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Light propagation in bianisotropic media: wave surfaces with 16 singular points

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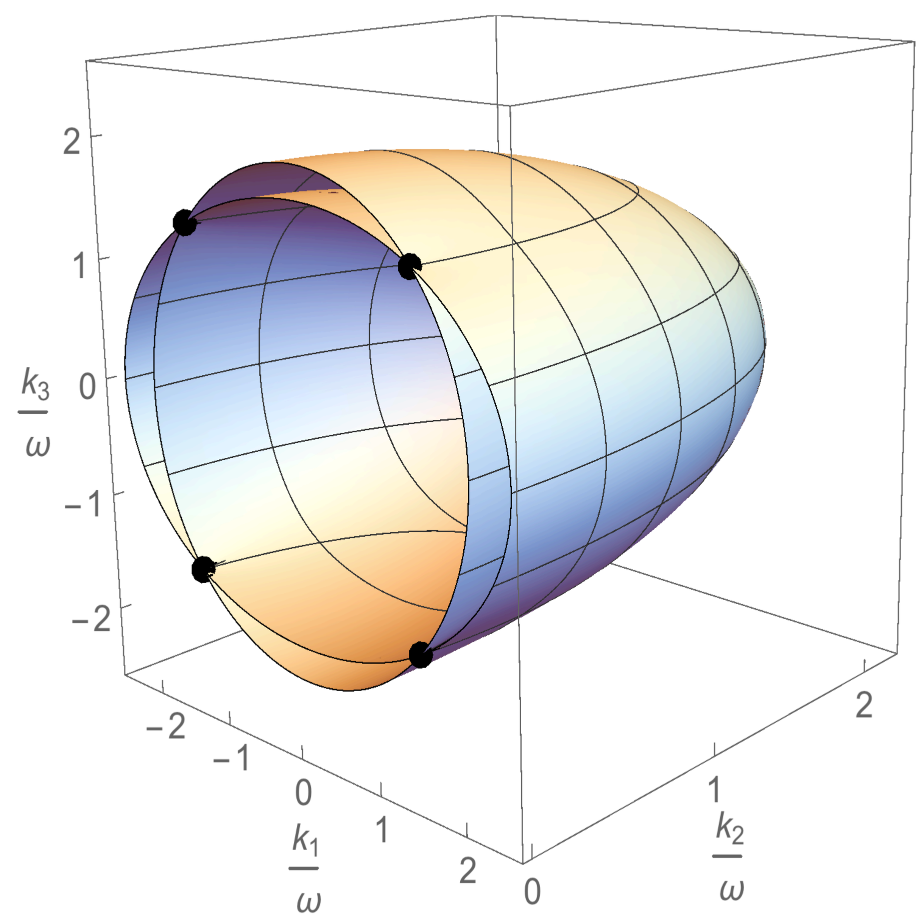
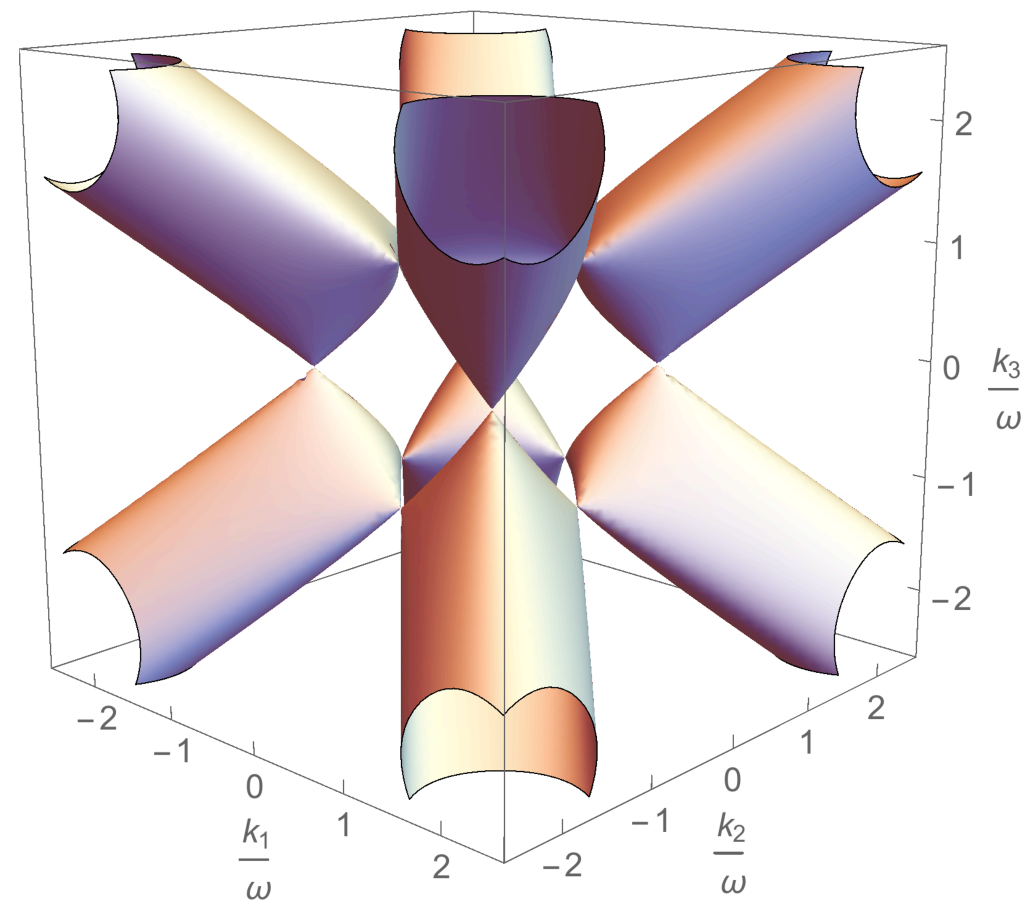
Biaxial media



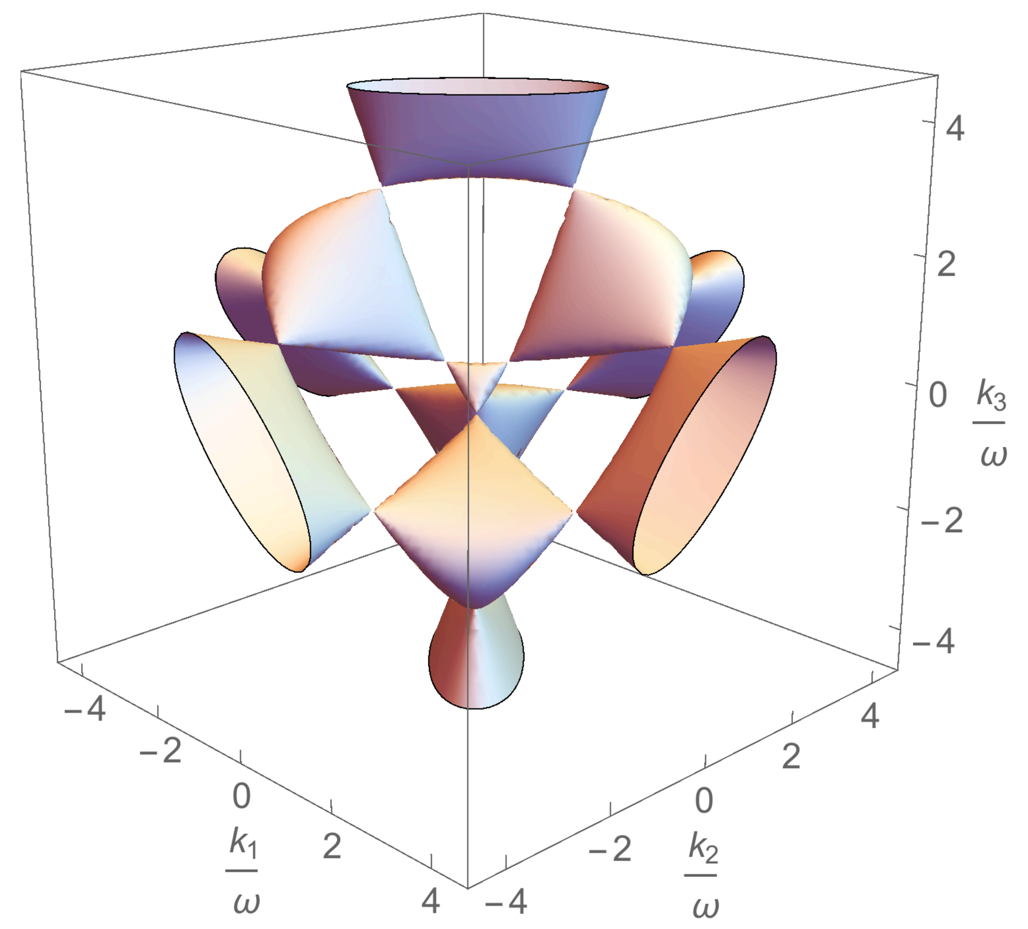
- The **Fresnel surface** determines the inverse phase velocity $\mathbf{k}/\omega$ of an electromagnetic wave propagating through a material in a given direction.
- Consider a biaxial medium whose permittivity and permeability tensors are
$$\underline{\underline{\varepsilon}} = \mathrm{diag}(3,4,6), \qquad \underline{\underline{\mu}} = \mathrm{diag}(1,1,1).$$
- Fresnel surface of this medium has **4 singular points**, located on two optical axes. At a boundary, the singularities can lead to internal **conical refraction**.
- Besides 4 singular points over real numbers, there are 12 singularities over the complex. Clarifying interpretation of the latter is a subject for future work.
- Question: **can the Fresnel surface of a linear medium exhibit more than 4 real singular points?**

