Nonlocal Gravity Simulates Dark Matter

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- FWH, B. Mashhoon, *Formal framework for a nonlocal generalization of Einstein's theory of gravitation,* Phys. Rev. D **79**, 064028 (2009)
- H.-J. Blome et al., *Nonlocal modification of Newtonian gravity,* Phys. Rev. D **80**, in press (2010).

Contents

1. Postulate of locality and its limits

- In SR, generalize Poincaré transformations from inertial observers to accelerated observers. For a Newtonian point particle, the initial configuration of which is specified by position and velocity, this is obvious (point coincidences):
- Postulate of locality: An accelerated observer (measuring device) along its *worldline* is at each instant physically equivalent to a hypothetical inertial observer (measuring device) that is otherwise identical and instantaneously *comoving* with the accelerated observer (measuring device).
- Acceleration lengths for an Earth-bound laboratory for translational acceleration $\ell_{\text{tr}} := c^2/g_\oplus \approx 1$ light year and for rotational acceleration $\ell_{\text{rot}} := c^2/\Omega_\oplus \approx 28$ AU. Dimension of experiment λ; then locality postulate fulfilled if $\lambda/\ell \ll 1$. Usually fulfilled (for point coincidences); however, becomes problematic for (extended) waves.
- Decaying muon in storage ring: $a \sim \gamma \frac{v^2}{r} \sim 10^{22} g$; $\ell \sim 10^{-6}$ m:

$$\tau_\mu = \gamma \tau_\mu^0 \left[1 + \frac{2}{3} \left(\frac{\lambda_{\text{Comp}}}{\ell} \right)^2 \right], \qquad \text{corrections} \sim 10^{-16}, \text{ negligible.}$$

- Radiating electron: classical charged particle accelerated by external force $\mathbf{f}$; typical wave length of radiation λ, thus $\lambda \sim \ell$ or $\lambda/\ell \approx 1$, locality violated: Abraham-Lorentz equation

$$m\frac{d\mathbf{v}}{dt} - \frac{2}{3}\frac{e^2}{c^3}\underbrace{\frac{d^2\mathbf{v}}{dt^2}}_{\text{"jerk"}} + \cdots = \mathbf{f}; \qquad \mathbf{x} \text{ and } \mathbf{v} \text{ no longer sufficient.}$$

  Wave phenomena tend to violate the locality postulate (unless we consider the eikonal limit).

- Accelerated system and in-coming electromagnetic wave: Accelerated frame $\mathbf{e}^\alpha = e_i{}^\alpha dx^i$ (with i as coordinate and α as frame index, both $= 0, 1, 2, 3$) obeys

$$\frac{de_i{}^\alpha}{d\tau} = \Phi_\beta{}^\alpha e_i{}^\beta \quad \text{with acc. tensor } \Phi_{\alpha\beta} = (-\mathbf{g}, \boldsymbol{\Omega}) = -\Phi_{\beta\alpha}.$$

  Plane electromagnetic wave with wave vector $k^i = (\omega, \mathbf{k})$. Observer rotates with Ω_0 around the wave. Then Bahram Mashhoon's analysis yielded $\hat{\omega} = \gamma(\omega \mp \Omega_0) = \omega_{\text{Dop}}(1 \mp \Omega_0/\omega)$. For a wave with spin $\mathbf{s}$, one finds $\sim \mp\gamma \mathbf{s} \cdot \boldsymbol{\Omega}_0$. This spin-rotation coupling, which has been experimentally verified (GPS), is of a non-local origin. When waves are involved, we are beyond point coincidences and nonlocality sets in.

2. Nonlocal theory of special relativity

- Acceleration induces nonlocality into SR.
- Mashhoon (1993) proposed a nonlocal theory of accelerated systems of the type (memory effect)

$$\mathfrak{F}_{\text{accelerated}}(t) = F_{\text{inertial}}(t) + \int_0^t K(t,t')F_{\text{inertial}}(t')dt'$$

- In 4d, for electrodyn., we have for the excitation $H = (\mathcal{D}, \mathcal{H})$ and the field strength $F = (E, B)$ in comp. (fwh + Yuri Obukhov, Foundations of Classical Electrodynamics, Boston, 2003):

$$H_{\alpha\beta}(\tau,\xi) = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} \int d\tau' \underbrace{K_{\alpha\beta}{}^{\gamma\delta}(\tau,\tau')}_{\text{nonlocal kernel}} F_{\gamma\delta}(\tau',\xi)\,.$$

