecture) and assume that the emitting observer is already located near the Schwarzschild radius.

Exercise 19 (12 credit points): Newtonian cosmology

19.1 In the Newtonian theory of gravity, how large is the total gravitational force of an infinitely extended mass distribution of mass density $\rho(\vec{x})$ on a mass $m$ that is located at the origin? What kind of problem occurs with an infinitely large and homogeneous mass distribution?

19.2 Consider a “universe” with a homogeneous mass density $\rho(t)$. Write out the Newtonian equation of motion for an arbitrary galaxy of mass $m$ whose radius vector with respect to the Earth be denoted by $\vec{x}(t)$. According to the Hubble law, we have

$$\ddot{x}(t) = a(t) \ddot{x}_0 \quad \text{with} \quad \frac{\dot{a}}{a} =: H(t).$$

($\ddot{x}_0$ denotes the location for an arbitrarily given time.)

Formulate the equation of motion for $a(t)$. Use the continuity equation to eliminate $\rho(t)$. Is a static universe ($a = \text{const}$.) possible? Show that integration leads to the “energy theorem”

$$a^2 - \frac{C}{a} + k = 0,$$

where $C > 0$ and $k$ are constants. How can one interpret $k$? Sketch $a(t)$ roughly for the three cases $k < 0$, $k = 0$ and $k > 0$.

19.3 Add ad hoc a repulsive force $m \ddot{x} \Lambda/3$ with “cosmological constant” $\Lambda > 0$ to the Newtonian gravitational force. Formulate the modified equation of motion for $a(t)$ as well as the modified “energy theorem”. Is it now possible to have a static universe?