Singularities in Generalized Chaplygin Gas model

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Outline

Our Universe

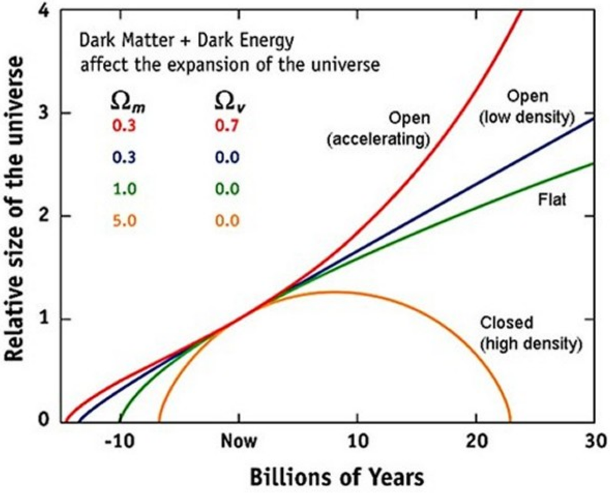
- Friedmann equations

$$\dot{H} = -\frac{\kappa^2}{2}(\rho + P) \tag{1}$$

$$H^2 = +\frac{\kappa^2}{3}(\rho) \tag{2}$$

They are the Einstein equations for a spatially flat, homogeneous, isotropic universe filled with an ideal fluid (Friedmann–Lemaître–Robertson–Walker (FLRW) model).

Where the $\kappa^2 = 8\pi G$, and $H = \frac{\dot{a}}{a}$ is the Hubble rate.
a is the scale factor ($\sim$ size of the universe).


Dark Matter + Dark Energy
affect the expansion of the universe
Ω_m
Ω_v
0.3 0.7
0.3 0.0
1.0 0.0
5.0 0.0
Open
(accelerating)
Open
(low density)
Flat
Closed
(high density)
Relative size of the universe
Billions of Years
0
1
2
3
4
-10
Now
10
20
30

Generalized Chaplygin Gas (GCG) model

– Density:

$$\rho = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} \tag{3}$$

– Equation of state:

$$P = -\frac{A}{\rho^{\alpha}} \tag{4}$$

GCG model: past is matter dominated, present/future is dark energy dominated

Singularities in GCG model

– A singularity happens when a variable blows up.
That leads to an unphysical situation.

– Depending of the choice of the constants in the equation of state, one or more energy conditions can be violated.
In that case one type of the singularities can occur.

– Energy conditions:

$$\rho \pm P \geq 0, \qquad \rho \geq 0, \qquad \rho + 3P \geq 0. \tag{5}$$

Classification of singularities predicted by GCG model

– Type I (Big rip)

$$\rho \to \infty, \quad |P| \to \infty, \quad a \to \infty \tag{6}$$

– Type II (Sudden)

$$\rho \to \rho_s, \quad |P| \to \infty, \quad a \to a_s \tag{7}$$

– Type III

$$\rho \to \infty, \quad P \to \infty, \quad a \to a_s \tag{8}$$

– Type IV

$$\rho \to 0, \quad P \to 0, \quad a \to a_s, \tag{9}$$

Divergence of higher derivative of a

FLRW universe with a scalar field

– The energy momentum tensor for a scalar field:

$$T_{00} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \mathrel{\hat{=}} \rho \tag{10}$$

$$T_{ii} = g_{ii}\left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right) \mathrel{\hat{=}} P \tag{11}$$

$\Rightarrow$ use the scalar field ϕ and its potential $V(\phi)$ to model the ρ and P of the GCG

– The Wheeler-DeWitt equation in terms of a, and ϕ:

$$\left[\frac{\hbar^2 G}{3\pi a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{4\pi^2 a^3}\frac{\partial^2}{\partial \phi^2} - \frac{3\pi\mathcal{K}}{4G}a + a^3\pi^2\left(2\mathcal{V}(\phi) + \frac{\Lambda}{4\pi G}\right)\right]\Psi(a,\phi) = 0 \tag{12}$$

Ψ can be understood as the wavefunction of the universe.

Type IV singularity

- Choice of variables:

$$A < 0, \quad B > 0, \quad -\frac{1}{2} < \alpha < 0, \quad \alpha \neq \frac{1}{2}(\frac{1}{n} - 1) \tag{13}$$

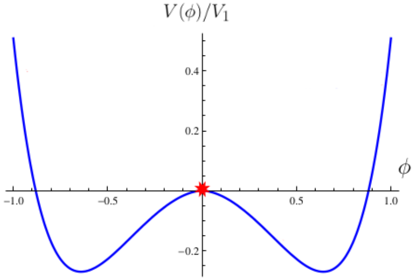
- The scalar field ϕ:

$$\dot{\phi}^2 = |A|^{\frac{1}{1+\alpha}} \frac{\left(\frac{a_{max}}{a}\right)^{3(1+\alpha)}}{\left[\left(\frac{a_{max}}{a}\right)^{3(1+\alpha)} - 1\right]^{\frac{\alpha}{1+\alpha}}} \tag{14}$$

- The scalar field potential:

$$V(\phi) = V_1 \left[\sinh^{\frac{2}{1+\alpha}} \left(\frac{\sqrt{3}}{2}\kappa|1+\alpha||\phi|\right) - \frac{1}{\sinh^{\frac{2\alpha}{1+\alpha}} \left(\frac{\sqrt{3}}{2}\kappa|1+\alpha||\phi|\right)}\right] \tag{15}$$

Shape of the potential in type IV



Ultimate goal

- Avoid such situations by two techniques:
  - Vanishing probability (wave function) at the singularity.
  - Tunneling through the singularity.

The latter case is considered for the type IV.

Summary

– We live in a flat universe described by FLRW model.

– The GCG fluid as a model can describe the past and the future of our universe.

– Depending on the choice of the equation of state, a singularity can occur at a certain time.

– The ultimate goal is to avoid this undefined physical situation.

– In case of the type IV singularity, the avoidance happens by tunneling of the wave function of the universe.

References

[1] M. Bouhmadi-Lopez, C. Kiefer and M. Kraemer, "Resolution of type IV singularities in quantum cosmology," arXiv:1312.5976 [gr-qc].

[2] C. Kiefer, "Quantum gravity," Int. Ser. Monogr. Phys. **124** (2004) 1 [Int. Ser. Monogr. Phys. **136** (2007) 1] [Int. Ser. Monogr. Phys. **155** (2012) 1].

[3] M. Bouhmadi-Lopez, P. F. Gonzalez-Diaz and P. Martin-Moruno, "On the generalised Chaplygin gas: Worse than a big rip or quieter than a sudden singularity?," Int. J. Mod. Phys. D **17** (2008) 2269 [arXiv:0707.2390 [gr-qc]].

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