Mashhoon's nonlocal kernel turned out to be

$$K_{\alpha\beta}{}^{\gamma\delta}(\tau,\tau') = \frac{1}{2}\,\epsilon_{\alpha\beta}{}^{\mu[\delta}\left(\delta_\mu^{\gamma]}\,\delta(\tau-\tau') - u\rfloor\Gamma_\mu{}^{\gamma]}(\tau')\right).$$

Connection $\Gamma_\mu{}^\gamma$ with respect to the accelerating frame.

- Conventionally, SR plus *equivalence principle* (EP) $\longrightarrow$ GR. Can we apply the EP to the *nonlocal* SR? No, this does not seem to be possible, probably since the EP is a strictly local principle.
- How can we then generalize general relativity which is a strictly local theory? Idea: We know electrodynamics is a *gauge theory;* we can make it nonlocal, as shown on the last slide. Take gravity as a gauge theory of translations; generalize it to a nonlocal theory in a similar way as in electrodynamics.
- Poincaré gauge theories of gravity, see arXiv:gr-qc/9602013: Gauging translations $\longrightarrow$ Cartan's torsion; gauging Lorentz rotations $\longrightarrow$ curvature; Riemann-Cartan spacetime:

GR Curvature	Poincaré gauge theory Curvature + Torsion	$GR_{\|\|}$ Torsion
$\uparrow$	$\longleftarrow$ equivalent $\longrightarrow$	$\uparrow$

- The subcase of the translational gauge theory $GR_{||}$ is equivalent to GR.

3. Analogy between (local) vacuum electrodynamics and 'Einsteinian' teleparallelism

Vacuum electrodynamics	$\mathrm{GR}_{\vert\vert}$ (in $\Gamma_\alpha{}^\beta \overset{*}{=} 0$): transl. gauge theory
Minkowski spacetime	Weitzenböck spacetime (torsion, flat)
$U(1)$ one generator	T^4 four generators
charge conserv. $\Rightarrow \quad dJ = 0$	$d\mathcal{T}_\alpha = (e_\alpha \rfloor C^\beta) \wedge \mathcal{T}_\beta \quad \Leftarrow$ energy conserv.
$dH = J$	$\boxed{dH_\alpha - E_\alpha = \mathcal{T}_\alpha} \quad \Leftarrow$ field eq. of gravity
def. of $F \Rightarrow \quad f_\alpha = (e_\alpha \rfloor F) \wedge J$	$f_\alpha = (e_\alpha \rfloor C^\beta) \wedge \mathcal{T}_\beta \quad \Leftarrow$ def. of C^β
$dF = 0$	$dC^\alpha = 0$
F is irreducible $\quad F = dA$	$C^\alpha = de^\alpha \overset{\text{red.}}{\Rightarrow} C^\alpha = \overbrace{{}^{(1)}C^\alpha}^{\text{tensor}} + \overbrace{{}^{(2)}C^\alpha}^{\text{vector}} + \overbrace{{}^{(3)}C^\alpha}^{\text{ax. vec.}}$
$H = \sqrt{\dfrac{\varepsilon_0}{\mu_0}}\, {}^\star F$	$H_\alpha = \dfrac{1}{2\kappa} {}^\star \underbrace{\left(a_1 {}^{(1)}C^\alpha + a_2 {}^{(2)}C^\alpha + a_3 {}^{(3)}C^\alpha\right)}_{\mathfrak{C}^\alpha}$
	$a_1 = -1, \quad a_2 = 2, \quad a_3 = \dfrac{1}{2}$
$-\overset{\text{elmg}}{\mathcal{T}}{}_\alpha = e_\alpha \rfloor \overset{\text{elmg}}{V} + (e_\alpha \rfloor F) \wedge H$	$E_\alpha = e_\alpha \rfloor V + (e_\alpha \rfloor C^\beta) \wedge H_\beta$

