University of Cologne
Institute for Theoretical Physics
Prof. Dr. Claus Kiefer
Manuel Krämer

5th June 2013

www.thp.uni-koeln.de/gravitation/courses/rcii13.html

8th exercise sheet on Relativity and Cosmology II

Summer term 2013

Deadline for delivery: Thursday, 13th June 2013 during the exercise class.

Exercise 17 (4 credit points): *Accretion disks*

Give an estimate for the characteristic energy that is emitted by an accretion disk with radius R around a compact spherically symmetric object. For simplicity (even though this is not totally realistic), assume that the luminosity is that of a black body of radius R and temperature T and that it amounts to a given fraction ε of the Eddington luminosity. (At the end, use $\varepsilon \approx 0.5$.)

Exercise 18 (4 credit points): *Redshift in case of a gravitational collapse*

Consider an observer on the surface of a collapsing spherical star who emits radial light signals in short proper time intervals Δs, i.e. with a constant frequency $\omega_* = 2\pi/\Delta s$. These signals are received by a stationary observer at large distance $r = r_{\mathrm{R}}$, i.e. with a frequency $\omega_{\mathrm{R}} = 2\pi/\Delta t_{\mathrm{R}}$, where Δt_{R} refers to the Schwarzschild time. Calculate the dependence of the frequency ratio $\omega_{\mathrm{R}}/\omega_*$ on t_{R}. Indicate the time scale of the redshift in terms of seconds if you measure M in solar masses.

Hint: Use Eddington–Finkelstein coordinates (as discussed in the lecture) and assume that the emitting observer is already located near the Schwarzschild radius.

Exercise 19 (12 credit points): *Newtonian cosmology*

19.1 In the Newtonian theory of gravity, how large is the total gravitational force of an infinitely extended mass distribution of mass density $\rho(\vec{x})$ on a mass m that is located at the origin? What kind of problem occurs with an infinitely large and homogeneous mass distribution?

19.2 Consider a "universe" with a homogeneous mass density $\rho(t)$. Write out the Newtonian equation of motion for an arbitrary galaxy of mass m whose radius vector with respect to the Earth be denoted by $\vec{x}(t)$. According to the Hubble law, we have

$$\vec{x}(t) = a(t)\,\vec{x}_0 \qquad \text{with} \qquad \frac{\dot{a}}{a} =: H(t)\,.$$

($\vec{x}_0$ denotes the location for an arbitrarily given time.)
Formulate the equation of motion for $a(t)$. Use the continuity equation to eliminate $\rho(t)$. Is a static universe ($a = \text{const.}$) possible? Show that integration leads to the "energy theorem"

$$\dot{a}^2 - \frac{C}{a} + k = 0\,,$$

where $C > 0$ and k are constants. How can one interpret k? Sketch $a(t)$ roughly for the three cases $k < 0$, $k = 0$ and $k > 0$.

19.3 Add ad hoc a repulsive force $m\,\vec{x}\,\Lambda/3$ with "cosmological constant" $\Lambda > 0$ to the Newtonian gravitational force. Formulate the modified equation of motion for $a(t)$ as well as the modified "energy theorem". Is it now possible to have a static universe?