16 real singular points: 4 at infinity

- A material is **magnetoelectric** if applied electric field induces a non-zero magnetization, and applied magnetic field induces a non-zero polarization. For non-dissipative linear media, this signifies
$$\mathbf{D} = \underline{\underline{\varepsilon}}\,\mathbf{E} + \underline{\underline{\alpha}}\,\mathbf{B},$$
$$\mathbf{H} = \underline{\underline{\mu}}^{-1}\mathbf{B} - \underline{\underline{\alpha}}^{T}\mathbf{E}.$$
- Select magnetoelectric, permittivity and permeability tensors according to:
$$\underline{\underline{\alpha}} = \mathrm{diag}\big(\sqrt{3}/2, -\sqrt{3}/2, 0\big), \qquad \underline{\underline{\varepsilon}} = \mathrm{diag}\big(1,1,-1/2\big), \qquad \underline{\underline{\mu}} = \mathrm{diag}\big(1,1,-2\big).$$
- Fresnel surface of this medium has **16 real singular points**. Nonetheless, 4 of these are located at infinity, and describe zero-frequency modes, $\omega = 0$.
- Question: **can the Fresnel surface of a linear medium exhibit 16 real singular points that are all finite?**



16 singular points: all finite



- Consider a magnetoelectric material as above. However, stipulate that
$$\underline{\underline{\alpha}} = \mathrm{diag}(3+\sqrt{3}, -3-\sqrt{3}, 0),$$
$$\underline{\underline{\varepsilon}} = \mathrm{diag}(-1-\sqrt{3}, -1-\sqrt{3}, -4+2\sqrt{3}), \qquad \underline{\underline{\mu}}^{-1} = \mathrm{diag}(1+\sqrt{3}, 1+\sqrt{3}, 4-2\sqrt{3}).$$
- The corresponding Fresnel surface has **16 real and finite singular points**.
- Question: **can the Fresnel surface of a linear medium exhibit more than 16 singularities over the real or the complex numbers?**
- Dispersion equations of linear materials are **quartic** homogeneous polynomial equations in the variables $(\omega, \mathbf{k})$ [F.W. Hehl and Y.N. Obukhov, "Foundations of Classical Electrodynamics", Birkhäuser, Boston, 2003]. Thereby, Fresnel surfaces **cannot** have more than 16 singular points.

Interesting facts…

- Fresnel surfaces of non-dissipative local and linear media are **Kummer surfaces**, whence the singularities give rise to a **(16,6)-configuration**. More explicitly, the singular points determine 16 planes, with each plane containing 6 points; in addition, the planes meet at the 16 singular points, with each point lying on 6 of the planes.
- **Metamaterial realization** of media with 16 singularities: appropriate combination of metal bars, split-ring resonators, and magnetized particles can generate correct permittivity, permeability, and magnetoelectric tensors.

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