4. Nonlocal gravitation generalizes GR

Introduce nonlocality into the framework of teleparallel gravity:

$$\mathcal{H}^{ab}{}_c(x) = \underbrace{\frac{1}{\kappa}\sqrt{-g(x)}\,[\mathfrak{C}^{ab}{}_c(x)}_{\text{this is equivalent to GR}} - \int \underbrace{\Omega^{ai}\Omega^{bj}\Omega_{ck}\,K(x,y)}_{\text{new nonlocal piece}}\,\mathfrak{C}_{ij}{}^k(y)\sqrt{-g(y)}\,d^4y]\,,$$

where $\Omega(x,y)$ is the world-function of Ruse-Synge (half the square of the geodesic distance connecting x and y); furthermore

$$\Omega_a(x,y) = \frac{\partial\Omega}{\partial x^a}\,,\quad \Omega_i(x,y) = \frac{\partial\Omega}{\partial y^i}\,;\quad \Omega \text{ satisfies}\quad 2\Omega = g^{ab}\Omega_a\Omega_b = g^{ij}\Omega_i\Omega_j\,,$$

and $\Omega_{ai}(x,y) = \Omega_{ia}(x,y)$ with $\lim_{y\to x}\Omega_{ai}(x,y) = -g_{ai}(x)\,.$

The causal scalar kernel $K(x,y)$ indicates the nonlocal deviation from GR. For $K(x,y) = 0$, we recover $\mathrm{GR}_{||}$ and thus, GR. $K(x,y)$ is in general a function of coordinate invariants, such as the quadratic torsion invariants (Weitzenböck)

$$\left({}^{(1)}C_{ij}{}^k\right)^2\,,\quad \left({}^{(2)}C_{ij}{}^k\right)^2\,,\quad \left({}^{(3)}C_{ij}{}^k\right)^2\,.$$

We chose the *simplest* nonlocal constitutive model involving a scalar kernel. The physical origin of this kernel will be discussed below.

5. Linear approximation

$$e_i{}^{\alpha} = \delta_i^{\alpha} + \psi^{\alpha}{}_i\,, \qquad |\psi^{\alpha}{}_i| \ll 1\,.$$

Indices become the same. Metric of the Weitzenböck spacetime:

$$g_{ij} = \eta_{ij} + h_{ij}\,, \qquad h_{ij} = 2\psi_{(ij)}$$

Gravitational field strength $C_{ij}{}^{k} = 2\psi^{k}{}_{[j,i]}$ and the modified one

$$\mathfrak{C}^{ij}{}_k = -\frac{1}{2}\left(h_k{}^{i}{}_{,}{}^{j} - h_k{}^{j}{}_{,}{}^{i}\right) + \psi^{[ij]}{}_{,k} + \delta_k^i\left(\psi_{,}{}^{j} - \psi_l{}^{j}{}_{,}{}^{l}\right) - \delta_k^j\left(\psi_{,}{}^{i} - \psi_l{}^{i}{}_{,}{}^{l}\right)\,,$$

where $\psi = \eta_{ij}\psi^{ij}$. Constitutive relation:

$$\kappa\mathcal{H}^{ij}{}_{\alpha}(x) = \underbrace{\mathfrak{C}^{ij}{}_{\alpha}(x)}_{\text{equiv. to GR}} + \int \underbrace{\mathcal{K}(x,y)}_{\text{new nonlocal piece}} \mathfrak{C}^{ij}{}_{\alpha}(y)d^4y\,,$$

$\mathcal{K}(x,y)$ is scalar kernel $K(x,y)$ evaluated in Mink. spacetime. Then,

$$\boxed{\partial_j\mathfrak{C}^{ij}{}_k + \int \frac{\partial\mathcal{K}(x,y)}{\partial x^j}\mathfrak{C}^{ij}{}_k(y)d^4y = \kappa\mathcal{T}_k{}^i\,.}$$

In linear approximation, $\partial_k\mathfrak{C}^{ik}{}_j = G^i{}_j$ (= linearized Einstein tensor $\doteq$ Fierz-Pauli for spin 2). $\partial_i\mathcal{T}_k{}^i = 0$ follows from lin. field eq. in box.

6. Reciprocal kernel

Assume that $\mathcal{K}(x, y)$ is a function of $x - y$; then

$$G_{ij}(x) + \int \mathcal{K}(x - y) G_{ij}(y) d^4y = \kappa \mathcal{T}_{ij}(x) \, ,$$

Fredholm integral equation of the 2nd kind. Solve formally by the *Liouville-Neumann* method of successive substitutions. Infinite series in terms of iterated kernels $\mathcal{K}_n(x, y)$, $n = 1, 2, 3, \ldots$:

$$\mathcal{K}_1(x, y) := \mathcal{K}(x, y) \, , \quad \mathcal{K}_{n+1}(x, y) := -\int \mathcal{K}(x, z) \mathcal{K}_n(z, y) d^4z \, .$$

If the resulting infinite series is uniformly convergent, we can define a *reciprocal kernel* $\mathcal{R}(x, y)$ given by

$$\mathcal{R}(x, y) := -\sum_{n=1}^{\infty} \mathcal{K}_n(x, y) \, .$$

Then the solution of the linearized field equation can be written as

$$G_{ij}(x) = \kappa \mathcal{T}_{ij}(x) + \kappa \int \mathcal{R}(x, y) \mathcal{T}_{ij}(y) d^4y \, .$$

7. Simulating dark matter

Assume that the constitutive kernel is of the form

$\mathcal{K}(x-y) = \delta(x^0-y^0)p(\mathbf{x}-\mathbf{y})$. Thus, $\mathcal{R}(x-y) = \delta(x^0-y^0)q(\mathbf{x}-\mathbf{y})$,

where p and q are reciprocal spatial kernels; the ansatz $\delta(x^0-y^0)$ should be sufficient for a non-rel. approx.; rewrite field eq. as

$$G_{ij} = \kappa\left(\mathcal{T}_{ij} + \Pi_{ij}\right) .$$

Interpret the new source for linearized GR as "dark matter". Π_{ij} is the symmetric energy-momentum tensor of "dark matter":

$$\Pi_{ij}(x) = \int \mathcal{R}(x,y)\mathcal{T}_{ij}(y)d^4y .$$

Dark matter is the integral transform of matter by the reciprocal kernel $\mathcal{R}(x,y)$. Thus, dark matter should be similar to actual matter. For example, dark matter associated with dust would be pressure-free, while Π_{ij} is traceless for radiation with $\mathcal{T}_k{}^k = 0$. For dust of density ρ, we find

$$\rho_{\mathrm{D}}(t,\mathbf{x}) = \int q(\mathbf{x}-\mathbf{y})\,\rho(t,\mathbf{y})d^3y ,$$

so that the density of dark matter ρ_{D} is in effect the convolution of ρ and q. In linear approximation, the kernel is *universal*.

8. Digression: Motion of stars in spir. galaxies

- Spiral galaxies, missing mass: Oort (1932), Zwicky (1933)...
- Structure of a spiral galaxy: bulge, disk, globular clusters.
  According to Kepler's third law (planets around the Sun):

$$\text{Kepler:} \qquad \frac{4\pi^2}{T^2} = \frac{GM}{r^3}\,, \quad v = \frac{2\pi r}{T} \quad \Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

- However, Rubin + Ford (1970) found flat rotation curves for stars of spiral galaxies: $v(r) \longrightarrow v_0 =$ constant, instead of $\sim 1/\sqrt{r}$
  Review: Sofue + Rubin, Ann. Rev. Astr. Ap. **39** (2001) 137
- If Newton-Einstein theory is OK:

$$M = M(r), \qquad M(r) \sim r \Rightarrow \text{ dark matter}$$

$$\text{From} \quad g = \frac{v_0^2}{r}\,, \quad \text{find} \quad \rho_{\text{Dark}} = -\frac{1}{4\pi G}\vec{\nabla}\cdot\vec{g} \;\Rightarrow\; \rho_{\text{Dark}} = \frac{v_0^2}{4\pi G}\frac{1}{r^2}\,.$$

$$\text{Hence} \quad M_{\text{Dark}} \underbrace{\sim}_{(\text{large } r)} \int \rho_{\text{Dark}} 4\pi r^2 dr = \frac{v_0^2}{G}\, r\,.$$

- Consider circular motion of star in a spiral. Outside the bulge, the Newtonian acceleration of gravity for each star at radius $|\mathbf{x}|$ is toward the galactic center with magnitude $v_0^2/|\mathbf{x}|$, where v_0 is the (approximately) constant speed of stars.
- We neglect the dimensions of the galactic bulge,

$$\rho(t, \mathbf{y}) = M\delta(\mathbf{y}) \,,$$

  where M is the effective mass of the galaxy. We derived on the last slide,

$$\rho_{\mathrm{D}}(t, \mathbf{x}) = \frac{v_0^2}{4\pi G} \frac{1}{|\mathbf{x}|^2} \,.$$

- Substitute both equations into

$$\rho_{\mathrm{D}}(t, \mathbf{x}) = \int q(\mathbf{x} - \mathbf{y}) \, \rho(t, \mathbf{y}) d^3 y \,,$$

$$\Rightarrow \qquad q(\mathbf{x}) = \frac{1}{4\pi\lambda} \frac{1}{|\mathbf{x}|^2} \,,$$

  where $\lambda := GM/v_0^2$ is a constant length parameter of order 5 kpc.

9. Modified Poisson equation

- With this reciprocal kernel $q(\mathbf{x}) = 1/(4\pi\lambda|\mathbf{x}|^2)$, the Newtonian limit of our linearized nonlocal theory is given by

$$\nabla^2\Phi = 4\pi G\left[\rho(t,\mathbf{x}) + \frac{1}{4\pi\lambda}\int\frac{\rho(t,\mathbf{y})d^3y}{|\mathbf{x}-\mathbf{y}|^2}\right]$$

  (Φ = Newtonian pot.).

- For a point mass m with $\rho(t,\mathbf{x}) = m\delta(\mathbf{x})$, we find

$$\Phi(t,\mathbf{x}) = Gm(\underbrace{-\frac{1}{|\mathbf{x}|}}_{\text{Newton}} + \underbrace{\frac{1}{\lambda}\ln\frac{|\mathbf{x}|}{\lambda}}_{\text{`dark matter'}}),$$

  the logarithmic dark matter term can be neglected in the solar system.

10. Recovering the Tohline-Kuhn system

- We recover the Tohline-Kuhn scheme that—apart from a disagreement with the empirical Tully-Fisher law $M \propto v_0^4$—has been quite successful in dealing with dark-matter issues in galaxies and clusters. However, the universality of our kernel $q(x)$ implies $M \propto v_0^2$; therefore, the Tully-Fisher relation is violated (Bekenstein, private communication). We recall:
- Joel Tohline (1983): $g = \frac{GM}{r^2}\left(1 + \frac{r}{\lambda}\right) = \frac{GM}{r^2} + \frac{v_0^2}{r}$

  In general: $\vec{g} = -\vec{\nabla}\Phi\,, \quad \Phi = -\frac{GM}{r} + \frac{GM}{\lambda}\ln(\frac{r}{\lambda})$

  $\lambda = \frac{GM}{v_0^2} =$ fixed constant $\sim$ 1 to 10 kpc (based on rotation curves of spiral galaxies)
- Jeffrey Kuhn et al. (1987)

  Modified Poisson's equation: $\nabla^2\Phi = 4\pi G\left[\rho + \frac{1}{4\pi\lambda}\int \frac{\rho(\vec{y})d^3y}{|\vec{x}-\vec{y}|^2}\right]$

  OK for spiral galaxies + cluster of galaxies

  Note: Integro-differential eq. relates Φ and ρ: nonlocal gravity
- Can this Tohline-Kuhn scheme be obtained from first principles?

11. Discussion

• The recovery of the Tohline-Kuhn scheme is a nontrivial feature of our theory.

• In our theory the gravitational potential is modified, but Newton's equation of motion (in contrast to MOND) is upheld.

• We should look for higher order approximations.

• Can the nonlocal kernel $q(x, y)$ be derived from first principles? One of us (BM) observed that the kernel fulfills $\nabla^2 q = 8\pi\lambda q^2$; this corresponds to the *semilinear wave equation* (Derrick 1964)

$$\Box\varphi = \varphi^2 \, .$$

• With H.-J. Blome and C. Chicone we studied the minute nonlocal modification of Newton's theory within the solar system; this paper has been accepted by PRD in the meantime. We find nonlinear as well as nonlocal modifications of Poisson's equation. We hope that with the new equation we can resolve the problem with Tully-Fisher.

• A comparison with Bekenstein's Tensor-Vector-Scalar theory (TeVeS) could be of value...