Negative index of refraction, perfect lenses and transformation optics – some words of caution.

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August 18, 2010
Overview: ’Negative refractive index ≠ Folding of space’.

From: J.B. Pendry et al., PRL, 90:2, 2003

From: U. Leonhardt et al., arXiv:1007.0078v2, 2010
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▶ Review why negative index (left) is often compared to folding of space (right) – wrongly so.

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Overview: 'Negative refractive index $\neq$ Folding of space'.

- Review why negative index (left) is often compared to folding of space (right) – wrongly so.
- Use conventional transformation optics consistently $\Rightarrow$ 'negative index $\neq$ folding of space'.

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- Review why negative index (left) is often compared to folding of space (right) – wrongly so.
- Use conventional transformation optics consistently ⇒ 'negative index ≠ folding of space'.
- Folding gives no perfect lensing, as it introduces an extra source, rather than amplifying evanescent fields.

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- Review why negative index (left) is often compared to folding of space (right) – \underline{wrongly} so.
- Use \underline{conventional} transformation optics consistently $\Rightarrow$ ’negative index $\neq$ folding of space’.
- Folding gives no \underline{perfect} lensing, as it introduces an extra source, rather than amplifying evanescent fields.
- Other ways to get a negative index do work, but is it really \underline{worth} it?

From: J.B. Pendry et al.,
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Often negative index is (wrongly) linked to Folding. Why?

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Right concepts.
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Thank-you.
Often negative index is (wrongly) linked to Folding. Why?

1. Start with vacuum.
2. Perform the folding.
3. Replicate field.
4. Remove folding.
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Impression: a negative index slab in vacuum...
Usual Space Tr. Optics: a refresher using Pendry’s cloak.

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◊ **Vacuum**: Grid $(x, y)$.

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- **Vacuum**: Grid \( (x, y) \).
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- **Interpretation as a material**: Grid \((x, y)\).
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So, let’s fold space... but get no negative index!
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Useful things:

▶ 3 stages: Vacuum, Transformation and Interpretation.
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- 3 stages: Vacuum, Transformation and Interpretation.
- Coord. change: $\gamma' = \Lambda^T \cdot \gamma \cdot \Lambda$, for a Jacobian matrix $\Lambda$. 

\[ \epsilon = \epsilon_0 \quad \text{and} \quad \mu = \mu_0. \]
So, let’s fold space . . . but get no negative index!

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Stage 1: $\gamma^{ij}$

Diag(1, 1, 1)
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Stage 1: $\gamma^{ij}$  
Diag(1, 1, 1)  

Stage 2: $\gamma'^{i'j'}$  
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Diag(1, 1, 1)  
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- Using the master formula: $\epsilon^{ij} = \epsilon_0 \left[ \frac{\det(\bar{\gamma}^{ij})}{\det(\gamma^{ij})} \right]^{-\frac{1}{2}} \bar{\gamma}^{ij}$

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Diag(1, 1, 1)  Diag(1, 1, 1)  Diag(1, 1, 1)

- Using the master formula: $\epsilon^{ij} = \epsilon_0 \left[ \frac{\det(\bar{\gamma}^{ij})}{\det(\gamma^{ij})} \right]^{-\frac{1}{2}} \bar{\gamma}^{ij}$
- Immediately: $\epsilon = \epsilon_0$ and $\mu = \mu_0$.
- A folding transformation on vacuum does nothing!
Aside: Don’t believe my formulae? Look at this!

\[
\text{Under parity (} \vec{r} \rightarrow -\vec{r} \text{), given } \epsilon = \text{Diag}(\epsilon, \epsilon, \epsilon):
\]

- **Myself (element-wise):**
  \[
  \epsilon(-\vec{r}) \sim \epsilon(\vec{r})
  \]

- **Opponent (element-wise):**
  \[
  \epsilon(-\vec{r}) \sim -\epsilon(\vec{r})
  \]

Crucially, for a centro-symmetric medium:

- **Myself:**
  \[
  \epsilon(\vec{r}) \neq 0
  \]

- **Opponent:**
  \[
  \epsilon(\vec{r}) = 0
  \]

\[\text{Wrong}\]

\[\text{⋄ Simple, but true: E.J. Post, North Holland, 1962.}\]

\[\text{⋄ Cf. Cartan’s "twist": F.W. Hehl, Birkhäuser, 2003.}\]
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Under parity \((\vec{r} \rightarrow -\vec{r})\), given \(\varepsilon = \text{Diag}(\varepsilon, \varepsilon, \varepsilon)\):

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\begin{align*}
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’Folding’ argument gives no perfect lens (preamble).

Fold X-axis into a slab (allegedly, a perfect lens).
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The field at a point...
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- The field at a point... is replicated at all intersections.
- Spike of a point source is tripled. Perfect lens?
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- Fold X-axis into a slab (allegedly, a perfect lens).
- The field at a point... is replicated at all intersections.
- Spike of a point source is tripled. Perfect lens?
- Contrary common belief: the answer is NO...
'Folding’ argument gives no perfect lens!

Compare: 'Fold' lens (left) with 'Pendry' lens (right).
'Folding’ argument gives no perfect lens!

◊ Compare: 'Fold' lens (left) with 'Pendry' lens (right).
  • 'Fold' lens ⇒ Source+Sink+Source
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- The middle “active sink”?
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◊ Compare: ’Fold’ lens (left) with ’Pendry’ lens (right).
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  • ’Pendry’ lens \(\Rightarrow\) Amplify evanescent field.

◊ Similar result can be obtained with traditional tools:

◊ The middle “active sink”? A carefully phased source...
The fish-eye lens needs an active sink... Physical? Useful?

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- Based on active sink: Blaikie, NJP, 12, 2010.

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The fish-eye lens needs an active sink… Physical? Useful?

- Based on active sink: Blaikie, NJP, 12, 2010.
- Meep FDTD simulation: no sink, no perfection.

The simulation shown here comes from a collaboration with P. Kinsler.
The fish-eye lens needs an active sink... Physical? Useful?

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- Hotly debated: active sinks are useful? physical?

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Folding fails? Other ways to get a negative index

Fundamental (non transf-based):

\[ \chi_{\mu\nu\alpha\beta} = -\left( \mu_0 / \varepsilon_0 \right) - \frac{1}{2} \left( g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha} \right) \]

- Just see: part is affected by coord-change, part is not.
- Insert a minus sign where unaffected \(\Rightarrow\) Negative index!
- Fundamental minus: not due to a coordinate change. \(\Rightarrow\) Not all optics is transformations! (cf. later...)

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Conclusions.

- Negative index often thought as a folding of space.
- **But** with this approach:
  - Rigorously, $\epsilon < 0$ and $\mu < 0$ are **not** obtained.
  - Perfect lensing does **not** occur, rather...
  - Carelessness generates extra sources/sinks.
- So... **do not** argue in terms of 'folding'!
- Other transformations work: but no real advantage.
- Further information:
  - Luzi Bergamin and Alberto Favaro, arXiv:1001.4655
  - And, of course, the EMTS proceedings!
Thank-you!