Poincaré gauge theory of gravity
— An Introduction

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Motivation or “what if...”?

History:
- field equations of General Relativity, Einstein (1915)
- discovery of spin, Pauli (1924); Uhlenbeck & Goudsmit (1925)
- relativistic description of spin, Dirac (1928)
- gravity as gauge theory: Utiyama (1956), Sciama (1960), Kibble (1961)

“Newton successfully wrote apple = moon, but you cannot write apple = neutron.”

– J. L. Synge

The Dirac equation, minimally coupled to gravity:

\[ i \gamma^\alpha \varepsilon^j_\alpha \left( \partial_j + \frac{i}{4} \Gamma_i \right) \psi + m\psi = 0 \]

Problem: the frame field has to be put into General Relativity by hand.

What if spin had been discovered before General Relativity?
Would Einstein have applied the equivalence principle to a neutron instead?
Physical interpretation of the frame field

The frame field $e^j_\mu$ supplies us with orthonormal basis vectors on the curved space: (necessary for spinor representation → it is a field of fundamental importance)
Physical interpretation of the frame field

Think about the frame field displayed in a random coordinate space: it rotates!

Is there a gauge principle involved?
Example: a brief description of U(1) gauge theory

Consider a complex field $\phi$ under a global U(1) transformation $\phi \mapsto e^{i\alpha} \phi$, with $\alpha \in \mathbb{R}$:

If the theory is invariant under this transformation, we call U(1) a **rigid symmetry**.
Example: a brief description of U(1) gauge theory

Now carry out a local transformation $\phi \mapsto e^{i\alpha(x)}\phi$:

Due to $x$-dependence, any dynamical theory is not invariant anymore.

How do we rescue this? We need a gauge potential $A$!
Example: a brief description of U(1) gauge theory

The gauge potential restores gauge invariance by $d \rightarrow d + ieA$:

What have we gained? We can construct a Lagrangian for $A$ using $F := dA$:

$$\mathcal{L} := F \wedge *F + j \wedge A$$

yields electrodynamics with conserved current $j$.
Towards a gauge theory of gravity

We saw: to describe electrodynamics as a gauge theory, we have to

1. **forget** about electrodynamics (!)
2. carry out a **gauge procedure** with a suitable group, here: \( U(1) \)
3. obtain electrodynamics **for free** from gauge curvature Lagrangian

To describe gravity as a gauge theory, we first **have to forget about gravity**.

What remains if we do that?

**special relativity,**

and fields propagating on flat Minkowski space

Note the difference: symmetries in **external** space, not in internal space.
Symmetries of Minkowski space

Translational invariance: four parameters
conserved energy momentum

Rotational invariance: six parameters
conserved spin-angular momentum

Total symmetry group: the **Poincaré group**  \( P(1, 3) = T^{1,3} \rtimes SO(1, 3) \)
After applying the gauge procedure, there are two gauge fields:

- the coframe \( \vartheta^\mu = e_i{}^\mu dx^i \), which is essentially the frame field
  translational invariance, four parameters
  field strength: torsion
  source for torsion: spin-angular momentum

- the Lorentz connection \( \Gamma_{\mu\nu} \), an additional gauge potential
  rotational invariance, six parameters
  field strength: curvature
  source for curvature: energy-momentum

These gauge potentials can be used to define a viable theory of gravity
(Einstein–Cartan theory in a spacetime with curvature and torsion).

We fall back to General Relativity for vanishing torsion.
Conclusions & Outlook

- Yes, it is possible to formulate gravity as a gauge theory.

- In Poincaré gauge theory, the frame field $e^j_\mu$ is the gauge potential of translations, and it is accompanied by the Lorentz connection $\Gamma_{\mu\nu}$ as the rotational potential.

- Gauge approach helpful for quantization?

- See also: Loop Quantum Gravity (but vanishing torsion)

Literature: