9th exercise sheet on Relativity and Cosmology II
Summer term 2013

Deadline for delivery: Thursday, 27th June 2013 during the exercise class.

Exercise 20 (6 credit points): Friedmann I

Consider a Friedmann model with $k \neq 0$ and present density parameters $\Omega_{m,0}$, $\Omega_{r,0}$ and $\Omega_{v,0}$ as well as $\Omega := \Omega_{m,0} + \Omega_{r,0} + \Omega_{v,0}$. Furthermore let

$$\rho_c(a) = \frac{3a^2}{8\pi G a^2}$$

be the critical density at a time when the scale parameter had the value $a$, and let $\Omega_m(a) = \rho_m(a)/\rho_c(a)$ etc. be the corresponding relative densities.

Determine the quantity $\Omega(a) - 1$ as a function of $\Omega_{m,0}$, $\Omega_{r,0}$, $\Omega_{v,0}$ and $a$. This quantity indicates how much the considered model “deviated” from a flat model at a certain time. What kind of “aesthetic problem” arises for a Friedmann model whose density parameter $\Omega$ differs only slightly from unity today?

Exercise 21 (8 credit points): Friedmann II

Solve the Friedmann equation for a universe that contains radiation as well as non-relativistic matter (dust). For this purpose, rewrite the Friedmann equation as a differential equation with respect to the conformal time $\eta$, solve this equation for the three possible values of $k$ and write out the result in the form $(a(\eta), t(\eta))$.

Show further how one can obtain the result for $k = 0$ from the results for $k \neq 0$ and analyze the implicitly defined functions $a(t)$ for large and small values of $t$.

Exercise 22 (6 credit points): Friedmann II

Current observations indicate that we live in a flat ($k = 0$) universe with positive cosmological constant, in which the contribution of radiation to the total energy density can be neglected.

Solve the Friedmann equation for this model. (Hint: The substitution $s^2 = (1/\Omega_{m,0} - 1) a^2$ could be helpful.)

Determine the age of the universe as a function of $H_0$ and $\Omega_{m,0}$. How does $a(t)$ behave for large and small values of $t$?

According to well-established models for stellar evolution, several globular clusters in our galaxy are at least 12 billion years old. Draw a $(h - \Omega_m)$-diagram ($h$ is the parameter in the definition of $H_0$) and sketch the contour lines for a constant age of the universe. Determine which parameter range is compatible with the above-mentioned observation. In doing so, only consider values $0.4 < h < 1$. 